

The ion tail is blue in color because CO^+ ions are both abundant and specifically absorb light and re-radiate near $4200 \text{ \AA}$.

Plot: Table of Cometary Constituents

- Many comets are of periods much longer than Halley, typically in the $10^5 - 10^6$ year range. Such long period comets are not aligned with the ecliptic.

Jan Oort postulated the existence of a comet cloud at $3000 \text{ AU} < r < 10^5 \text{ AU}$ with $\sim 10^{12}$ comets constituting around $100M_{\oplus}$. The **Oort cloud** has never been confirmed, though its existence seems virtually certain to astronomers.

* the present locale of the cloud is probably not at the radius of their formation — perhaps they were catapulted there by close encounters with Jovian planets.

Chemical Species in Comets

Table 21.1 of Carroll & Ostlie,
Modern Astrophysics.

Atoms	Molecules	Ions
H	CH	H ⁺
C	C ₂	C ⁺
O	CN	Ca ⁺
Na	CO	CH ⁺
Mg	CS	CN ⁺
Al	NH	<u>CO⁺</u>
Si	N ₂	N ₂ ⁺
S	OH	OH ⁺
K	S ₂	H ₂ O ⁺
Ca	H ₂ O	H ₂ S ⁺
Ti	HCH	CO ₂ ⁺
V	HCN	H ₃ O ⁺
Cr	HCO	H ₃ S ⁺
Mn	NH ₂	CH ₃ OH ₂ ⁺
Fe	C ₃	
Co	OCS	
Ni	H ₂ CO	
Cu	H ₂ CS	
	NH ₃	
	NH ₄	
	CH ₃ OH	
	CH ₃ CN	
	(H ₂ CO) _n	

- Short period comets ($\lesssim 200 - 500$ years) like Halley are aligned with the ecliptic and are thought to be associated with the **Kuiper belt** of comets at $30 \text{ AU} < r < 100 \text{ AU}$. Objects in the Kuiper belt were first detected at 23rd magnitude in 1992 (radii $\sim 200 \text{ km}$)

Debris trails from short period comets in the solar neighbourhood form prominent **meteor showers** arising at different times throughout the year during Earth's passage around the sun; see the Table below.

Shower	Approximate Date	Parent Body
Quadrantid	January 3	(unknown)
Lyrid	April 21	Comet 1861 I
Eta Aquarid	May 4	Comet Halley
Delta Aquarid	July 30	(unknown)
Perseid	August 11	Comet Swift–Tuttle
Draconid	October 9	Comet Giacobini–Zinner
Orionid	October 20	Comet Halley
Taurid	October 31	Comet Encke
Andromedid	November 14	Comet Biela
Leonid	November 16	Comet 1866 I
Geminid	December 13	Asteroid 3200 Phaeton

3 Asteroids

Sometimes referred to as minor planets [chunks of rock], they mostly lie in a belt between Mars and Jupiter.

C & O,
Sec. 22.3

Plot: Radial Distribution of Asteroids

- Their radial distribution is highly structured with distinct underabundance zones (known as **Kirkwood gaps**), often occurring at orbital resonances with Jupiter, whose period is 11.86 years, so that $a_J = 5.2$ AU. Prominent resonances are

3.3AU $\rightarrow$ 2:1 period resonance

2.5AU $\rightarrow$ 3:1 period resonance

Plot: Orbital Resonances (hand-drawn)

