

1.1 Photo-electric Effect

The photo-electric effect (PE effect) is when a photon is absorbed by an atom and expels an electron. It was the first confirmation of quantization of photon energies: explained by Einstein.

C & O,
Sec. 5.2

- * e^- assumes a continuous spectrum of energies from zero upwards;
- * The maximum kinetic energy $K_{e,\max}$ does not depend on the brightness of the light, only on its wavelength;
- * each material (metal) has a cutoff frequency ν_c (or $\lambda_c = c/\nu_c$) below which the PE effect ceases;
- Einstein proposed that light consisted of particles called **photons** with energy quantized according to

$$E_\gamma = h\nu = \frac{hc}{\lambda} \quad , \quad (1)$$

where h is Planck's fundamental constant.

- Each metal possesses a minimum **binding energy** ϕ (also called the **work function**), usually of the order of a few eV, so that

$$\nu_c = \frac{\phi}{h} \quad , \quad \lambda_c = \frac{hc}{\phi} \quad . \quad (2)$$

The maximum kinetic energy of the electron resulting from the PE effect is

$$K_{e,\max} = E_\gamma - \phi = \frac{hc}{\lambda} - \phi = h\nu - \phi \quad . \quad (3)$$

Obviously, $K_{e,\max} = 0 \Leftrightarrow \nu = \nu_c, \lambda = \lambda_c$.

- This linear relationship, and the associated cutoff, was the main reason for Einstein's Nobel Prize in 1921.

- Photo-electric absorption is maximized in the ultra-violet and soft X-ray band. It has to be corrected for when performing X-ray spectroscopy in compact objects such as AGNs and neutron stars. The cross section for the PE effect above the threshold frequency ν_c scales roughly as $(\nu/\nu_c)^{-7/2}$.

* The severity of the source spectral attenuation below around 5 keV is used to measure the integrated **column density** $n_e d$ along the line of sight to the source at distance d .

- Since the direction that an ejected electron emerges depends on the photon polarization (i.e. the orientation of its electric field vector), the photo-electric effect can be used as a tool for **X-ray polarimetry**.

Plot: Photo-electron Ejection Dependence on Light Polarization

1.2 Compton Scattering

A second watershed development in quantum theory was the identification by Compton in 1922 of the coupling between wavelength changes and the scattering angle θ in the collision between a photon and an electron.

**C & O,
Sec. 5.2**

* the analytic relationship Compton identified is a direct consequence of light quantization as photons: $E_\gamma = hc/\lambda$.

