

# 3. TIDES, COMETS AND ASTEROIDS

Matthew Baring – Lecture Notes for ASTR 350, Fall 2026

*Reading assignment:* Section 19.1 from C & O, outlining the constituents and broad structure of the solar system.

## 1 Tidal Forces

The dynamics of heavenly bodies are subject to greater complexity than has been described heretofore: non-point masses can exhibit tidal forces *if they are aspherical*. “Fluid” bodies that rotate can become aspherical (prolate or oblate). If these bodies are of uniform mass density  $\rho$ , their shapes are known as **Maclaurin spheroids**, and their cross sectional eccentricities depend only on the ratio  $\Omega/\sqrt{G\rho}$ , where  $\Omega = 2\pi/P$  is the rotational angular frequency. In the presence of other bodies, these spheroids experience a differential gravitational force across them, known as a **tidal force**.

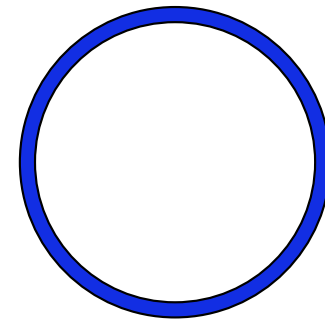
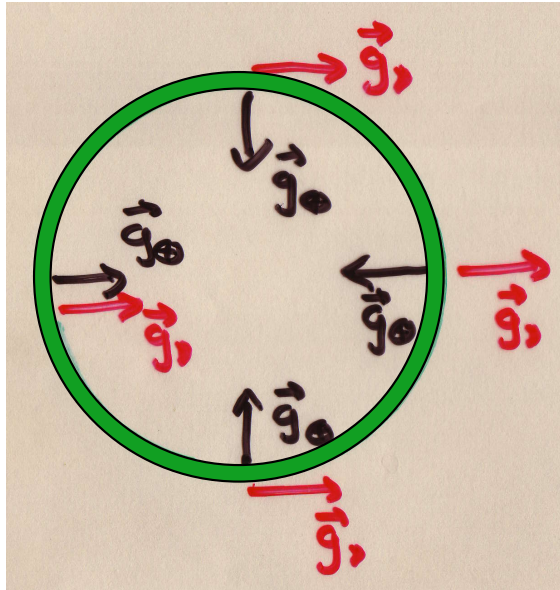
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Sec. 19.2

**Plot:** Tidal Forces: Earth-Moon System

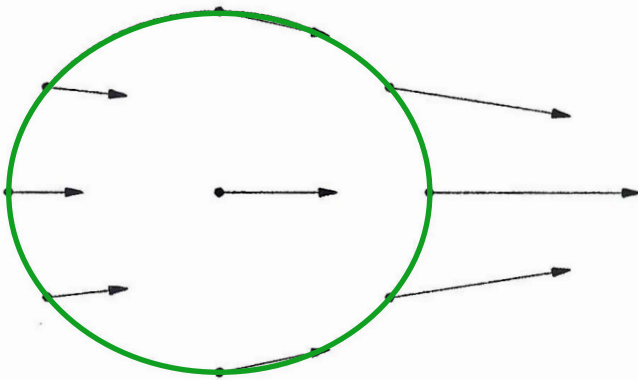
- Since the net force differential is not aligned with Earth’s rotation vector, rotational torques result, *leading to a change in the rate of rotation*.
  - \* The day was shorter in the age of the dinosaurs.
  - \* The associated spin-down is a gravitational analogue of electromagnetic spin-down in pulsars (later on in the course).

