3 Stellar Pulsation

A profound characteristic of the late stages of evolution of stars more massive than the sun is the phenomenon of **stellar pulsation**.

• It is the marker of the epoch just prior to the death throes of stars, and an indicator of large mass loss through stellar winds.

3.1 Variable Stars

Variable stars have been known for 4 centuries. The initial discovery event was o Ceti, which was later renamed **Mira**, or miraculous. By 1660, the 11 month period of its variability had been established.

C & O, Sec. 14.1

Plot: Light Curve of Mira

∗ On top of the principal period, there are a multitude of harmonics.

• Mira is a **pulsating star** that is an example of a **long-period variable** (LPV), which typically have periods of 100–700 days. It has an irregular light curve.

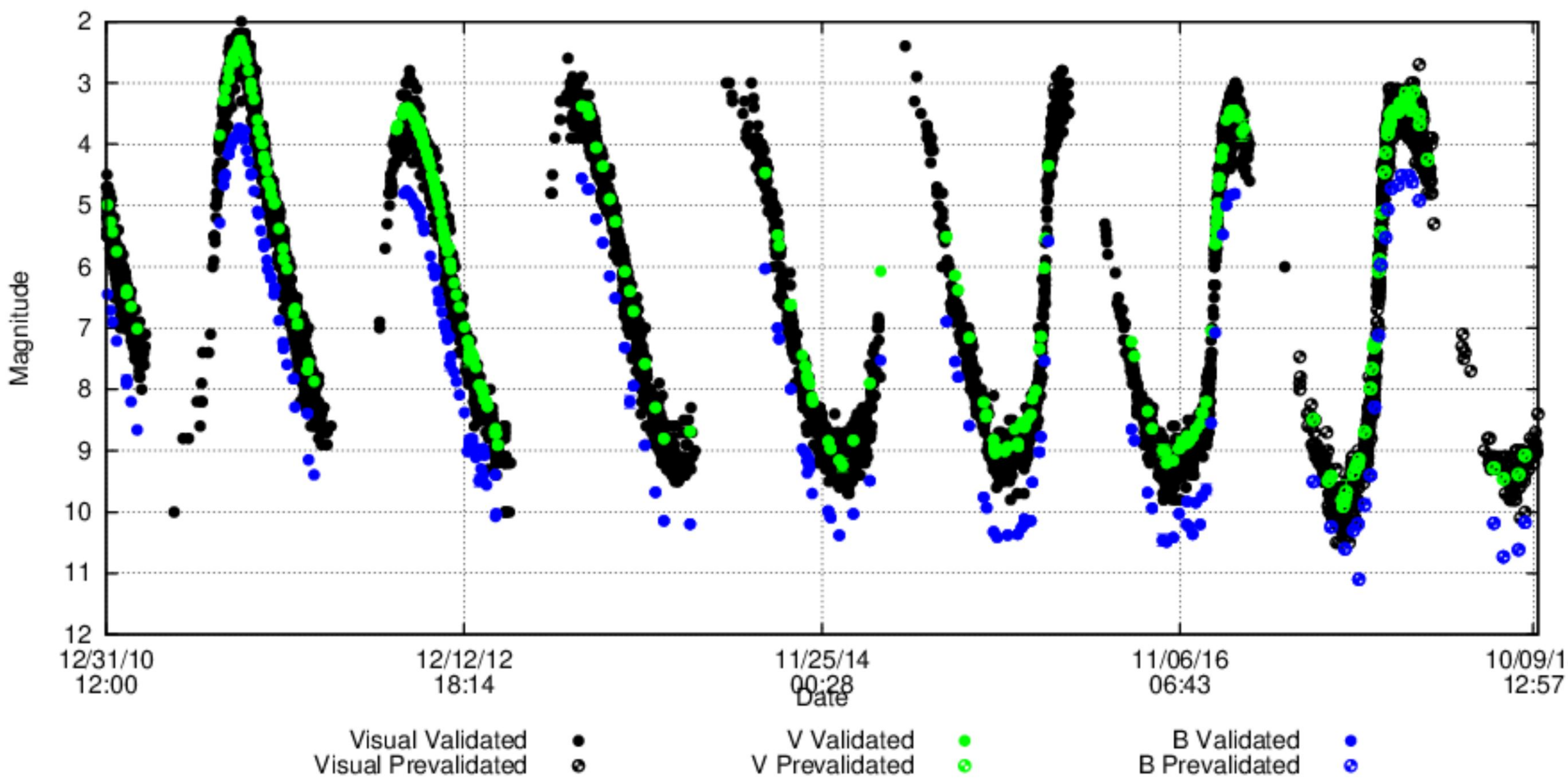
• It was another century before another pulsating star was discovered, δ Cephei, a classical **Cepheid variable** (1784, Goodricke). This star was less dramatic in its variations.