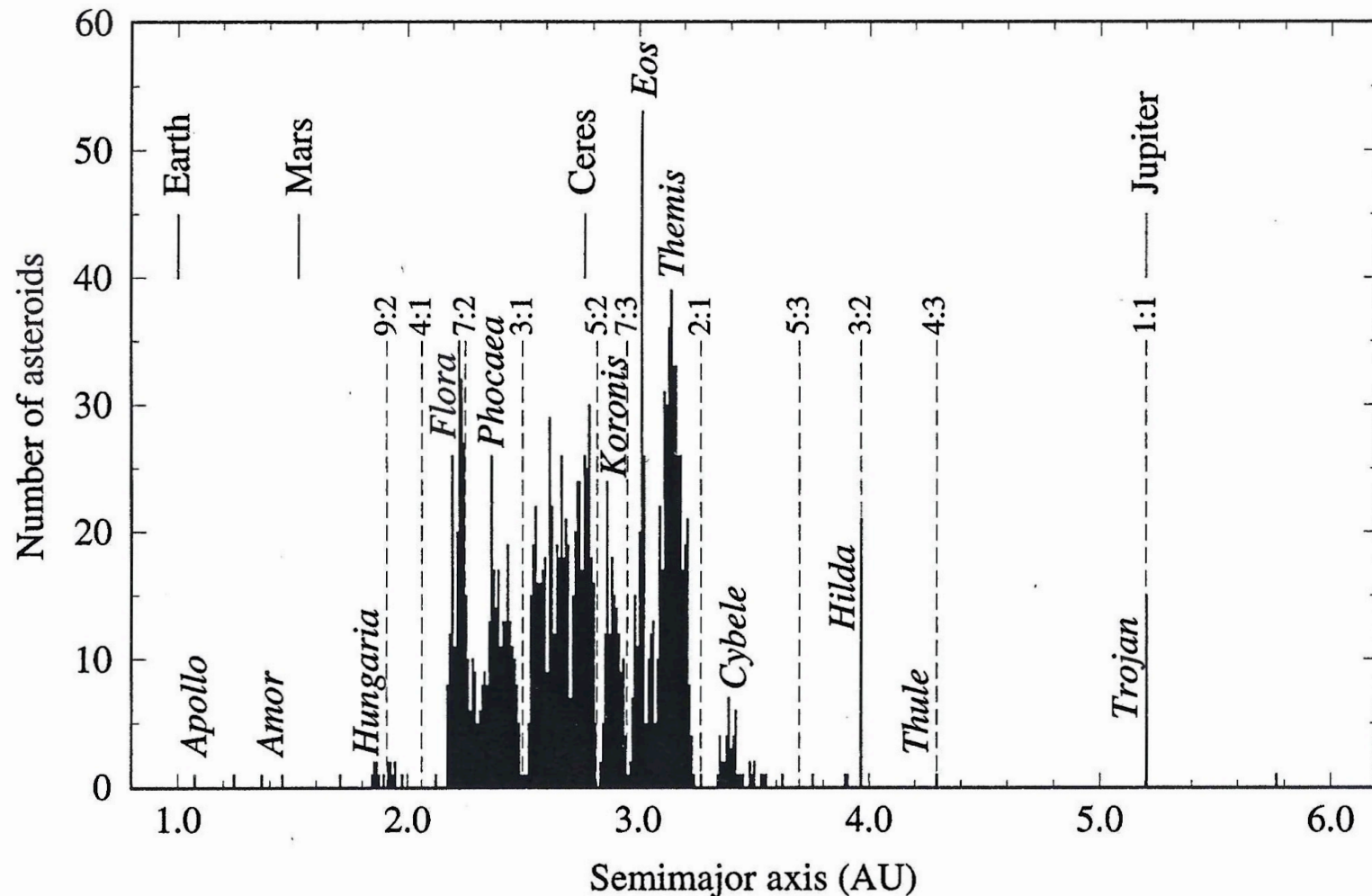
Resonances yield more frequent asteroidal oppositions (nearest approaches) to Jupiter. Of these, *2:1 periods provide the most dramatic effect* due to both the proximity ($a_A/a_J \sim 2^{-2/3}$) to Jupiter and the frequency of encounters.

- * Oppositions perturb asteroidal orbits and render them more elliptical.

Plot: Orbital Eccentrization during Oppositions (hand-drawn)

Their orbital eccentricities are not large like short-period comets, but are still significant, leading to transient smearing of gaps.

Asteroid Orbit Distribution



Distribution of 1796 asteroids, highlighting Kirkwood gaps.
Figure 21.10 of Carroll & Ostlie, *Modern Astrophysics*.

- In 1918, Hirayama first identified **asteroidal families** with nearly identical orbits. Nearly 100 such families are now known, perhaps with each family formed by collisional disintegration of a larger, parent asteroid.

- * Asteroidal masses sum to less than $10^{-3}M_{\oplus}$ in total, implying that they are dynamically unimportant in the solar system.