# Tidal Forces: Earth-Moon System

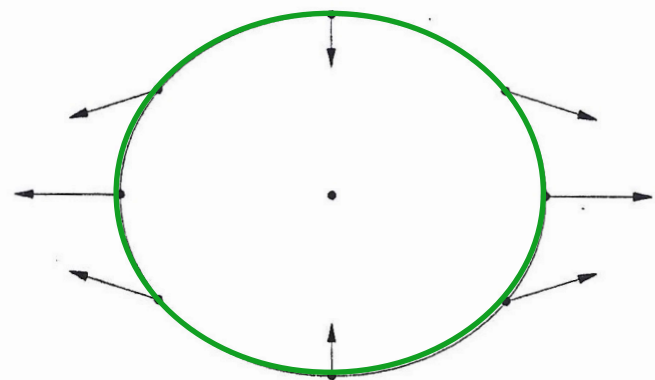
Earth



Moon



(a)



(b)

Fig. 18.4  
C&O

- (a) Gravitational force of moon on Earth;
- (b) Differential gravitational force, relative to its center.

Consider the tidal force geometry for the Earth-moon gravitational interaction, for a point mass  $m$  on the Earth's surface.

**Plot:** Tidal Force Geometry

The force components along the Earth-Moon central line at some point P (x-direction) and orthogonal to this (y-direction) are ( $M$  is the lunar mass)

$$\frac{GMm}{s^2} \cos \phi \hat{x} \quad \text{and} \quad -\frac{GMm}{s^2} \sin \phi \hat{y} \quad , \quad (1)$$

where  $\phi$  is the angle between the  $P\vec{M}$  vector and the  $C\vec{M}$  vector (see plot). The distance between point P and the Moon's center is  $s$ . At the center of Earth the components are  $(GMm/r^2) \hat{x}$  and  $0 \hat{y}$ . The differential force between Earth's surface at P and its center is therefore

$$\overrightarrow{\Delta F} \equiv \vec{F}_P - \vec{F}_C = GMm \left( \frac{\cos \phi}{s^2} - \frac{1}{r^2} \right) \hat{x} - \frac{GMm}{s^2} \sin \phi \hat{y} \quad . \quad (2)$$

Now, for moon distances far exceeding the Earth radius  $R$ , i.e.,  $r \gg R$ ,

$$s^2 = (r - R \cos \theta)^2 + (R \sin \theta)^2 \approx \frac{r^2}{1 + 2R \cos \theta / r} \quad (3)$$

so that

$$\overrightarrow{\Delta F} \approx \frac{GMm}{r^2} \left[ \cos \phi \left( 1 + \frac{2R}{r} \cos \theta \right) - 1 \right] \hat{x} - \frac{GMm}{r^2} \left( 1 + \frac{2R}{r} \cos \theta \right) \sin \phi \hat{y} \quad . \quad (4)$$

Hence, since  $\phi \approx 0$ , we have  $\cos \phi \approx 1$  and  $r \sin \phi \approx s \sin \phi = R \sin \theta$ ,

