9. STELLAR EVOLUTION I

Matthew Baring – Lecture Notes for ASTR 350, Fall 2025

1 Star Formation

The fact that the solar system is of an age considerably less than the Hubble time and that it has an unusually high metallicity suggests that it was formed from products of stellar evolution, i.e. preprocessing. **C & O, pp. 405–8** All of us are made from stardust.

Mass loss from stars can generate large clouds of gas and dust. Much of this can be neutral hydrogen, which can be mapped via the 21cm (1420 MHz) spin-flip transition in the radio. This hyperfine transition is used to map galactic rotation curves. Estimating its frequency will be a homework problem!

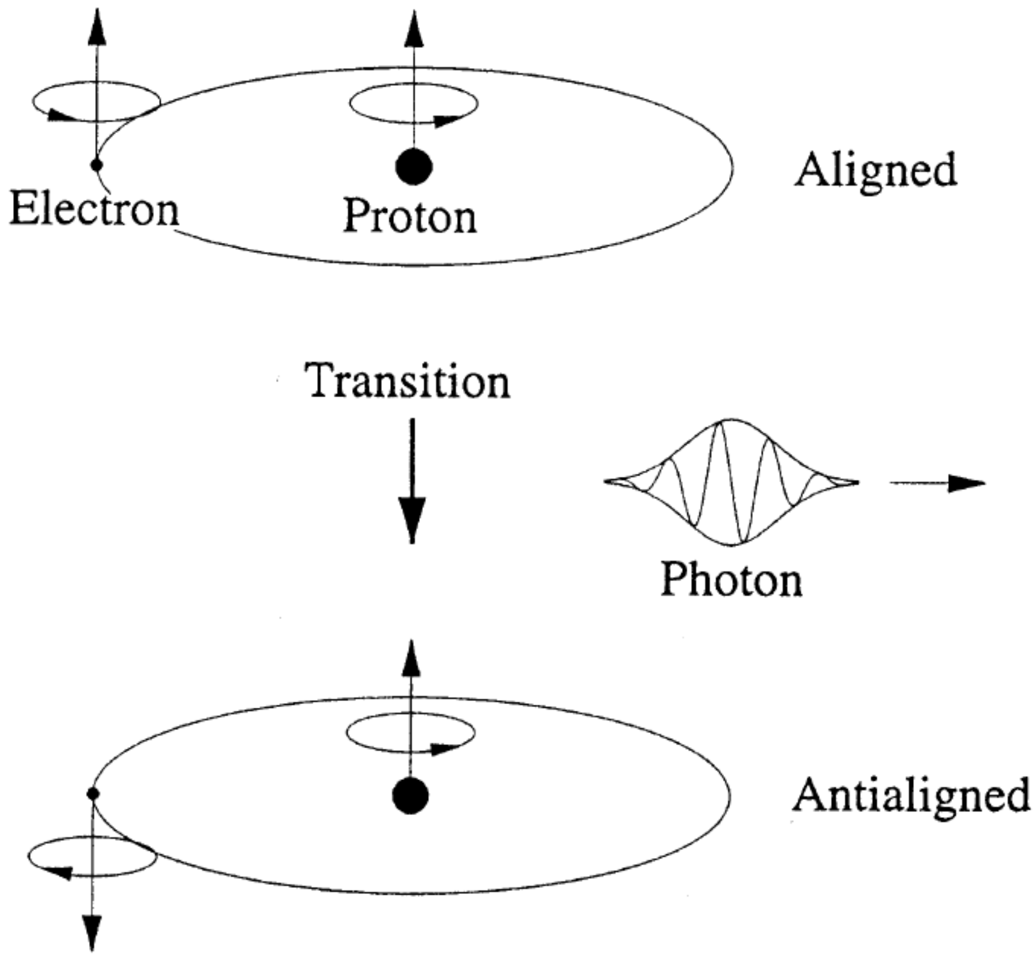
Plot: 21cm Spin-Flip Transition

There are many **diffuse hydrogen clouds** in the galaxy, known also as **H I regions**, and they possess $T \sim 30 - 80\,\mathrm{K}$, number densities $n \sim 100 - 800$ cm^{-3} and masses $1 - 100 M_\odot$.

