

3 Blackbody Radiation

Blackbody radiation is the principal continuum spectrum that is inherently thermal in nature, an idealized form that is never a perfect reality.

- A blackbody absorbs all light incident upon it and re-radiates it with a characteristic spectrum that is determined by quantum mechanics.

3.1 Wien and Stefan-Boltzmann Laws

- The blackbody spectrum peaks at a wavelength $\lambda_{\max}$ that is given by **Wien's displacement law**:

**C & O,
Sec. 3.4**

$$\lambda_{\max} T = 0.290 \text{ cm } K \quad . \quad (13)$$

e.g. for $T_{\odot} = 5.77 \times 10^3$ as the surface solar temperature, Wien's law gives $\lambda_{\max} = 0.290/5.77 \times 10^3 \equiv 5030 \text{ \AA}$ (Angstroms).

- * Hence, hot stars are blue, and cool stars are red.

The total luminosity of a blackbody was shown by Stefan in 1879 to obey

$$\boxed{L = A\sigma T^4} \quad , \quad (14)$$

for A being the surface area of the blackbody. Eq. (14) is known as the **Stefan-Boltzmann Law**, and

$$\sigma = 5.67 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} K^{-4} \quad (15)$$

is the Stefan-Boltzmann constant.

- Boltzmann explained Stefan's experimental determination in terms of thermodynamics and electromagnetism.

e.g. For a sphere like the sun, with a radius $R_{\odot} = 6.96 \times 10^{10} \text{ cm}$,

$$L_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4 = 3.826 \times 10^{33} \text{ erg/sec} \Rightarrow T_{\odot} = 5770 K \quad (16)$$

is the effective temperature of the solar surface.

3.2 Planck Spectrum

The actual shape of the spectrum was explained by Planck, who found an empirical formula for the **intensity**

**C & O,
Sec. 3.5**

$$B_\lambda(T) = \frac{a}{\lambda^5 (e^{b/\lambda T} - 1)} \quad , \quad (18)$$

where $B_\lambda(T) d\lambda dA_\perp d\Omega$ is the radiation energy per unit area ($dA_\perp = dA \cos \theta$), per unit wavelength $d\lambda$, per unit solid angle $d\Omega$.

* The units of $B_\lambda(T)$ are $\text{erg cm}^{-3} \text{ sec}^{-1} \text{ sr}^{-1}$, or often, to avoid unit confusion, $\text{erg cm}^{-2} \text{ \AA}^{-1} \text{ sec}^{-1} \text{ sr}^{-1}$.

Plot: B_λ solid angle geometry

- Consider standing waves or *harmonic modes* in a cavity of length L . Planck theorized such a gedanken experiment, where waves that do not interfere with themselves destructively must possess wavelengths

$$\lambda = \frac{2L}{n} \quad , \quad n = \text{integer}. \quad (19)$$

Classical thermodynamics could yield an infinite amount of energy for standing waves, due to high-frequency light (the so-called *UV problem*).

- Planck proposed that the waves could only have discrete, integer multiples of a minimum wave energy. The quantum of minimum energy was

$$h\nu = \frac{hc}{\lambda_i} \quad \Rightarrow \quad \varepsilon_\gamma = \frac{2\pi n_i \hbar c}{\lambda_i} \quad . \quad (20)$$

These can be created or destroyed *ad libitum*, corresponding to **Bose-Einstein** statistics. Hence, expressing the number of states n_i in terms of wavenumbers $k_i = 2\pi/\lambda_i = 2\pi\nu/c$, we can write down the change in the **number of states** in 3D for a wave vector element $d^3k = dk_x dk_y dk_z$ as

$$dN \equiv 2 dn_x dn_y dn_z = 2 \frac{L_x L_y L_z}{(2\pi)^3} d^3k \rightarrow \frac{8\pi V}{c^3} \nu^2 d\nu \quad . \quad (21)$$

The factor of 2 accounts for the two independent polarization states.