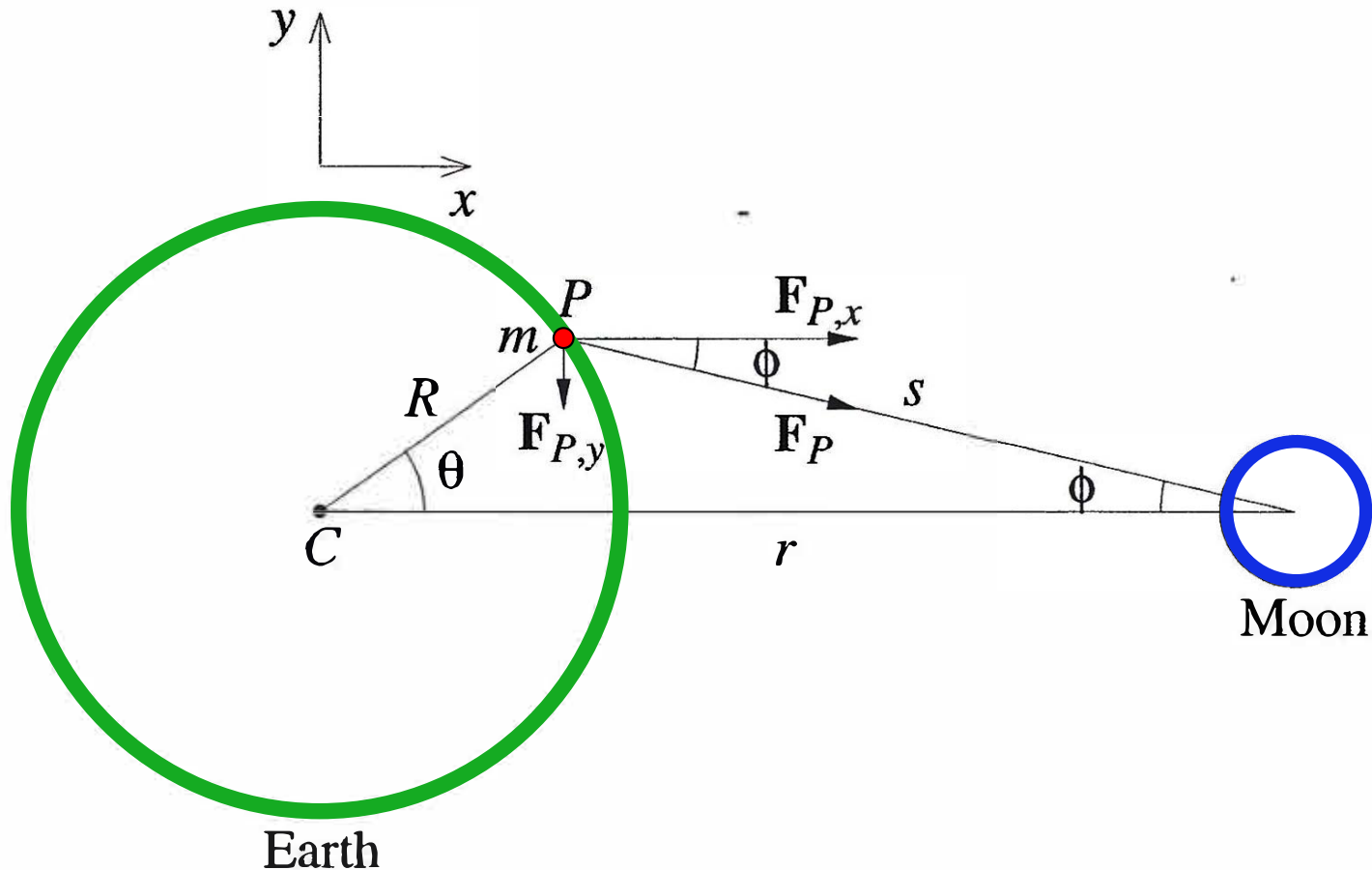
$$\overrightarrow{\Delta F} \approx \frac{GMm}{r^2} \frac{R}{r} \left( 2 \cos \theta \hat{x} - \sin \theta \hat{y} \right) \quad . \quad (5)$$

Observe the  $1/r^3$  dependence (perturbation theory expansion) that applies to both isotropic and anisotropic mass distributions. The  $-\hat{y}$  term implies a quadrupole nature for the distortion.

**Plot:** Differential Tidal Forces - Revisited

- Tidal bulges in the Earth's crust amount to around 10cm (c.f. *water in driven environments* such as the Bay of Fundy and the English Channel) — on the Moon, the crustal bulges are of magnitude  $\sim 20$  m!

# Earth-Moon Tidal Force Geometry



- Figure 18.3 of Carroll & Ostlie, *Modern Astrophysics*

Note that the tidal bulges are not directly aligned to the line to the Moon: the bulge axis leads the Earth-Moon line.

**Plot:** Moon-Earth Torque

Frictional dissipation drags the AB axis ahead of the Earth-moon line  $\Rightarrow$  loss of kinetic energy due to the gravitational torque. Accordingly, the **Earth's rotation slows**, at the rate of  $dP/dt \approx 0.0016 \text{ sec/century}$ ;

\* this then establishes that the *current* timescale for doubling the duration of the Earth's day is  $5.4 \times 10^9$  years, i.e. *Earth has only modestly spun down over its lifetime*.