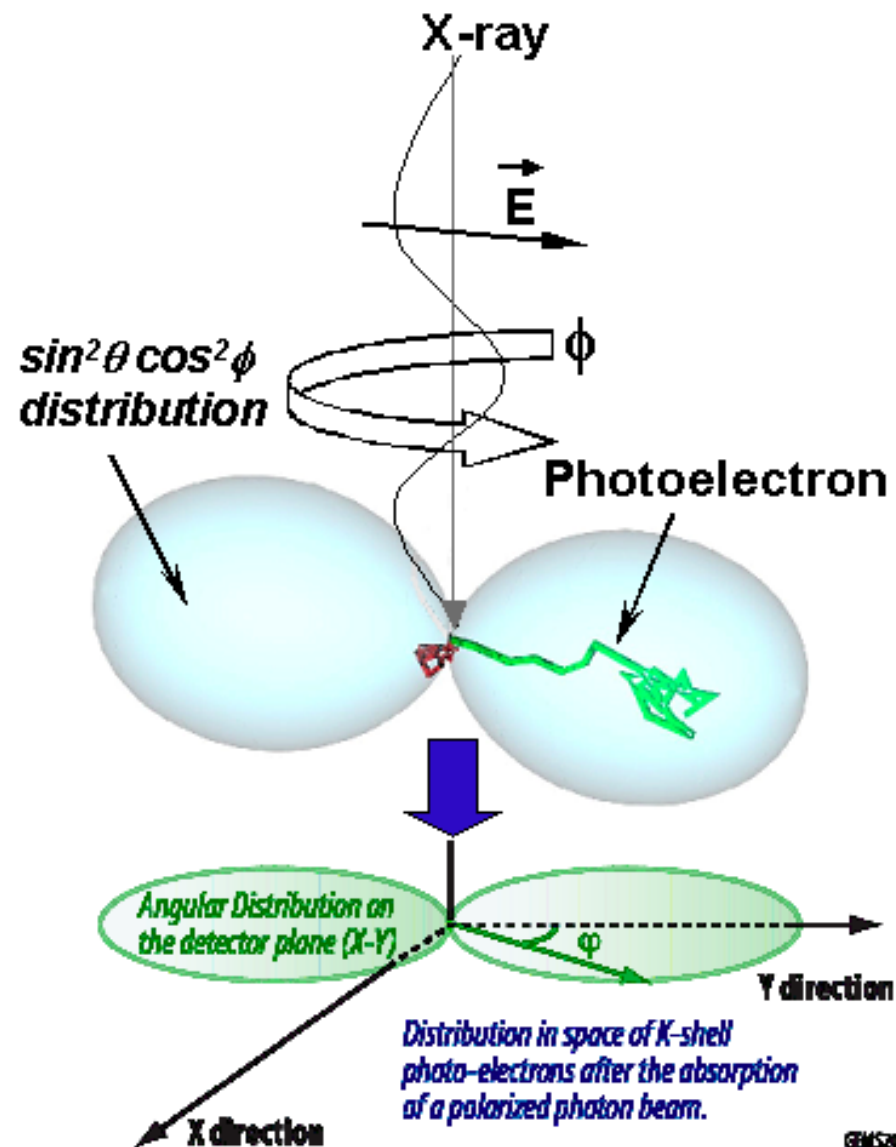
Plot: Compton scattering

- If $E_\gamma = hc/\lambda = h\nu = p_\gamma c$ collides with an electron at rest, then conservation of (relativistic) energy and momentum gives