• Dust in such clouds can act as a site for hydrogen atoms to meet and coalesce into molecules. Moreover, the dust can shield the molecular hydrogen from radiative dissociation. Dusty clouds that have much of their hydrogen in molecular form are called **molecular clouds**.

∗ H_2 does not have any palpable radiative tracers in the radio or optical, unlike atomic hydrogen, so it is hard to discern directly. Instead, it is usually associated with other molecules, such as CO, so that these are used as tracers. Hence, molecular clouds are usually mapped in 2.6mm CO emission.

21 cm Spin-Flip Transition



- When the spins of the electron and proton in hydrogen transition from being aligned to anti-aligned, a 21cm (1420 MHz) wavelength photon is emitted.

• **Giant molecular clouds** (GMCs) are enormous complexes of dust and gas, typically with radii $r \sim 50\,\text{pc}$, with $T \sim 20-30\,\text{K}$ and number densities $n \sim 100-300\ \text{cm}^{-3}$. Residing within such clouds are **cores** (*draw schematic*) of radii $r \sim 0.1 - 1\,\text{pc}$ and $T \sim 100 - 200\,\text{K}$ and $n \sim 10^7 - 10^9\ \text{cm}^{-3}$.

$\boxed{\textbf{Plot:}}$ Orion and Monoceros Molecular Clouds

∗ The existence of **fragmentation** into such cores, with masses typically around $10 - 1000 M_\odot$, indicates that they are the sites of star formation.

∗ Thousands of GMCs are known in our galaxy, mostly in the spiral arms.

1.1 Gravitational Collapse: Jeans Criterion

If molecular clouds are the sites for star formation, what conditions must guarantee collapse? Obviously, gravity must outweigh kinetic motions. In the following, neglect rotation and the influence of magnetic fields. **C & O, Sec. 12.2**

• The gravitational potential of a spherical cloud of uniform density and mass M_c and radius r_c is

$$U \sim -\frac{3}{5}\,\frac{GM_c^2}{r_c} \quad , \tag{1}$$

and this must exceed twice the virial kinetic temperature $K = 3NkT/2$ to seed collapse. Here $N = M_c/\mu m_H$. Hence $2K < |U|$ and the criterion is

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\,\frac{GM_c^2}{r_c} \quad . \tag{2}$$

If we set $r_c = (3M_c/4\pi\rho_c)^{1/3}$, then we arrive at the **Jeans criterion**

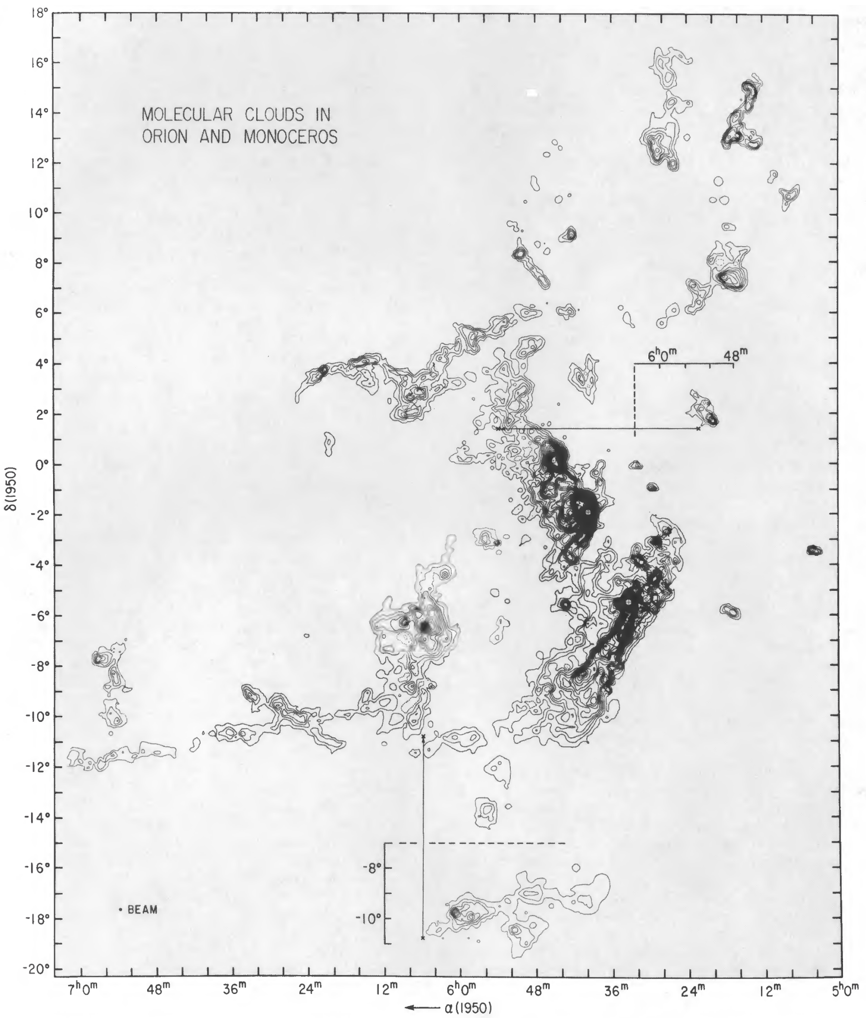
$$M_c > M_{\text{J}} = \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho_c}\right)^{1/2} \tag{3}$$