The Specific Intensity

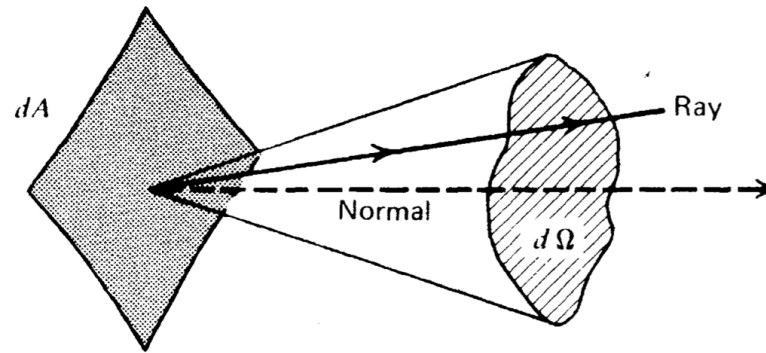


Figure 1.2 Geometry for normally incident rays.

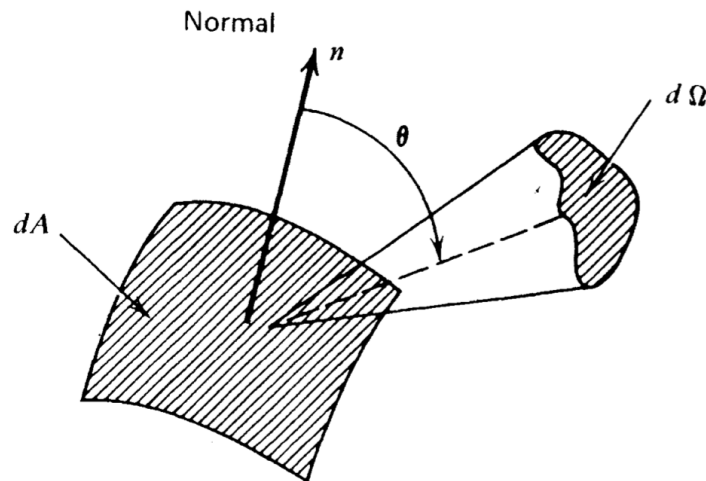


Figure 1.3 Geometry for obliquely incident rays.

This must be combined with thermal statistics of a Bose gas: occupation numbers are exponential Boltzmann factors, and are summed over all states:

$$\sum_{n=1}^{\infty} e^{-\varepsilon_n/kT} \equiv \sum_{n=1}^{\infty} e^{-nhc/\lambda kT} \quad . \quad (22)$$

These ingredients led Planck to the semi-quantum formula:

$$B_{\lambda}(T) = \frac{2hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)} \quad . \quad (23)$$

This is the famous **Planck spectrum** of blackbody radiation. The value of **Planck's constant** h was determined to be

$$h = 6.626 \times 10^{-27} \text{erg sec} \quad . \quad (24)$$

The quantum theory of light was ultimately established by Einstein's analysis of the photoelectric effect. In frequency space, the Planck spectrum is

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2(e^{h\nu/kT} - 1)} \quad . \quad (25)$$

Plot: Blackbody spectrum in ν space

Planck's quantization of light energies fixed the UV problem because it “suppressed” high frequency modes via the Boltzmann factors $e^{-h\nu/kT}$ by ascribing quantum energies $h\nu$ to photons.