- At the same time, Earth's tides pull the moon along, accelerating it and pushing it to larger orbital distances. This rate can be connected to the Earth's spin-down rate via simple calculations of the rate of exchange of angular momentum between the two bodies.

\* laser ranging measurements establish  $dr_{\text{Moon}}/dt \approx 3\text{--}4 \text{ cm/year}$ .

Given the order of the tidal force in Eq. (5), the torque can be quickly estimated:

$$\frac{dL}{dt} \sim \left| \vec{R} \times \overrightarrow{\Delta F} \right| \sim GMm \frac{R^2}{r^3} \Rightarrow \frac{d\omega}{dt} \sim \frac{1}{mR^2} \frac{dL}{dt} \sim \frac{GM}{r^3} . \quad (6)$$

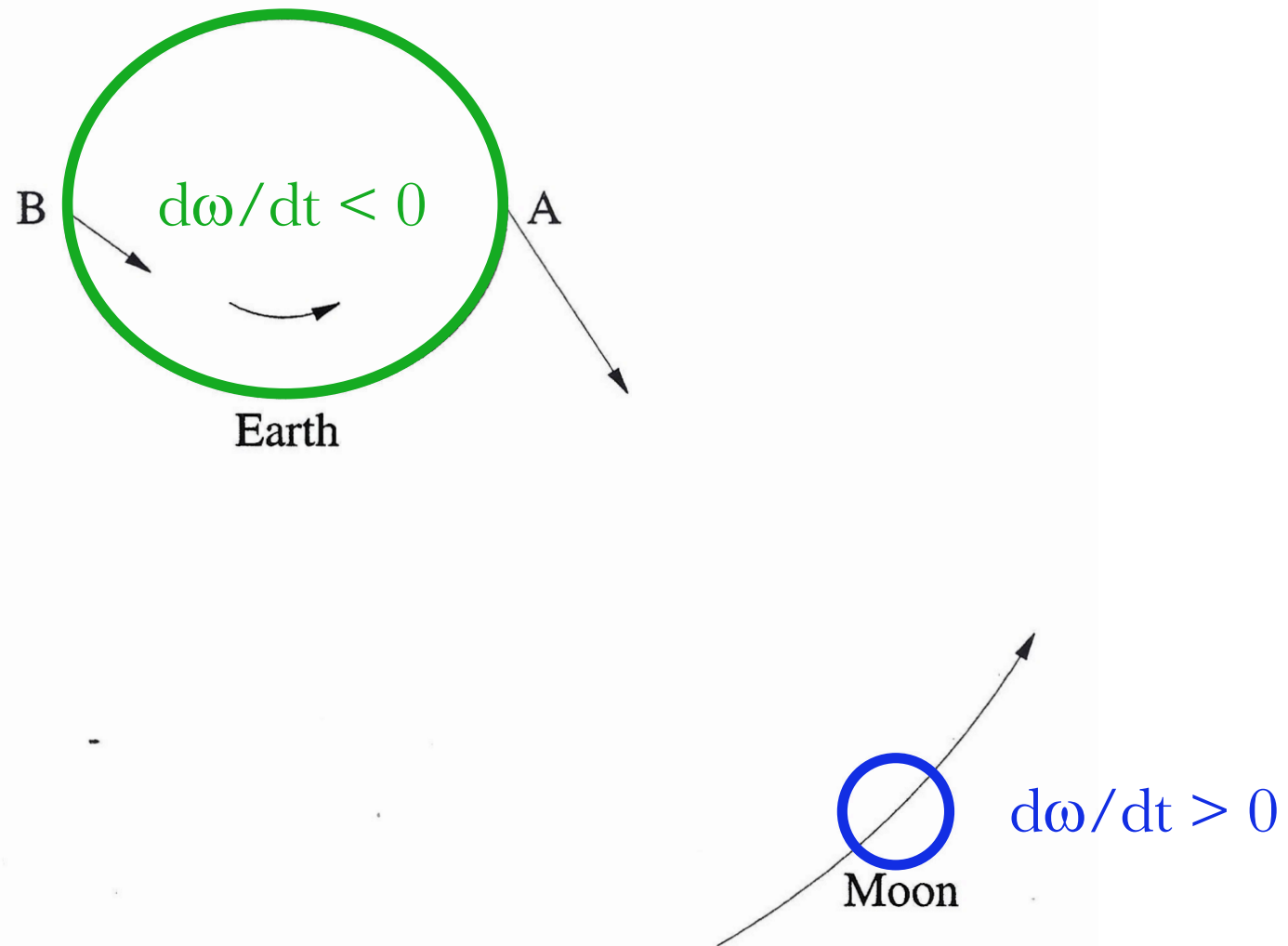
This equation can be applied equally to either the Earth or the Moon, and leads to the inference that