where M_{J} is called the **Jeans mass**, or equivalently

$$r_c > R_{\text{J}} = \left(\frac{15kT}{4\pi G\mu m_H \rho_c}\right)^{1/2} \tag{4}$$

where R_{J} is called the Jeans length.

Orion-Monoceros Molecular Cloud Complex



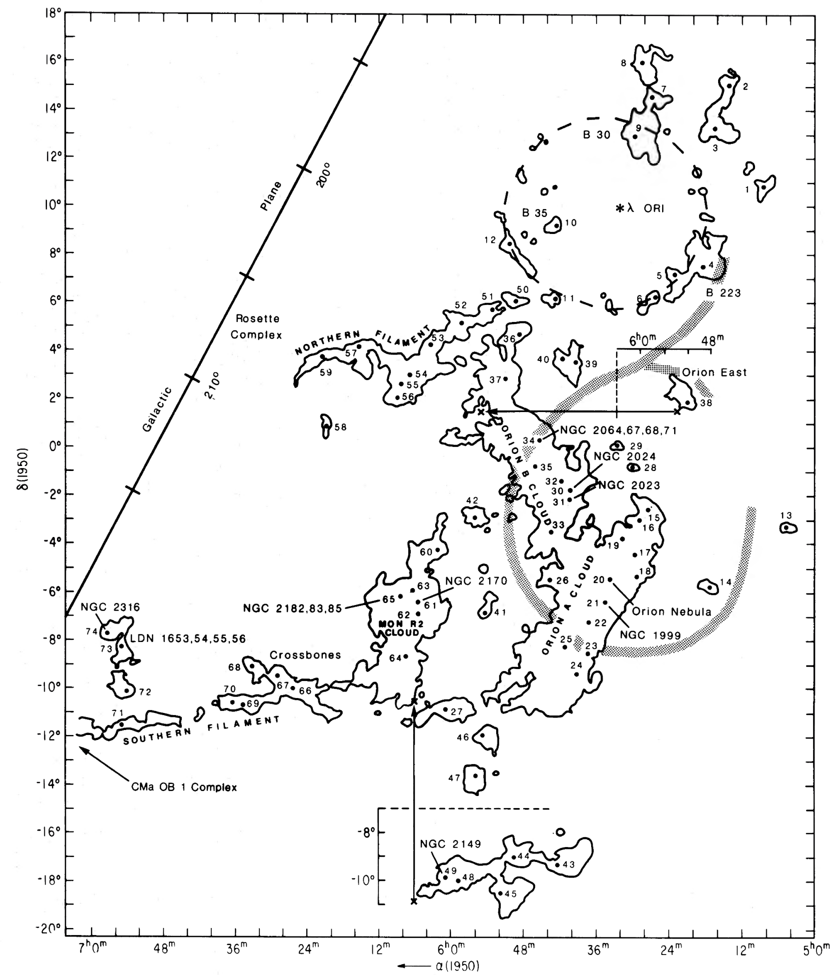


FIG. 3.—Schematic diagram of the molecular clouds: the lowest contour from Fig. 2. Dots with numbers, corresponding to those in Table 1, indicate locations of CO emission peaks. Some NGC numbers indicate the optically prominent objects coincident with CO peaks. The extent of UV emission from Barnard's loop is indicated by the shaded arc (from O'Dell, York, and Henize 1967; Isobe 1973). The dashed line roughly indicates the extent of the λ Ori ring of clouds.