• Cepheids are powerful tools for distance calibration for nearby extragalactic scales. Henrietta Leavitt (worked for Pickering) catalogued 2400 Cepheids in the SMC, a fixed distance locale.

• Leavitt noticed a strong correlation between the absolute magnitude of classical Cepheids and their oscillation periods. Eventually, the normalization of this was calibrated using Polaris, the North Star at $d = 200\,\text{pc}$.

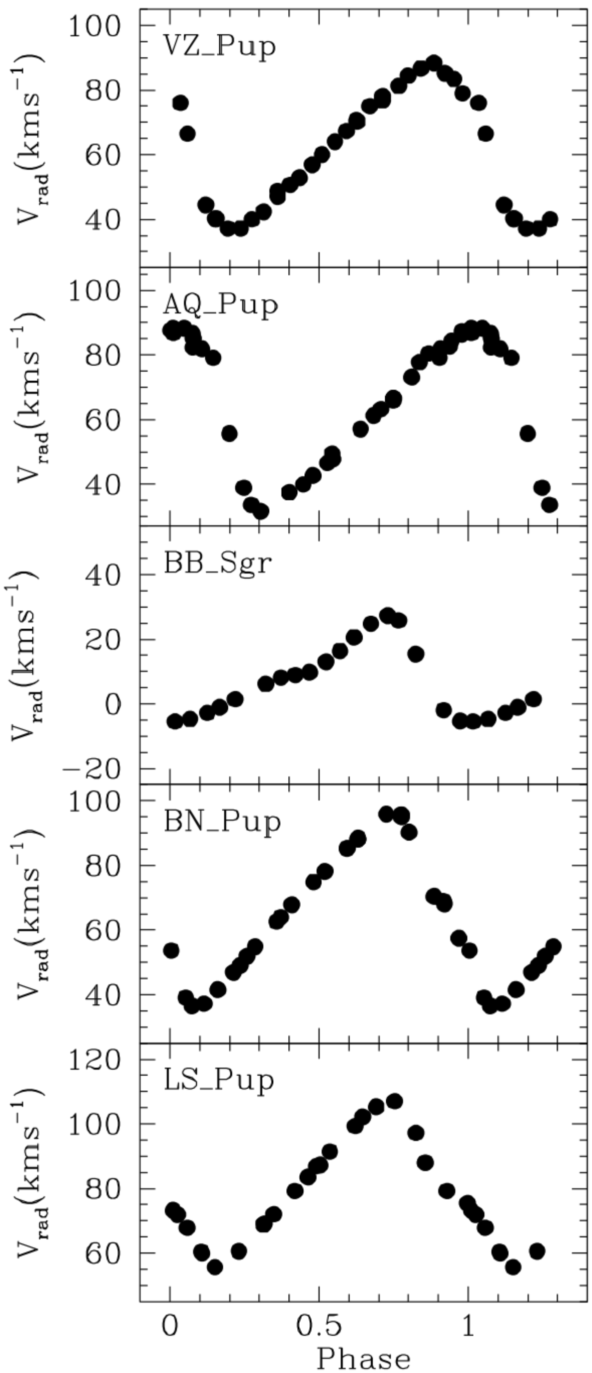
Plot: Period-Luminosity Relation for Classical Cepheids