$$\left. \frac{d\omega}{dt} \right|_{\text{Moon}} \sim \frac{M_{\oplus}}{M_{\text{Moon}}} \left. \frac{d\omega}{dt} \right|_{\oplus} \sim 81 \left. \frac{d\omega}{dt} \right|_{\oplus} , \quad (7)$$

modulo differences in frictional dissipation.

- Hence, the moon has had plenty of time to relax into a 1–1 synchronous rotation, which is *the hallmark end state of tidal friction* (an effect common to moons of Mars, Jupiter and Saturn).

# Moon-Earth Torque



- Figure 18.5 of Carroll & Ostlie, *Modern Astrophysics*. Earth's bulge A is closer to the moon than is bulge B, resulting in a net torque on the planet.

## 2 Comets

The most famous comet is Halley's, with a period of 76 years, and whose last perihelion passage was in 1986. Comets are characterized by high  $e$  orbits in the ecliptic, and distinguished by a prominent head, corona and extremely lengthy tails. Halley's comet has  $e \approx 0.967$ .

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**Plot:** Comet Halley: Schematic

- Whipple (1950) proposed the “dirty snowball” model, with the comet being a mixture of ice and dust in the nucleus.

- \* Ice sublimates when heavily irradiated by the sun, leading to a large ( $10^{12}$  cm) hydrogen *halo* about a *coma*.

- \* Comets typically possess two tails — an **ion tail**, pointing directly away from the sun, and a *dust tail*, inclined at some non-radial angle.

The snowball concept of comets was confirmed by the Giotto and Vega fly-bys of Halley in 1986.

**Plot:** Comet Halley – Nucleus Schematic

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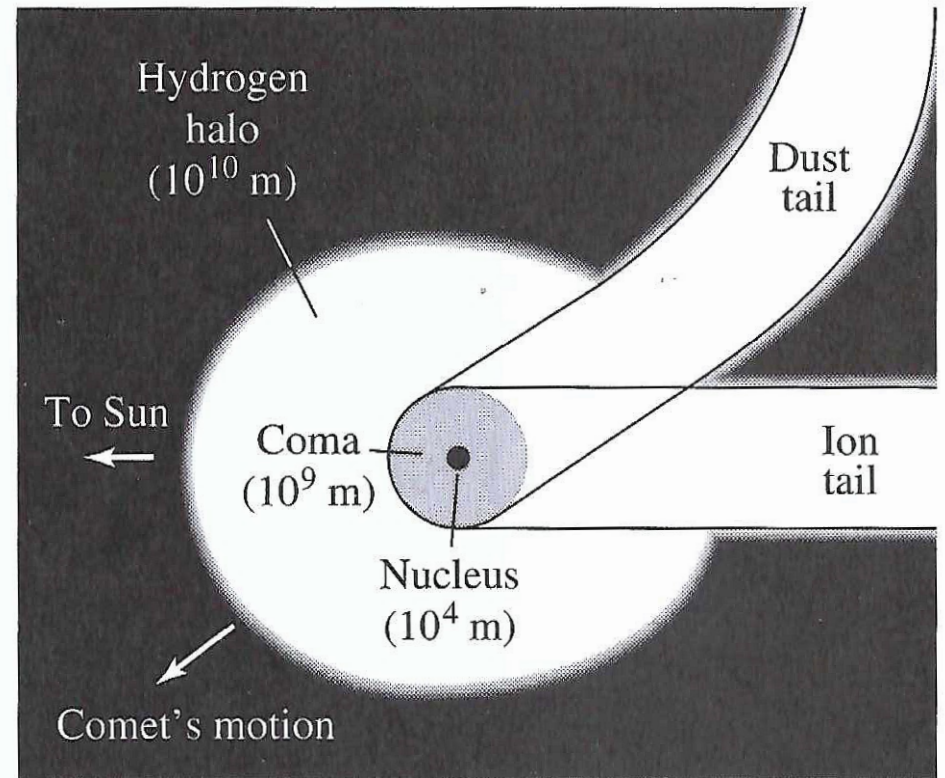
- Consider the dust tail — its character is determined by radiation pressure. Suppose that dust grains have a radius  $R_g$  at distance  $d$  from the sun. The radiation energy flux at  $r$  from the sun is (in erg/sec/cm<sup>2</sup>)