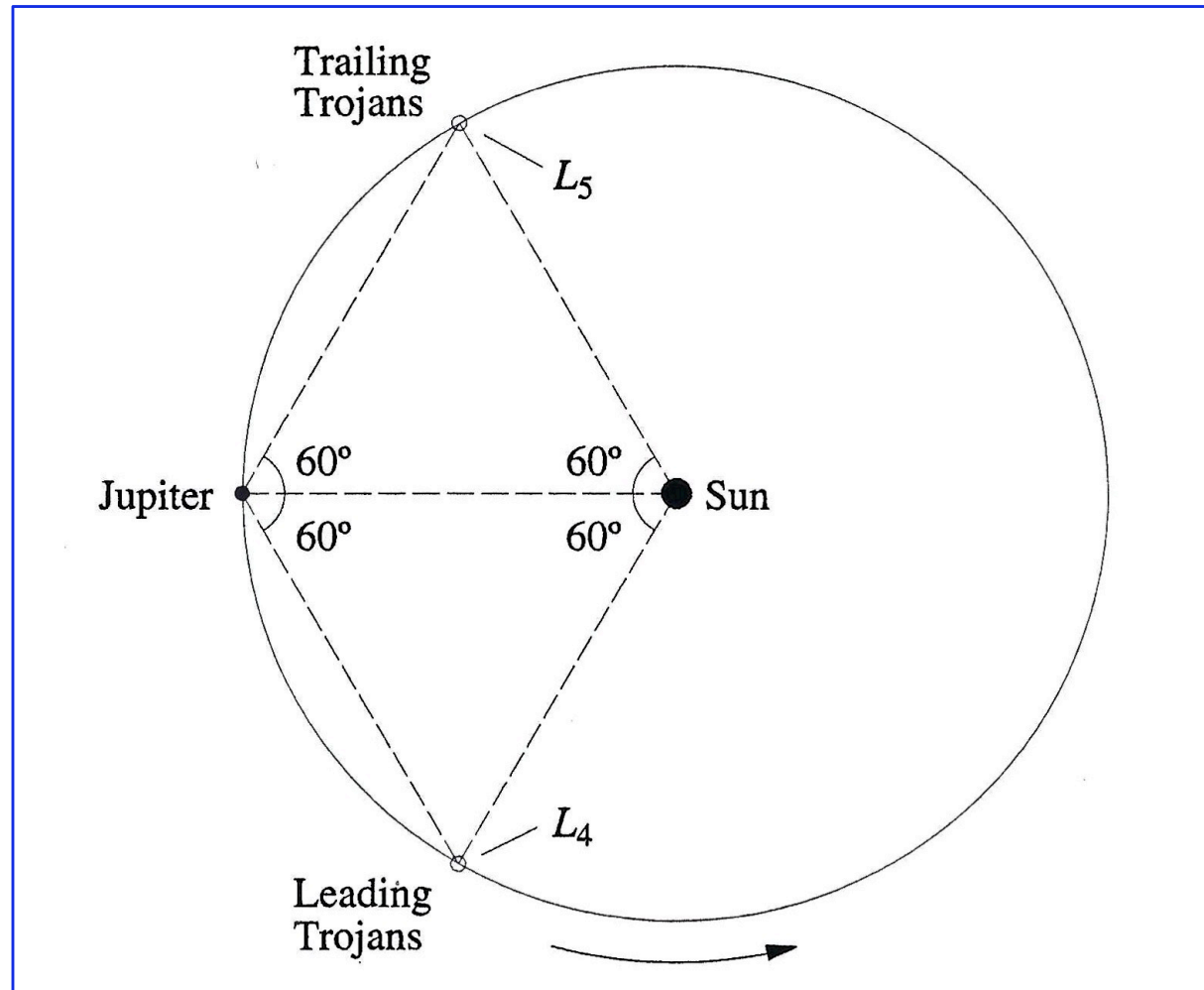
- **Trojan asteroids** that exist in the same orbit as Jupiter are particularly interesting. Trojans are located at the L4, L5 Lagrangian points in the sun-Jupiter potential. These are locales of exceptional stability in a rotating system (gravitational force plus centripetal acceleration).

C & O,
Sec. 17.3

Plot: Trojan Asteroid Locales

[*Reading Assignment: Lagrangian Points - sun-Jupiter system*]

Trojan Asteroid Locales



Trojan asteroids are located at the L_4 and L_5 Lagrangian points in the sun-Jupiter system; they lead or trail Jupiter by 60 degrees in orbital phase. Figure 21.11 of Carroll & Ostlie, *Modern Astrophysics*.

4. PHOTOMETRIC CONCEPTS AND RADIATION

Matthew Baring – Lecture Notes for ASTR 350, Fall 2026

1 Magnitudes

The awareness that some stars are brighter than others dates back to the Greeks, who devised an **apparent magnitude** scale:

**C & O,
Sec. 3.2**

- * $m = 1$ (bright) $\Rightarrow$ 6 (faint)
- In the 19th century, astronomical developments enabled determination that such a range corresponded to a factor of 100 in brightness or **flux** $\mathcal{F}$.
- * $\mathcal{F}$ = energy in photons/area/time at detector.
- The flux differs from **luminosity** L , which is the total radiated power of a star or source (i.e. the wattage of the light bulb!):
- * L = energy in photons/time.