- *Left panel*: CO map. *Right panel*: schematic highlighting cloud cores.
- From: R. Maddalena et al. (1986, ApJ **303**, 375)

$*$ e.g. For diffuse hydrogen clouds of $T \sim 50\,\mathrm{K}$ and $n \sim 500\,\mathrm{cm}^{-3}$, we have $M_{\mathrm{J}} \sim 1500 M_{\odot}$, in excess of the $1 - 100 M_{\odot}$ masses in such clouds; i.e. they are stable against gravitational collapse.

$*$ e.g. Contrast with cores of GMCs, where $T \sim 150\,\mathrm{K}$ but the densities are much higher, $n \sim 10^8\ \mathrm{cm}^{-3}$. These have $M_{\mathrm{J}} \sim 20 M_{\odot}$, implying that *GMC cores are unstable to collapse*, suggesting them as sites of star formation.

Plot: Orion and Monoceros Star Associations

$*$ There is also frequent physical association between GMCs and young O and B main-sequence stars, again indicating a star formation connection.

$*$ Note that the existence of multiple cores in GMCs suggests that stars should commonly form in groups, as is observed.

- Just as in structure formation calculations in the early universe, here there is a power spectrum of density perturbations with collapse seeded once the Jeans criterion is met.

Now we estimate the **collapse timescale** for cores of clouds, assuming that pressure gradients don't influence the infall (i.e. $|dP/dr| \ll GM_c \rho_c / r_c^2$). For optically thin clouds, the temperature remains nearly constant, so that the collapse is *isothermal*. Newton's law for this hydrodynamic system is

$$\frac{d^2 r}{dt^2} \; = \; -\frac{GM}{r^2} \quad , \tag{5}$$

cancelling out mass/density factors and letting r denote the time-dependent radius of the cloud, initially r_c.

(homologous collapse)

Since mass shells do not cross during collapse in this simplified scenario (i.e., there is hydrodynamic turbulence), the enclosed mass is a constant of the motion, so we can set $M = M_c = 4\pi \rho_c r_c^3 / 3$. Multiplying by dr/dt leads to a perfect derivative, so that the ODE integrates to

$$\frac{dr}{dt} \cdot \frac{d^2 r}{dt^2} \; = \; -\frac{GM}{r^2} \cdot \frac{dr}{dt} \quad \Rightarrow \quad \frac{1}{2} \left(\frac{dr}{dt} \right)^2 \; = \; \frac{4\pi}{3} \, G \rho_c r_c^3 \left(\frac{1}{r} - \frac{1}{r_c} \right) \quad . \tag{6}$$

Here we set $dr/dt = 0$ initially, i.e. at $r = r_c$. Then setting $\theta = r/r_c$, and

$$t_{\rm ff} \;=\; \sqrt{\frac{3\pi}{32}\,\frac{1}{G\rho_c}} \quad , \tag{7}$$

we solve

$$\frac{d\theta}{dt} \;=\; -\frac{\pi}{2t_{\rm ff}}\,\sqrt{\frac{1}{\theta}-1} \tag{8}$$

via the substitution $\theta = \cos^2\zeta$ to yield (*plot* $\rho = \rho(t)$)

$$-2\sin\zeta\cos\zeta\,\frac{d\zeta}{dt} \;=\; -\frac{\pi}{2t_{\rm ff}}\,\frac{\sin\zeta}{\cos\zeta} \quad\Rightarrow\quad \frac{\pi}{2t_{\rm ff}}\,dt \;=\; (1+\cos 2\zeta)\,d\zeta \tag{9}$$

which integrates to

$$\zeta + \frac{1}{2}\sin 2\zeta \;=\; \frac{\pi t}{2t_{\rm ff}} \quad . \tag{10}$$

This cycloidal solution has an analogy of bouncing closed universes in a closed Newtonian cosmology.