$$\mathcal{F} \sim \frac{L_{\odot}}{4\pi d^2} \Rightarrow F_{rad} = \frac{\pi R_g^2 \mathcal{F}}{c} \sim \frac{L_{\odot}}{4c} \left( \frac{R_g}{d} \right)^2, \quad (8)$$

where  $F_{rad}$  is the radiation force on the dust grain (in erg/cm). If the grain has a mean density  $\rho_g$  and mass  $m_g = 4\pi R_g^3 \rho_g / 3$ , then the solar gravitational field generates an inward force of

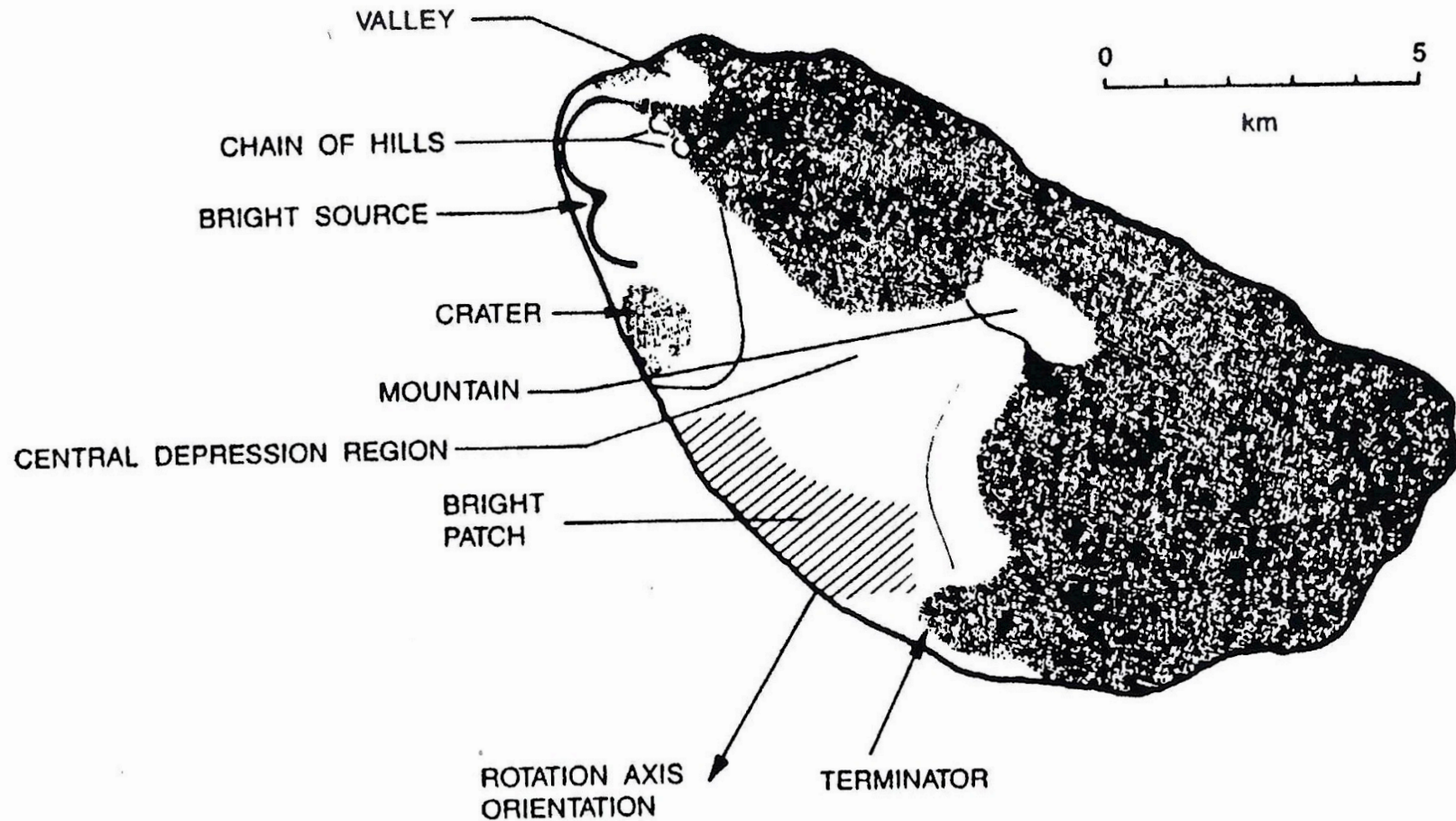
$$F_g = \frac{GM_{\odot} m_g}{d^2} = \frac{4\pi GM_{\odot} \rho_g R_g^3}{3d^2}. \quad (9)$$