Hence, the inverse square law of photon dilution establishes

$$\boxed{\mathcal{F} = \frac{L}{4\pi d^2}} \tag{1}$$

for a source at distance d from Earth.

Accordingly,

$$\mathcal{F}_{\odot} = \frac{L_{\odot}}{4\pi(1\text{AU})^2} = 1.36 \times 10^6 \text{erg sec}^{-1} \text{cm}^{-2} \quad (2)$$

is the solar flux at Earth and is called the **solar constant**.

* Note the cgs to SI unit conversion: $1 \text{ erg} = 1 \text{ g cm}^2 \text{ sec}^{-2} = 10^{-7} \text{ kg m}^2 \text{ sec}^{-2} = 10^{-7} \text{ Joules}$.

The flux and luminosity both depend on the waveband under consideration.

• Observational equivalents of $\mathcal{F}$ and L are the **apparent magnitude** m and the **absolute magnitude** M . Using the Greek scale as a guideline led astronomers to the *definitions*

$$\mathcal{F} \propto 10^{-2m/5} \quad \text{and} \quad L \propto 10^{-2m/5} d^2 \propto 10^{-2M/5} \quad (3)$$

The constants of proportionality were “benchmarked” by setting $m = M$ for sources at a distance of $d = 10 \text{ pc}$, a typical distance scale for nearby stars. Hence,

$$\boxed{m - M = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right)} \quad , \quad (4)$$

and we call this quantity a source’s **distance modulus**.

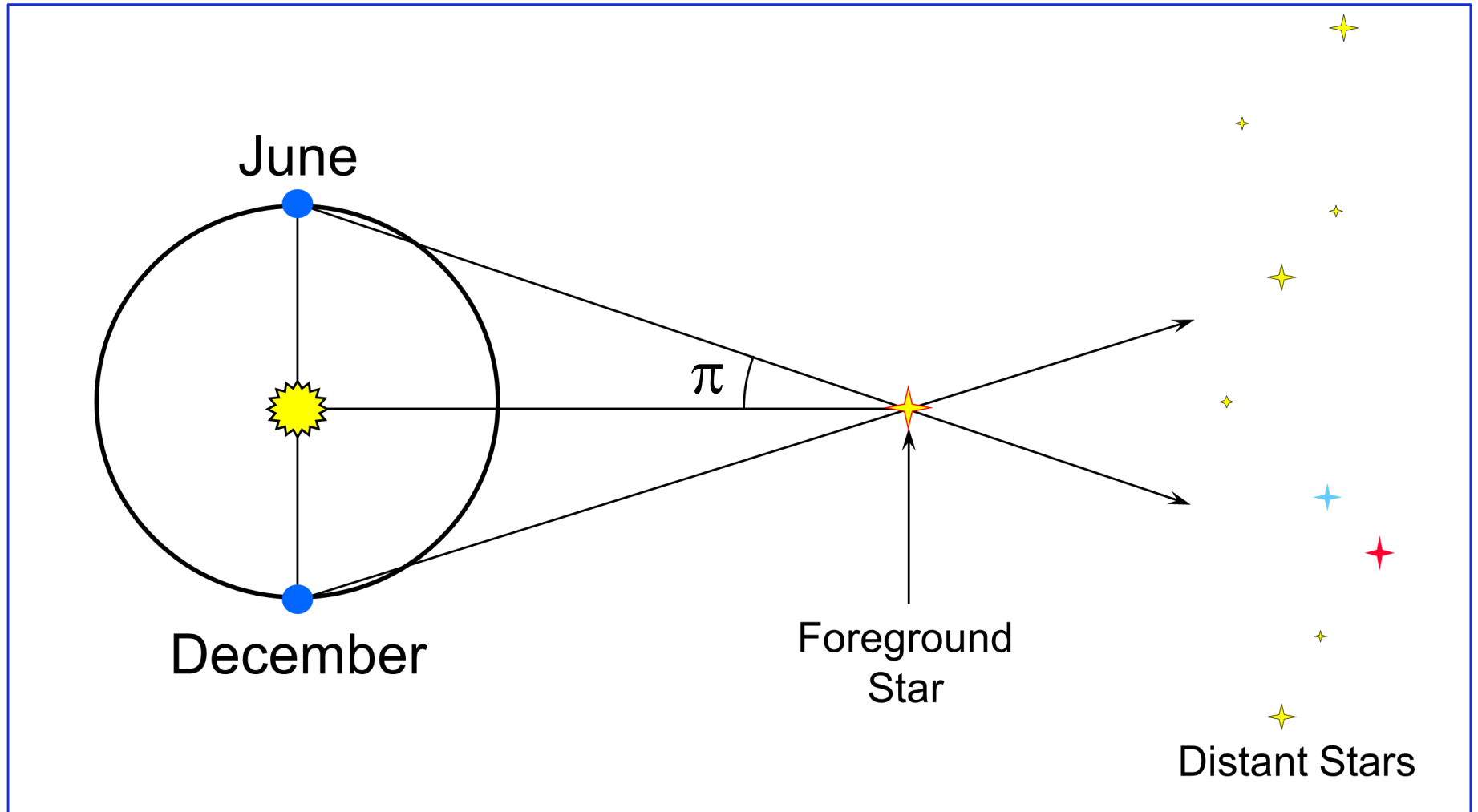
* Observe that for the sun, the brightest object in the sky, a complete outlier with $m_{\odot} = -26.81$, however its absolute magnitude is $M_{\odot} = 4.76$, typical of other main sequence stars.