• Since initially $\zeta = 0$ corresponding to $\theta = 1$, the radius of the sphere reaches zero when $\zeta = \pi/2$, so that $t_{\rm ff}$ is the **free-fall timescale**. This could be established using dimensional analysis only under isothermal assumptions.

• e.g. For cores of GMCs at $n \sim 10^8$ cm^{-3} that satisfy the Jeans criterion, $\rho_c \sim 2\times 10^{-16}$ g cm^{-3}, and we arrive at $t_{\rm ff} \sim 5000$ years; *once collapse starts, it is very quick*, i.e. inevitable.

• **More massive clouds collapse faster**, for given radius.

• Now let us explore the impact of temperature evolution. If the collapse is truly adiabatic, then $T \propto \rho^{\gamma-1}$, where $\gamma = C_P/C_V$ is the ratio of specific heats. We thus deduce that as collapse proceeds and the density rises, so does T, providing hydrodynamic pressure support. Then, using Eq. (3),

$$M_{\rm J} \;\propto\; \rho^{(3\gamma-4)/2} \quad , \tag{11}$$

i.e. $M_{\rm J} \propto \rho^{1/2}$ for $\gamma = 5/3$. Hence the Jeans mass increases during collapse to infinity for an ideal gas. (for a non-relativistic, ideal gas)

Obviously, this is an oversimplification, since it would imply all collapses *would cease due to pressure support* if they evolve into an adiabatic phase.

The physical resolution of this paradox is that the heating during collapse is curtailed when the gas becomes dense enough to become radiative, so that the effective γ approaches $4/3$, pressure is relieved and *the Jeans mass again becomes independent of density.*

$\boxed{\textbf{Plot:}}$ Jeans Mass Evolution during Free-Fall

• Essentially the fragmented mass corresponds to the minimum mass at the point when the collapse *transitions from isothermal to adiabatically radiative* character, bypassing an adiabatic, but non-radiative phase that would halt collapse. We can estimate this minimum mass as follows.

• The energy liberated in the collapse is clearly $\Delta E \approx 3GM_{\rm J}^2/(10R_{\rm J})$. Averaging this over the collapse time $t_{\rm ff} = \sqrt{3\pi/(32\,G\rho_{\rm J})} \propto R_{\rm J}^{3/2}/M_{\rm J}^{1/2}$ gives a gravitational luminosity (which could be tapped by radiative processes) of

G^(3/2)

$$L_{\rm ff} \;\sim\; \frac{\Delta E}{t_{\rm ff}} \;\sim\; \frac{GM_{\rm J}^2}{R_{\rm J}}\,\frac{M_{\rm J}^{1/2}}{R_{\rm J}^{3/2}} \;=\; G^{3/2}\left(\frac{M_{\rm J}}{R_{\rm J}}\right)^{5/2} \quad . \tag{12}$$

This can be set equal to a radiative luminosity of $4\pi R_{\rm J}^2\sigma T^4$ times a radiative efficiency factor ϵ, signalling the epoch when adiabatic evolution starts, i.e. the minimum mass is achieved.

$$G^{3/2}\left(\frac{M_{\rm J}}{R_{\rm J}}\right)^{5/2} \;\sim\; L_{\rm ff} \;\sim\; L_{\rm rad} \;\approx\; 4\pi\epsilon M_{\rm J}^2\sigma T^4\left(\frac{M_{\rm J}}{R_{\rm J}}\right)^{-2} \quad . \tag{13}$$

Eqs. (3) and (4) can effect elimination of $R_{\rm J}$ via $M_{\rm J}/R_{\rm J} = 5kT/(G\mu m_H)$, yielding an estimate to the minimum Jeans mass realized in collapses:

$$M_{\rm J}|_{\rm min} \;=\; 0.03\,T^{1/4}\,M_\odot \quad , \tag{14}$$

for temperatures in Kelvin and $\epsilon = 1$ and $\mu = 1$.

∗ This sets the rough lower bound to the mass scale for **protostar** formation: with $T \sim 10\,$K we get $M_{\rm J}|_{\rm min} \sim 0.05 M_\odot$. No main sequence stars are observed with lower masses! None are!

JEANS MASS EVOLUTION DURING FREE-FALL