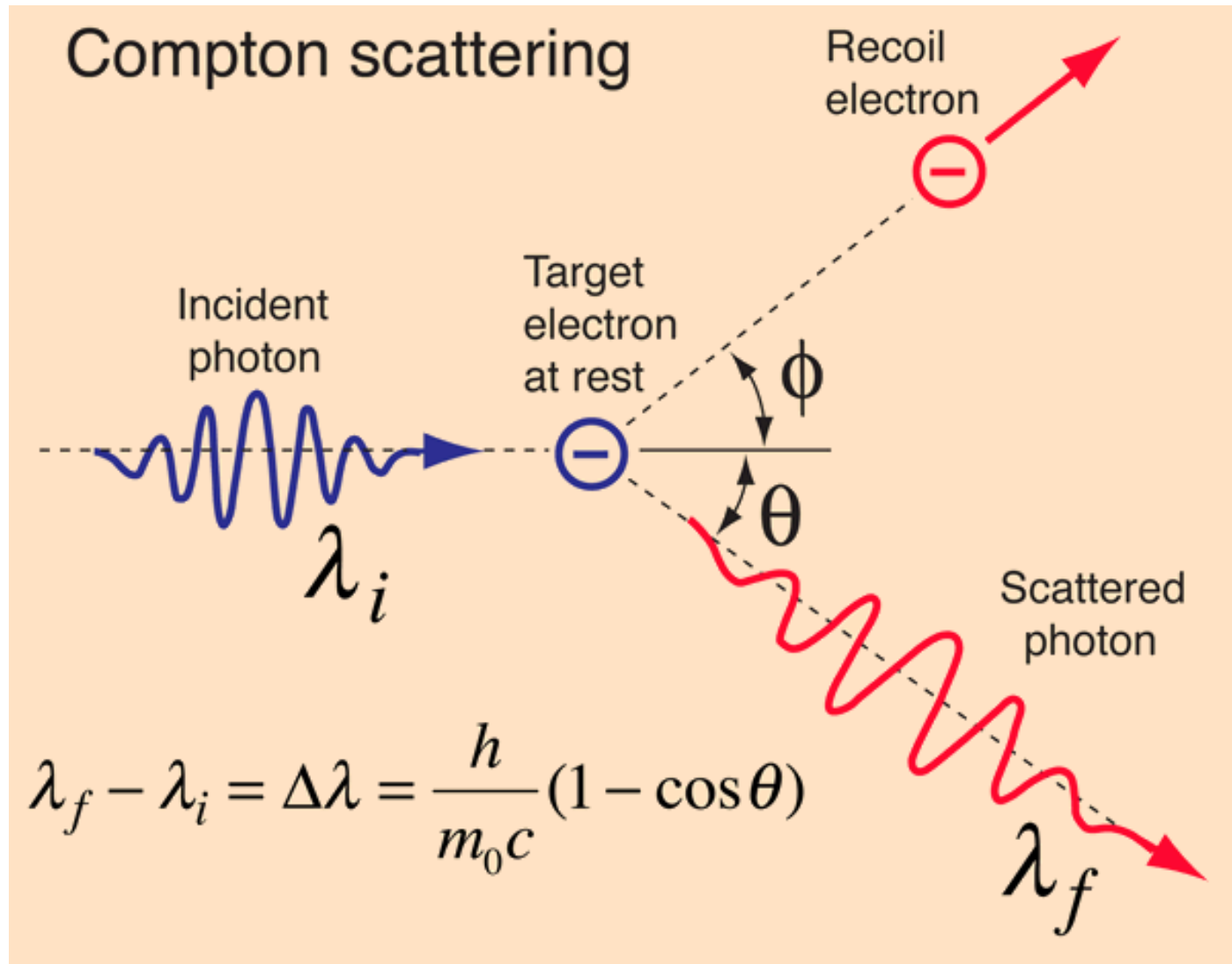
$$\Delta\lambda \equiv \lambda_f - \lambda_i = \frac{h}{m_e c} (1 - \cos \theta) \quad . \quad (4)$$

- This change in wavelength is a purely quantum phenomenon, known as the **Compton effect**. Its discovery provided a major impetus for the development of quantum theory.

Polarization in the Photoelectric Effect



Electron-photon Compton Scattering



- Here, $h/(m_e c) = 2.426 \times 10^{-10} \text{ cm}$ is known as the **Compton wavelength** of the electron; it represents the quantum spatial scale of the electron.

- * if we send $h \rightarrow 0$, so that quantum effects disappear, classical **Thomson scattering** still arises. This is realized in the long wavelength limit.

- The essentials of the Thomson process can be described using **Larmor formalism** for the radiation of accelerating charges. The power P per unit solid angle (averaged over all photon polarizations) is given by

$$\frac{dP}{d\Omega} = \frac{q^2}{8\pi c^3} (\dot{\mathbf{v}} \cdot \dot{\mathbf{v}}) \left\{ 1 + \cos^2 \theta \right\} \Rightarrow P = \frac{2}{3} \frac{q^2 |\dot{\mathbf{v}}|^2}{c^3} . \quad (5)$$

Here θ is the scattering angle of the photon. This differential power must be divided by the **Poynting flux** $S = \{|\mathbf{E}|^2 + |\mathbf{B}|^2\}c/(8\pi) \equiv |\mathbf{E}|^2 c/(4\pi)$ (light energy passing through unit area per unit time) to yield the **differential cross section** for scattering:

$$\frac{d\sigma}{d\Omega} = \frac{1}{S} \frac{dP}{d\Omega} = \frac{4\pi}{|\mathbf{E}|^2 c} \frac{dP}{d\Omega} . \quad (6)$$