• It is appropriate to now define the distance scale of a **parsec**, abbreviated **pc**. It is the distance at which 1 AU on the sky subtends 1 arcsec, i.e. the distance at which the **parallax** of a star is $\pi = 1''$:

$$1 \text{ pc} = \frac{1.496 \times 10^{13} \text{ cm}}{\pi/180/3600} = 3.09 \times 10^{18} \text{ cm} = 3.26 \text{ lt yr}. \quad (5)$$

Plot: Stellar parallax

Trigonometric Stellar Parallax



Geometry for trigonometric parallax. Credit: R. W. Pogge, Ohio State Univ.

A brief list of apparent magnitudes as benchmarks is as follows:

- * full moon: $m = -12.5$
- * Jupiter: $m = -2$
- * naked eye limit: $m = +6$
- * binocular limit: $m = +9$
- * 8m telescope: $m = +29$
- The distance modulus is a tool that can be used to estimate the distance to stars that are beyond the parallactic limit (which admittedly has moved further away in the Gaia era). A star of known spectral type can have its absolute magnitude M fairly well known. Its apparent magnitude is measured and its distance d inferred.
- Magnitude depends on color too, so astronomers use magnitude differentials to provide indications of color.
- * e.g. $B - V \equiv M_B - M_V$ is a color marker so that a red star has typically $B - V = 1.0$, whereas a blue star has typically $B - V = -0.5$.

**C & O,
Sec. 3.6**

Plot: Color bands and a putative stellar spectrum

- **Bolometric** magnitudes average over all wavebands, i.e. represent the mean magnitude of a source:

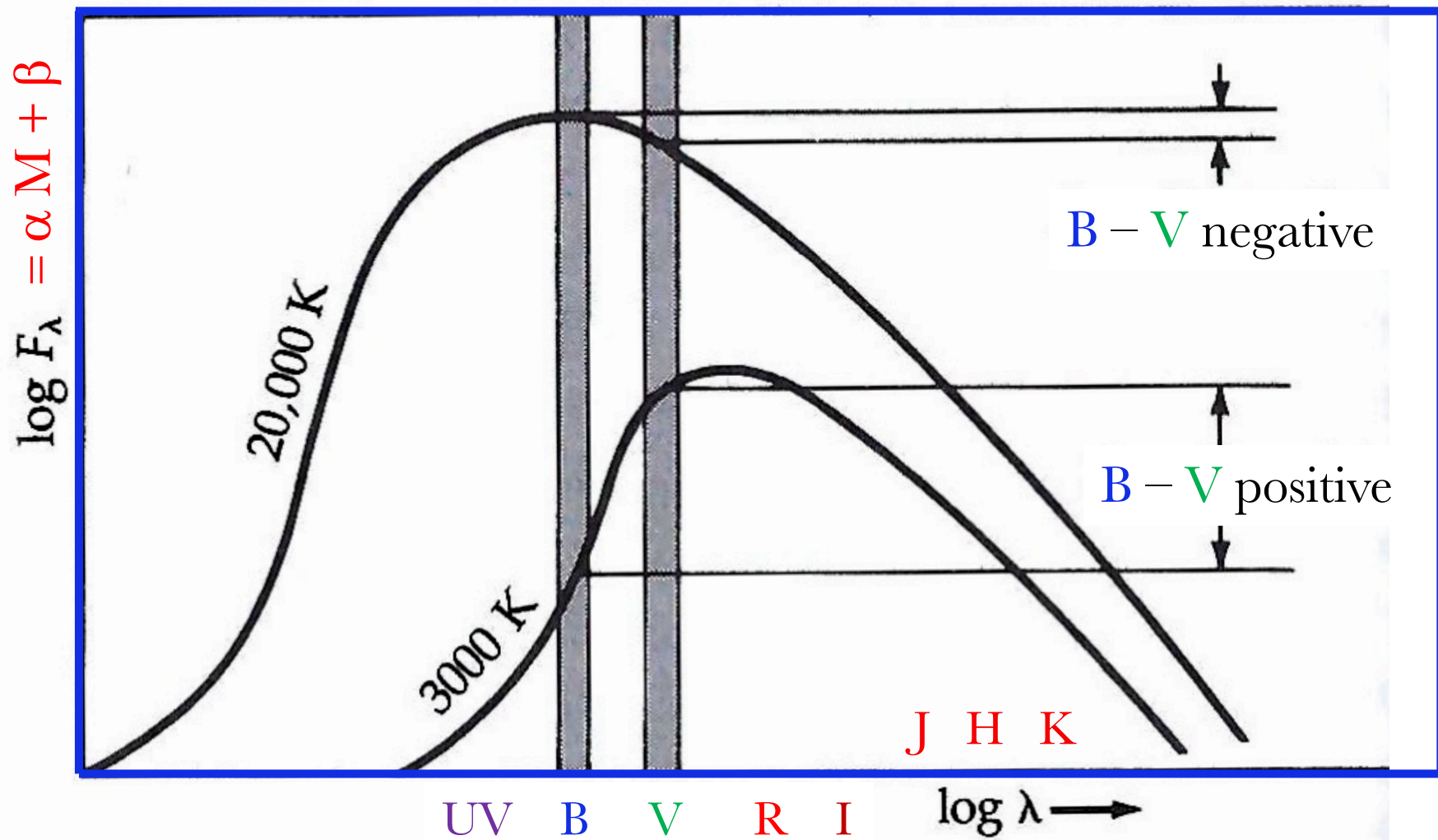
$$M_{bol} = M_V + \text{corr}_V \equiv M_B + \text{corr}_B \quad , \quad (6)$$

where the difference of correction factors $\text{corr}_V - \text{corr}_B \equiv M_B - M_V$ represents the spectrum or color of a star.

- * This difference is more *positive* for bluer (i.e. hotter) stars, and more negative for redder stars.

Plot: Photometric Color Band Wavelengths

Photometric Color Bands



Color index in the B-V system. Blackbody curves for 20,000K and 3,000K are shown, along with their intensities at B and V wavelengths.

Fig. 11-4: Zeilik, Gregory & Smith, *Introductory Astronomy & Astrophysics*.

Photometric Color Bands

Filter Letter	Effective Wavelength Midpoint λ_{eff} for Standard Filter ^[2]	Full width at half maximum ^[2] ^[c] (archetypal Bandwidth) ($\Delta\lambda$) ^[d]
Ultraviolet		
U	365 nm	66 nm
Visible		
B	445 nm	94 nm
G ^[3]	464 nm	128 nm
V	551 nm	88 nm
R	658 nm	138 nm
Near-Infrared		
I	806 nm	149 nm
Z	900 nm ^[4]	
Y	1020 nm	120 nm
J	1220 nm	213 nm
H	1630 nm	307 nm
K	2190 nm	390 nm
L	3450 nm	472 nm
Mid-Infrared		
M	4750 nm	460 nm
N	10500 nm	2500 nm
Q	21000 nm ^[5]	5800 nm ^[5]

2 Extinction and Reddening

Observed fluxes or magnitudes cannot be taken at face value as the exact measure of brightness of a star; they must be corrected for **interstellar extinction**, the attenuation of radiation on its path from source to Earth due to scattering or absorption in the interstellar medium (ISM).

C & O,
pp. 259-61

Plot: Absorption column and intensity attenuation

- From the absorption column set-up, the differential change in **intensity** $dI_\lambda = I_\lambda - I_0$ is related to path length ds by

$$\frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda ds \quad , \quad (7)$$

where κ_λ is called the **absorption coefficient**. This simply integrates to

$$I_\lambda = I_0 e^{-\tau_\lambda} \quad , \quad \tau_\lambda = \kappa_\lambda \Delta s \quad . \quad (8)$$

The *intensity is the energy of light passing through unit area in unit solid angle per unit time*. Here τ_λ is called the **optical depth**, and is the key measure of how opaque the ISM is. $\tau_\lambda \ll 1$ corresponds to **optically thin** conditions, whereas $\tau_\lambda \gg 1$ defines **optically thick** environs.