- The **Wien limit** is obtained for $h\nu \gg kT$, for which

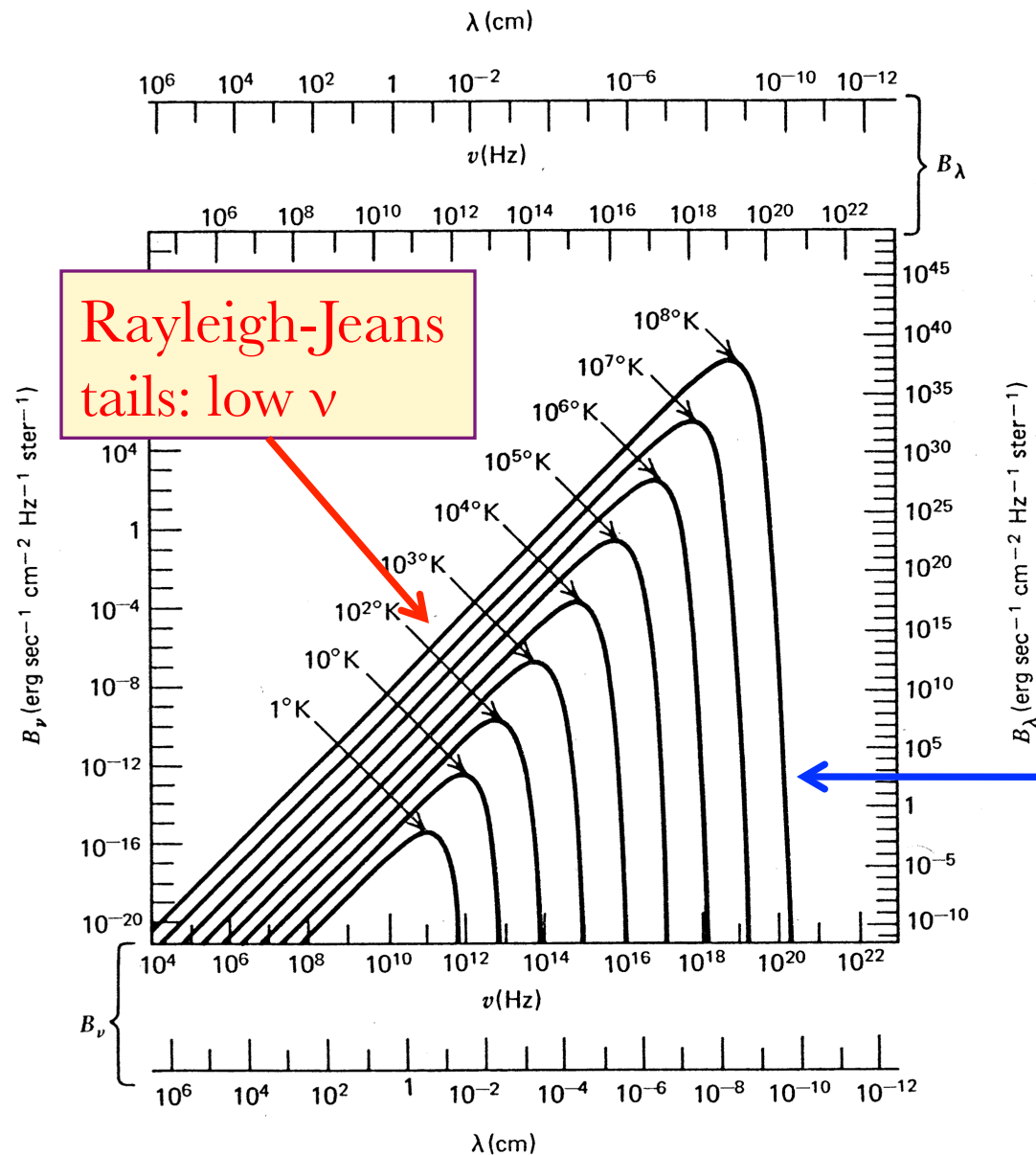
$$B_{\nu}(T) \approx \frac{2h\nu^3}{c^2} \exp\left\{-\frac{h\nu}{kT}\right\} \quad . \quad (26)$$

In the long wavelength **Rayleigh-Jeans** limit, $h\nu \ll kT$, we have

$$B_{\nu}(T) \approx \frac{2\nu^2 kT}{c^2} \quad . \quad (27)$$

This was first derived by Rayleigh in 1900 using classical physics.