# Comet Halley



*Left:* Comet Halley exhibited an ion tail (straight) and a dust tail (curved). Image is from Michigan Schmidt telescope at Cerro Telolo Interamerican Observatory (CTIO) on April 12, 1986. *Right:* anatomy of a comet.  
Figure 22.3 of Carroll & Ostlie, *Modern Astrophysics*.

# Comet Halley Nucleus



Schematic of the irregular surface of Comet Halley.  
Figure 21.8 of Carroll & Ostlie, *Modern Astrophysics*.

Observe that both the gravitational and radiation pressure forces vary as  $1/d^2$ . Their ratio can therefore be written as

$$\frac{F_g}{F_{rad}} \approx \frac{16\pi GM_\odot R_g \rho_g c}{3L_\odot} = \frac{R_g}{R_{cr}} \quad (10)$$

where

$$R_{cr} = \frac{3L_\odot}{16\pi GM_\odot \rho_g c} = 1.9 \times 10^{-5} \text{cm} = 1900 \text{\AA} \quad (11)$$

for  $\rho_g = 3 \text{g/cm}^3$ . Observe that this critical radius is slightly shorter than the wavelength of optical light from the sun. Hence, *grains with  $R_g < R_{cr}$  will be pushed outwards, forming a cometary tail.*

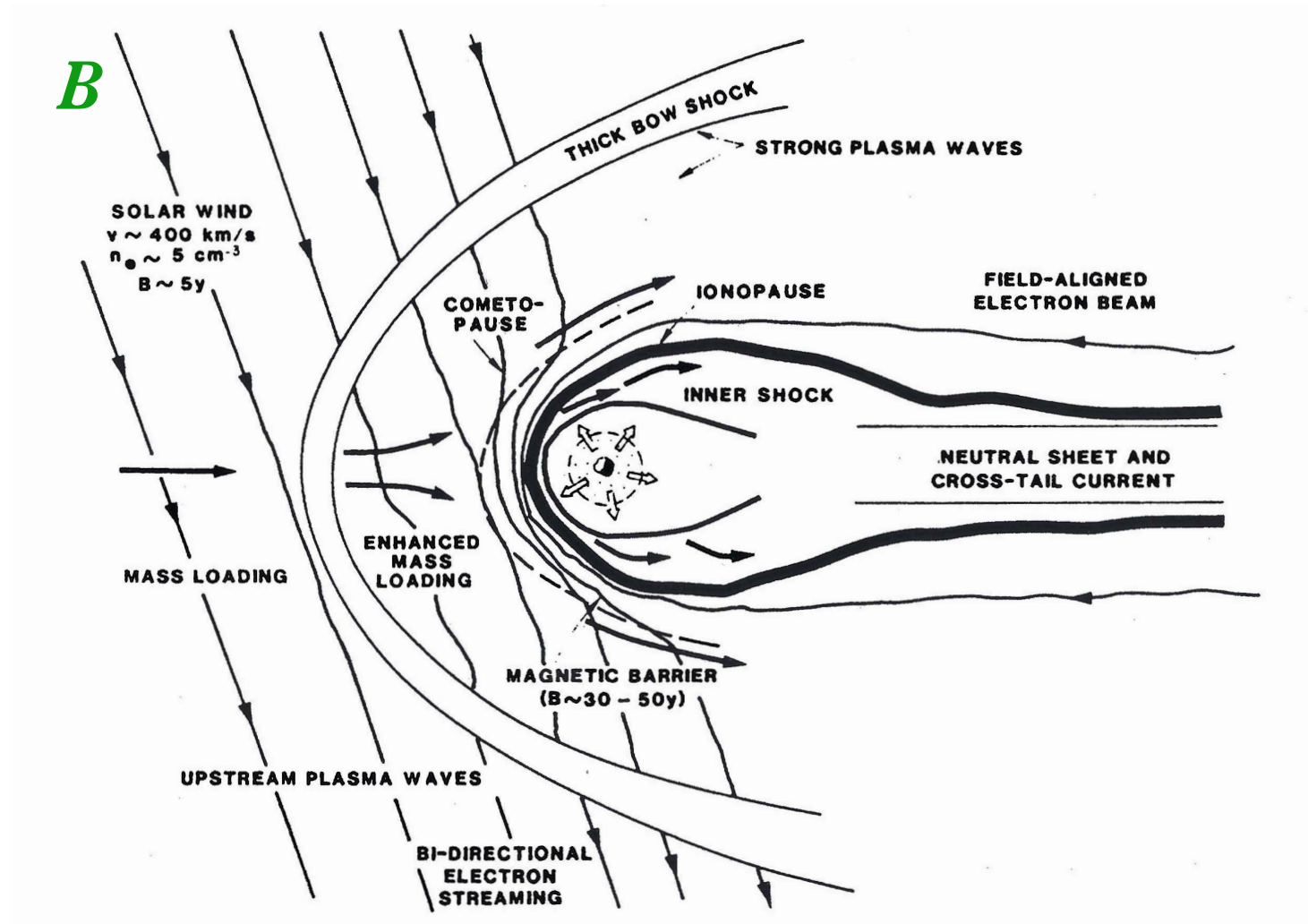
- Larger grains can, in fact orbit the sun. Yet they slowly spiral inwards due to the **Poynting-Robertson** effect:

- \* Grains absorb solar light (in the radial direction), and preferentially re-radiate it in their non-radial direction of motion; this momentum kick effects deceleration and a loss of orbital  $L$ . Note that the dependence of absorption on  $\lambda$  and potential of scattering complicate the picture.

- In contrast, the ion tail is composed of a plasma, much like Earth's magnetospheric tail, the **magnetotail**.

**Plot:** Cometary Bow Shock and Ion Tail

# Cometary Bow Shock and Tail



A model for the formation of a comet's ion tail.  
Mendis, *Ann. Rev. Astron. Astrophysics*. **26**, 11 (1988)