- In general, $\tau_\lambda = \tau(\lambda)$, and blue light is preferentially scattered (Rayleigh scattering) by dust, i.e. lost from the line of sight. This is manifested in the scattering of light by bound electrons that oscillate at some natural frequency ω_0 , with a cross section

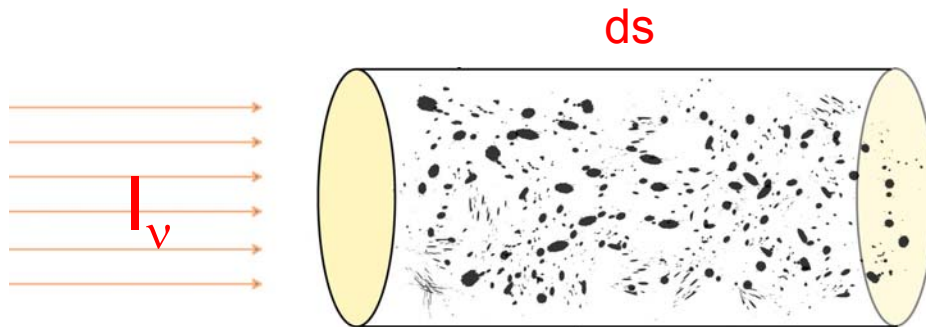
$$\sigma_{\text{Ray}} \propto \left(\frac{\omega}{\omega_0} \right)^4 \quad . \quad (9)$$

* This phenomenon is evinced in the blue nature of the sky via the scattering of sunlight. Like most scattering processes, it is polarization dependent, and so the sky can be darkened by a polaroid.

- Since $\tau_{\text{red}} < \tau_{\text{blue}}$, the ISM generates **interstellar reddening** of starlight as well as extinction. Both need to be corrected for in transit to Earth.

Interactions between photons and matter

absorption of radiation



$$dI_\nu = -\kappa_\nu I_\nu ds$$

κ_ν : absorption coefficient

$$[\kappa_\nu] = \text{cm}^{-1}$$

microscopical view: $\kappa_\nu = n \sigma_\nu$

loss of intensity in the beam (true absorption/scattering)

Over a distance s :

$$I_\nu^o \xrightarrow{s} I_\nu(s)$$

$$I_\nu(s) = I_\nu^o e^{-\int_0^s \kappa_\nu ds}$$

Slide Courtesy of
R-P Kudritzki, IFA,
U. Hawaii

Convention: $\tau_\nu = 0$ at the outer edge
of the atmosphere, increasing
inwards

$$\tau_\nu := \int_0^s \kappa_\nu ds$$

optical depth
(dimensionless)

or: $d\tau_\nu = \kappa_\nu ds$

- In magnitudes, the wavelength-dependent extinction is expressed as an attenuation magnitude:

$$A_\lambda = m_\lambda - m_\lambda(\text{no dust}) = 2.5 \log_{10} \frac{I_0}{I} = \tau_\lambda (2.5 \log_{10} e) = 1.08 \tau_\lambda \quad . \quad (10)$$

The observed magnitude is then corrected by A_λ to find the true magnitude. One can then define **color excesses**:

$$E_{B-V} = \underbrace{(B - V)}_{\text{observed}} - \underbrace{(B_0 - V_0)}_{\text{intrinsic}} \quad . \quad (11)$$

The intrinsic values are inferred from a star's spectral class and comparison with other stars.

- Interstellar extinction is not the whole story: **atmospheric extinction** complicates the picture. It can be extracted by performing a zenith comparison, a check that can be performed on a star during a night's observation.