Planck Spectra: ν Representation



- Spectra of **blackbody radiation** at various temperatures. Taken from the book by [Kraus, J. D. *Radio Astronomy*](#) (1966, McGraw-Hill Book Company)

- What is the peak of $B_\lambda(T)$? Setting $\chi = hc/(\lambda kT)$, we have

$$\frac{d \log B_\lambda(T)}{d\lambda} = 0 \quad \Rightarrow \quad \frac{d}{d\chi} \left\{ \frac{\chi^5}{e^\chi - 1} \right\} = 0 \quad \Rightarrow \quad \chi = 4.965 \quad . \quad (28)$$

The Wien displacement follows:

$$\lambda_{\max} = \frac{0.2898}{T(^{\circ}K)} \text{ cm} \quad . \quad (29)$$

- The Planck spectrum integrates to yield the total flux:

$$\mathcal{F} = \pi \int_0^\infty B_\nu(T) d\nu = \frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1} = \frac{2\pi(kT)^4}{c^2 h^3} \int_0^\infty \frac{\chi^3 d\chi}{e^\chi - 1} \quad (30)$$

Note that the factor of π represents the flux-weighting factor $\cos \theta$ integrated over a hemisphere, giving $2\pi \times 1/2$.

The integral evaluates to $\pi^4/15$, so that the Stefan-Boltzmann law is reproduced, with σ expressed in terms of fundamental constants:

$$\mathcal{F} = \sigma T^4, \quad \sigma = \frac{2\pi^5 k^4}{15c^2 h^3} \equiv \frac{\pi^2 k^4}{60c^2 h^3} = 5.67 \times 10^{-5} \text{ erg sec}^{-1} \text{ cm}^{-2} \text{ K}^{-4} \quad . \quad (31)$$

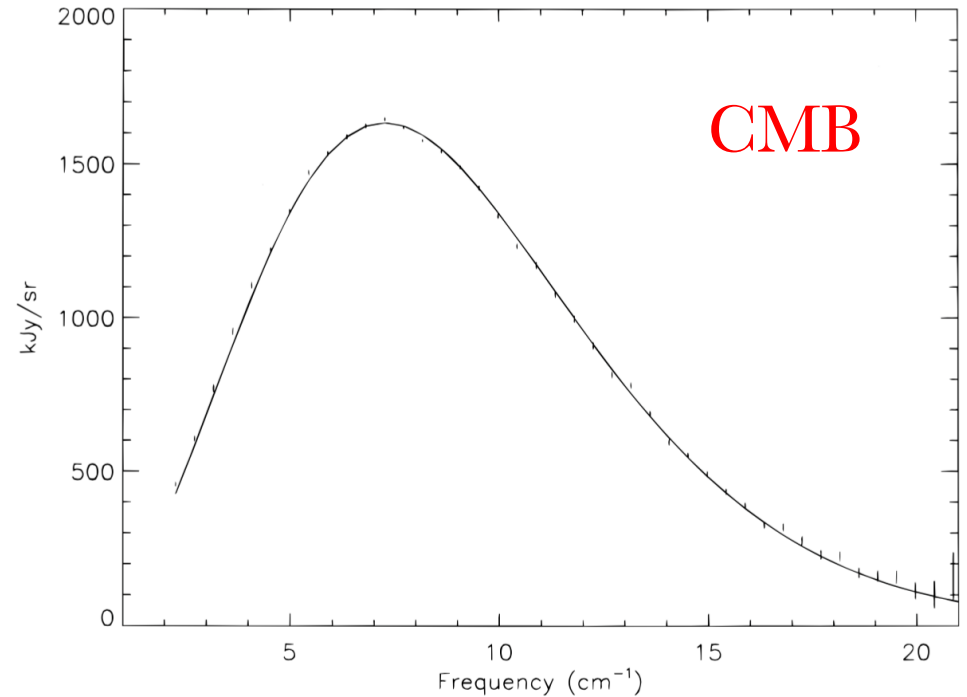
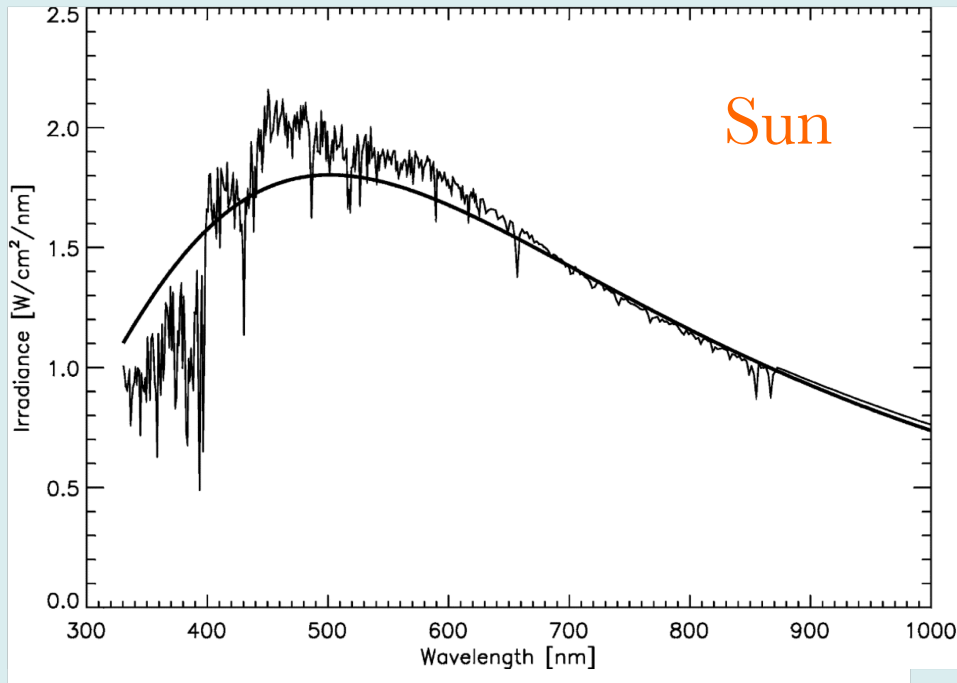
The number density of Planck photons can be obtained by dividing through by $h\nu$ before the integration over frequencies.