The mean electric field of the wave scales the electron acceleration through the Lorentz force, so that $m |\dot{\mathbf{v}}| = q|\mathbf{E}|$. One can then arrive at the **Thomson scattering cross section**. For unpolarized scattering, per unit solid angle, one has (for $q \rightarrow -e$)

$$\frac{d\sigma_T}{d\Omega} = \frac{r_0^2}{2} \left\{ 1 + \cos^2 \theta \right\} , \quad r_0 = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13} \text{ cm} . \quad (7)$$

Here r_0 is the **classical electron radius**, and gives a measure of the “size” of the point charge in classical electromagnetic theory. Integrating over θ ,

$$\sigma_T = \frac{8\pi}{3} r_0^2 = \frac{8\pi}{3} \alpha_f^2 \left(\frac{\hbar}{m_e c} \right)^2 . \quad (8)$$

Here $\alpha_f = e^2/\hbar c \approx 1/137.08$ is the **fine structure constant**, the fundamental coupling for interactions in **quantum electrodynamics** (QED).

- For $\lambda_i \lesssim h/(m_e c)$, extreme changes in the wavelength occur. This corresponds to photon energies $E_\gamma = hc/\lambda \sim m_e c^2$, and electron recoil becomes significant in the so-called **Klein-Nishina limit**.

1.3 Cyclotron and Synchrotron Radiation

- Cyclotron motion in a uniform magnetic field is described by the Lorentz force, which for a four-momentum $p^\mu \equiv (E, \mathbf{p}) = (\gamma mc^2, \gamma m \mathbf{v})$ yields space and time contributions of

$$\frac{d}{dt}(\gamma m \mathbf{v}) = \frac{q}{c} \mathbf{v} \times \mathbf{B} \quad , \quad \frac{d}{dt}(\gamma mc^2) = q \mathbf{v}_\perp \cdot \mathbf{B} = 0 \quad . \quad (9)$$

No work is done on the charge, so γ remains constant in time, neglecting radiation reaction. The equation of motion can then be separated to isolate the pieces for components of velocity parallel ($\mathbf{v}_\parallel \propto \mathbf{B}$) and perpendicular ($\mathbf{v}_\perp$) to the field:

$$\frac{d\mathbf{v}_\parallel}{dt} = 0 \quad , \quad \frac{d\mathbf{v}_\perp}{dt} = \frac{q}{\gamma mc} \mathbf{v}_\perp \times \mathbf{B} \quad . \quad (10)$$

The first of these is most easily deduced by writing $\mathbf{v} = \mathbf{v}_\parallel + \mathbf{v}_\perp$, and expanding the cross product, noting the trivial result $\mathbf{v}_\parallel \times \mathbf{B} = \mathbf{0}$.

- It follows that $\mathbf{v}_\parallel$ is constant, and therefore so also is $|\mathbf{v}_\perp|$, and the motion follows a helical path. The gyration is at the **cyclotron frequency**:

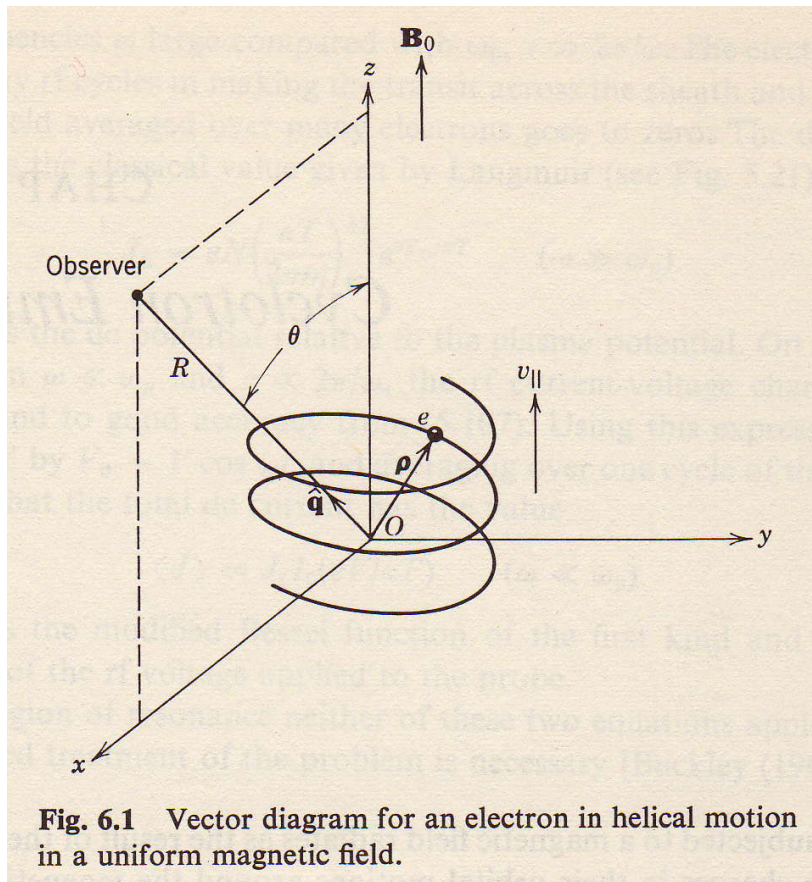
$$\omega_B = \frac{qB}{\gamma mc} \quad . \quad (11)$$

For a truly non-relativistic charge, all the radiation emerges at the cyclotron frequency. Yet, harmonics appear at the multiples $n\omega_B$, for integers $n = 2, 3, \dots$, and these are suppressed in their power to the order of $(v/c)^{2n}$.

Plot: Cyclotron Motion and Radiation

- For warm electrons, if $kT/m_e c^2 \sim v^2/c^2 \ll 1$, then the cyclotron fundamental is dominant, yet it is **Dopper broadened**. Eq. (11) would suggest that thermal motions might just redshift the fundamental at the $O(v^2/c^2)$ level, since $1/\gamma \approx 1 - v^2/(2c^2)$. Yet there is also blueshift+redshift in the $\mathbf{v}_\parallel$ component for an isotropic thermal gas, and this is of $O(v/c)$; this is what is dominant and defines the line broadening, with $\delta\nu/\nu \sim \sqrt{kT/m_e c^2}$.

Cyclotron and Synchrotron Radiation



- From Bekefi (1966): “*Radiation Processes in Plasmas*”