Plot: Atmospheric extinction geometry and zenith variation

The ratio of optical depths for zenith angle θ relative to the zenith value is simply

$$\frac{\tau(\theta)}{\tau(0)} = \frac{s}{h} = \sec \theta \quad (12)$$

to first order, with $\sec \theta \geq 1$. This result neglects the altitudinal distribution of attenuation. Converting this to magnitudes then results in

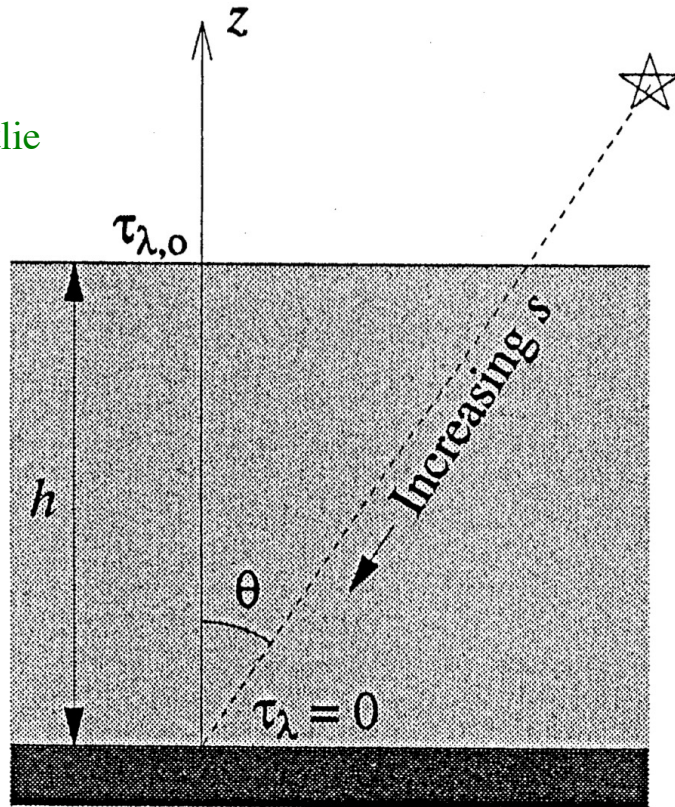
$$m_\theta = m_0 + 1.08 \tau_0 \sec \theta \quad , \quad (13)$$

so that the intersection of the m_θ - $\sec \theta$ plot with the ordinate axis yields the upper atmosphere magnitude (incident) m_0 ($\equiv m_\lambda$ above, to be corrected for interstellar extinction and reddening), and the slope $k_\lambda = 1.08 \tau_0$ is the extinction coefficient per unit airmass.

[Reading Assignment: C & O Sec. 3.3: the wave nature of light]

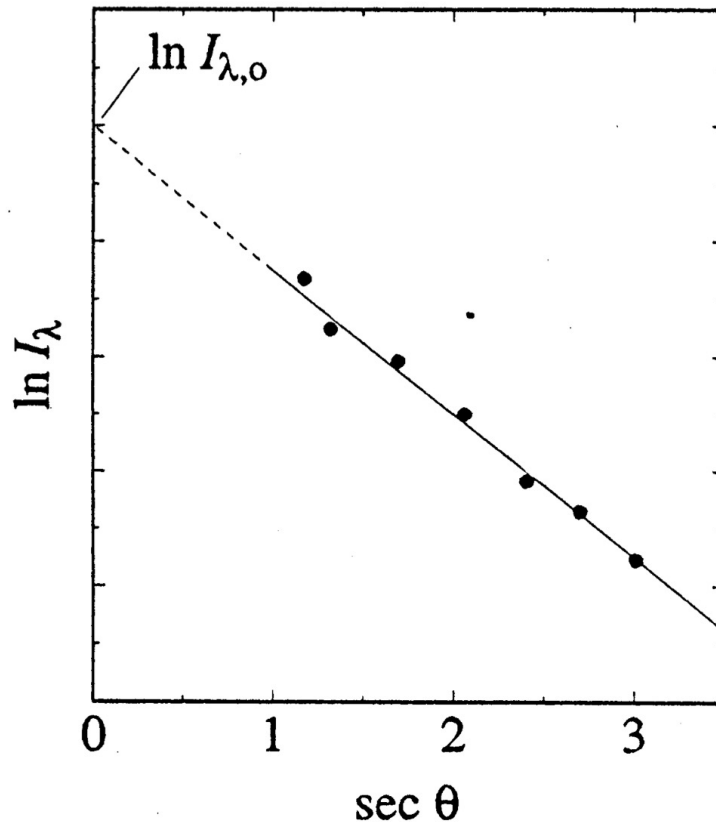
Atmospheric Extinction

Figure 9.8
Carroll & Ostlie



(a)

$$\ln I_\lambda = -\alpha m + \beta$$



(b)

- Left panel:* Light rays entering the Earth's atmosphere at an angle θ to the zenith.
- Right panel:* Logarithm of the intensity I_λ as a function of zenith angle proxy $\sec \theta$. This can be expressed linearly in terms of apparent magnitude m .