- The determination of stellar spectra leads to an evaluation of the *effective temperature* T_e of a star. Since stellar spectra are not true blackbodies, T_e is that needed to produce L_* , and will differ slightly from the true temperatures in stellar photospheres, as will become apparent in due course.

Plot: Solar spectrum and CMB blackbody

Thus, the Stefan-Boltzmann law $L_* = 4\pi R_*^2 \sigma T_e^4$ can be employed to determine stellar radii, implying that hotter and bigger stars are more luminous.

Spectra of Sun and Microwave Background



- *Left:* the solar spectrum in visible and IR (300-1000nm) as measured by Neckel & Labs (1984, Sol. Phys. 90, 205). The fitted solid curve is a blackbody at 5770° K, the effective temperature of the photosphere.
- *Right:* dB_J/dT spectrum of the cosmic microwave background (CMB) from COBE, with $T=2.728 \pm 0.004^\circ$ K Planck fit and 1 σ error bars. From Dixsen et al. (1996, ApJ 473, 576).

5. RADIATION PROCESSES AND THE QUANTUM ATOM

Matthew Baring – Lecture Notes for ASTR 350, Fall 2026

1 Continuum Mechanisms

In astrophysics, there are other sorts of continuum radiation spectra, some of which originate from thermal electrons, yet all of which do not correspond to thermal equilibrium between photons and particles. Examples are:

**C & O,
pp. 245-7**

- * free-bound and bound-free (photoelectric effect) transitions;
- * Thomson/Compton scattering of photons by electrons;
- * synchrotron radiation in magnetic fields;
- * free-free (bremsstrahlung) radiation.

Note that the last three of these arise from accelerating charges; and therefore are predicted by classical electromagnetism. Yet they do have quantum modifications that are mostly needed for compact object studies.

- We now consider each of these in some detail.