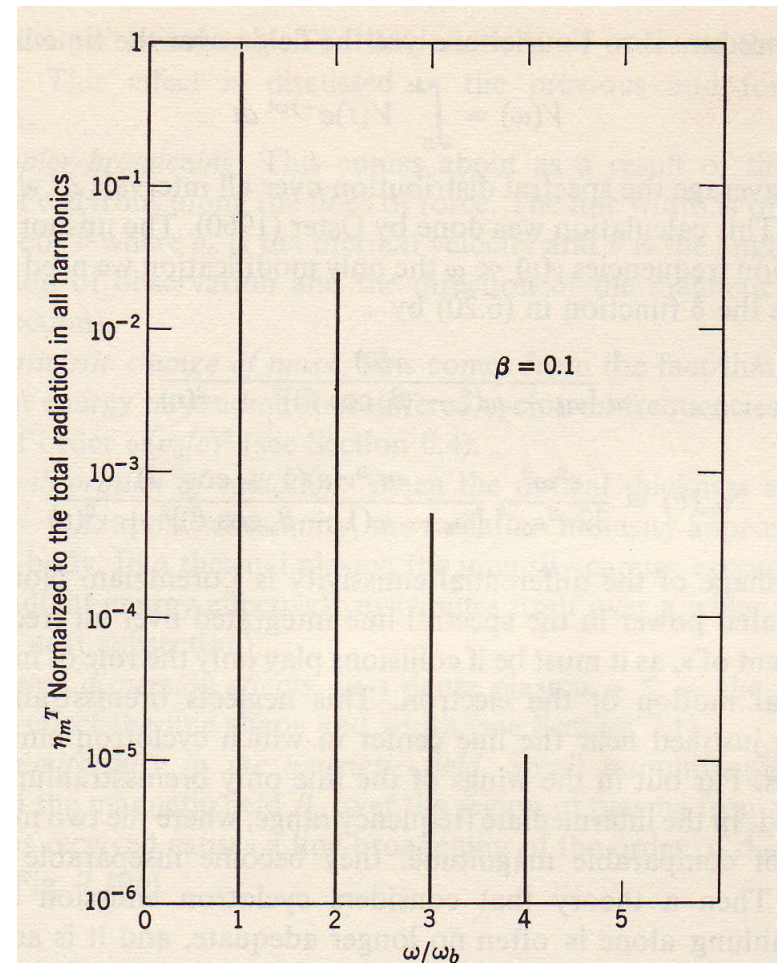


Fig. 6.2 Sketch of the spectrum of cyclotron radiation by a nearly nonrelativistic electron ($\beta_{||} = 0$).

Cyclotron radiation is elliptically polarized in general, but is circularly polarized when viewing perpendicular to the plane of gyration, i.e. along $\mathbf{B}$.

- Total radiated power for acceleration is given by the Larmor formula in Eq. (5), but adapted for this problem, i.e., using $|\dot{\mathbf{v}}| = |\dot{\mathbf{v}}_{\perp}| = qB/\gamma m$:

$$P \equiv \frac{dE}{dt} = \frac{2}{3} \frac{q^2}{c^3} \gamma^4 (\dot{\mathbf{v}} \cdot \dot{\mathbf{v}}) = \frac{2}{3} \frac{q^4}{m^2 c^3} \gamma^2 B^2 \quad . \quad (12)$$

The extra factor of γ^4 is needed to describe the Doppler boosting of the radiated power (luminosity). Where does it come from? There are three contributions:

- the light frequency (and energy E) is blueshifted by γ ,
- the emitting time interval is shrunk by $1/\gamma$,
- the radiation is collimated (**beamed**) into a solid angle $\propto 1/\gamma^2$.

Clearly, for large enough γ , the radiation must rapidly cool the electrons and modify the helical motion: this we call **radiation reaction**.

- When the electron's motion is ultra-relativistic, with $\gamma \gg 1$, many cyclotron harmonics contribute, and the Doppler broadening is so substantial that the harmonics overlap. The result is a continuum spectrum for the process of **synchrotron radiation**.

Plot:

 Synchrotron Radiation

- Synchrotron radiation is the principle emission mechanism from optically thin, magnetized plasmas with non-thermal electrons present, likely accelerated by astrophysical shocks or magnetic reconnection. It is seen in radio waves, optical, X rays and gamma rays. Manifestations include solar coronal field loops, supernova remnants, and relativistic jets near black holes.

* Often, these settings involve power-law distributions of electrons, and so the radiation evinces approximately a non-thermal power-law spectrum.

* Synchrotron radiation is highly polarized in general, though tangling of plasma magnetic fields can depolarize it significantly.

