

HOMework PROBLEMS 3

ASTR 350

Photometric Concepts and Radiation

Due date: Saturday, 10/3/26 — 6pm

Submission protocol: Please submit a scanned PDF copy (single file) of your homework by email to Prof. Baring at *baring@rice.edu* by the deadline. Retain your original. **AI cannot be used in working this homework.**

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1. Carroll and Ostlie problem 3.1.

[18 points credit]

2. Carroll and Ostlie problem 5.8.

[24 points credit]

3. Consider stellar orbits around the Galactic Center, for which the mass of the central supermassive black hole M_{BH} has been inferred to around $2 \times 10^6 M_{\odot}$ from the dynamics of stars such as S2.

(a) If the Galactic Center is 8.5 kpc distant, approximately what angular resolution capability is required in a telescope in order to clearly resolve the orbit of a star like S2, which has a period of 15.2 years? Assume that the eccentricity is typically smaller than that for S2, which has $e = 0.87$.

(b) Suppose that geometrical information on orbits of stars proximate to the black hole is not available, i.e. cannot be resolved on the sky, but that spectroscopic information from several stars with high orbital inclinations $i \approx 90^\circ$ (to the sky plane) is.

The relativistic Doppler effect for the change of frequency of light emitted by a star is given by (see material in Chapter 4 of Carroll and Ostlie)

$$\frac{\nu_{obs}}{\nu_0} = \sqrt{\frac{c + v_{los}}{c - v_{los}}} \quad , \quad (1)$$

where ν_0 is the light frequency in the frame of reference of a particular star, and ν_{obs} is the corresponding frequency detected by an observer on Earth. The speed $v_{los} < c$ is the component of the velocity along the line of sight *towards* Earth: i.e. negative v_{los} corresponds to motion away from Earth. Also, c is the speed of light.

- Derive a mathematical formula that describes the apparent luminosity of a star, as measured by an observer on Earth, in terms of its intrinsic luminosity L_0 in its own frame of reference, and the speed v_{los} . Assume that the star emits a Planck spectrum.

(c) Use this to derive an expression for the *maximum* change $\Delta m_{\max}$ in apparent magnitude the star can exhibit during its orbit around the supermassive black hole, if its orbital period is P . Assume that the orbit is circular and with inclination $i = 90^\circ$ to the plane of the sky.

(d) What condition constraining P and M_{BH} must be satisfied for the ordinary, non-relativistic Doppler formula to apply in this orbital problem? Specialize to this case, and thereby derive the simpler formula relating $\Delta m_{\max}$ to P when the orbital speeds are non-relativistic. What is $\Delta m_{\max}$ for a star with $P = 10$ years and a black hole of mass $M_{\text{BH}} = 4.3 \times 10^6 M_\odot$?

(e) Describe how you can use this approach to ascertain an unknown black hole's mass just using photometry/magnitude variations in stars near the Galactic Center. Mention any problems/limitations you might identify with your approach.

[58 points credit]