Synchrotron Beaming and Radio Sources

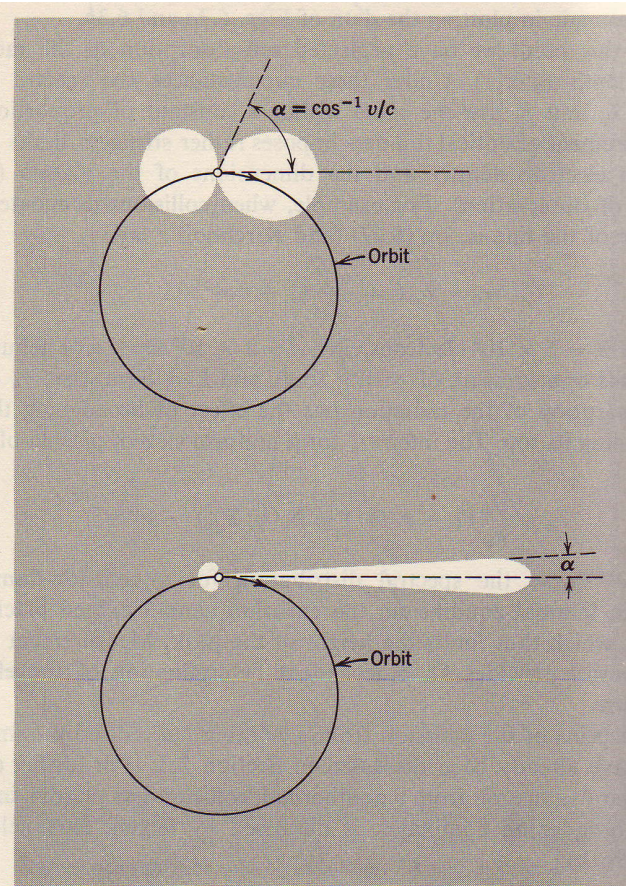


Fig. 6.5 A sketch of the polar diagram of the instantaneous radiation intensity of a nonrelativistic electron (above) and a highly relativistic electron (below). In the nonrelativistic case ($v/c \rightarrow 0$) the angular distribution is that of a dipole (3.11); the polar plot shown in the figure is a cross section of a doughnut-shaped object. In the highly relativistic case the polar plot shown is a cross section of a searchlight beam; the emission is predominantly along the instantaneous velocity vector of the charge. It is assumed that $\beta_{\parallel} = 0$. When $\beta_{\parallel} \neq 0$ the cone of radiation is again directed along the resultant velocity vector of the charge. [After Balwanz (1959).]

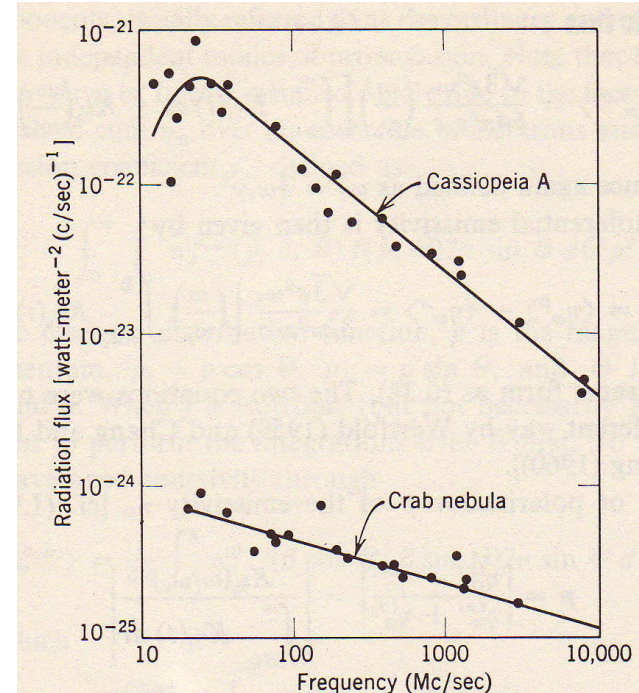


Fig. 6.8 Radiation flux from two astronomical objects. The experimental points represent observations of many workers in different observatories. [After Shklovsky (1960).]

- Synchrotron radiation is highly beamed and **highly polarized (linear)**;
- High efficiency makes it prime emission mechanism in many sources (e.g. pulsars, Crab PWN, Cassiopeia A SNR)