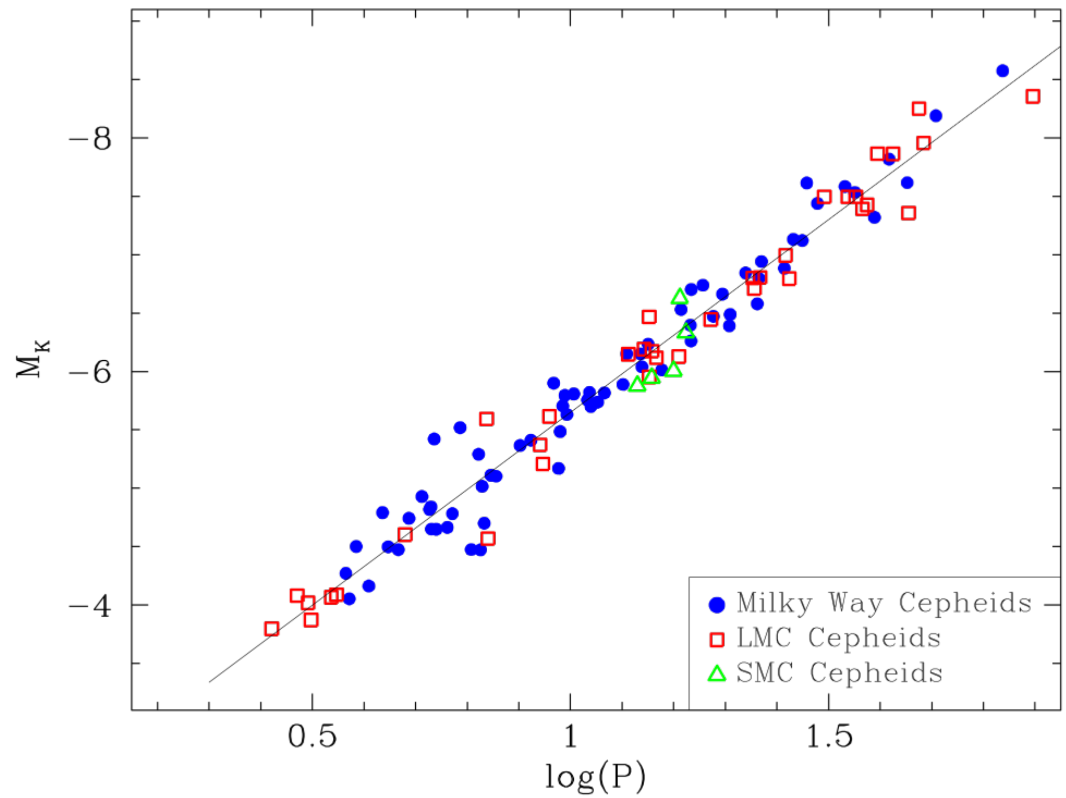
Light Curve of Mira



- Compiled from the database of the American Association of Variable Star Observers. www.aavso.org

Cepheid Period-Luminosity Relation





- *Left*: radial velocity curves for Milky Way Cepheids.
- *Top*: The Period-Luminosity relation in the K-band for the complete sample of Milky Way, LMC and SMC Cepheids having IRSB-determined distances.
- Storm et al. A&A (2011).

• The correlation is now known as the Cepheid's **period-luminosity relation**, which can be used to measure the distance to any Cepheid:

$$\boxed{\log_{10}\frac{\langle L\rangle}{L_\odot} \;=\; 1.15\log_{10}\Pi^d + 2.47 \quad ,} \tag{6}$$

where Π^d is the period in days. In observer's units this can be expressed as:

$$M_{\langle V\rangle} \;=\; -2.80\log_{10}\Pi^d - 1.43 \quad . \tag{7}$$

• Cepheids are used as **standard candles** to measure extragalactic distances. They are useful because, in addition to this relationship, they are *large and bright, and can be seen at large distances.*

• Variable stars lie in a confined **instability strip** in the Hertzsprung-Russell diagram, ranging from LPVs at the top down to δ Scuti stars near the main sequence.

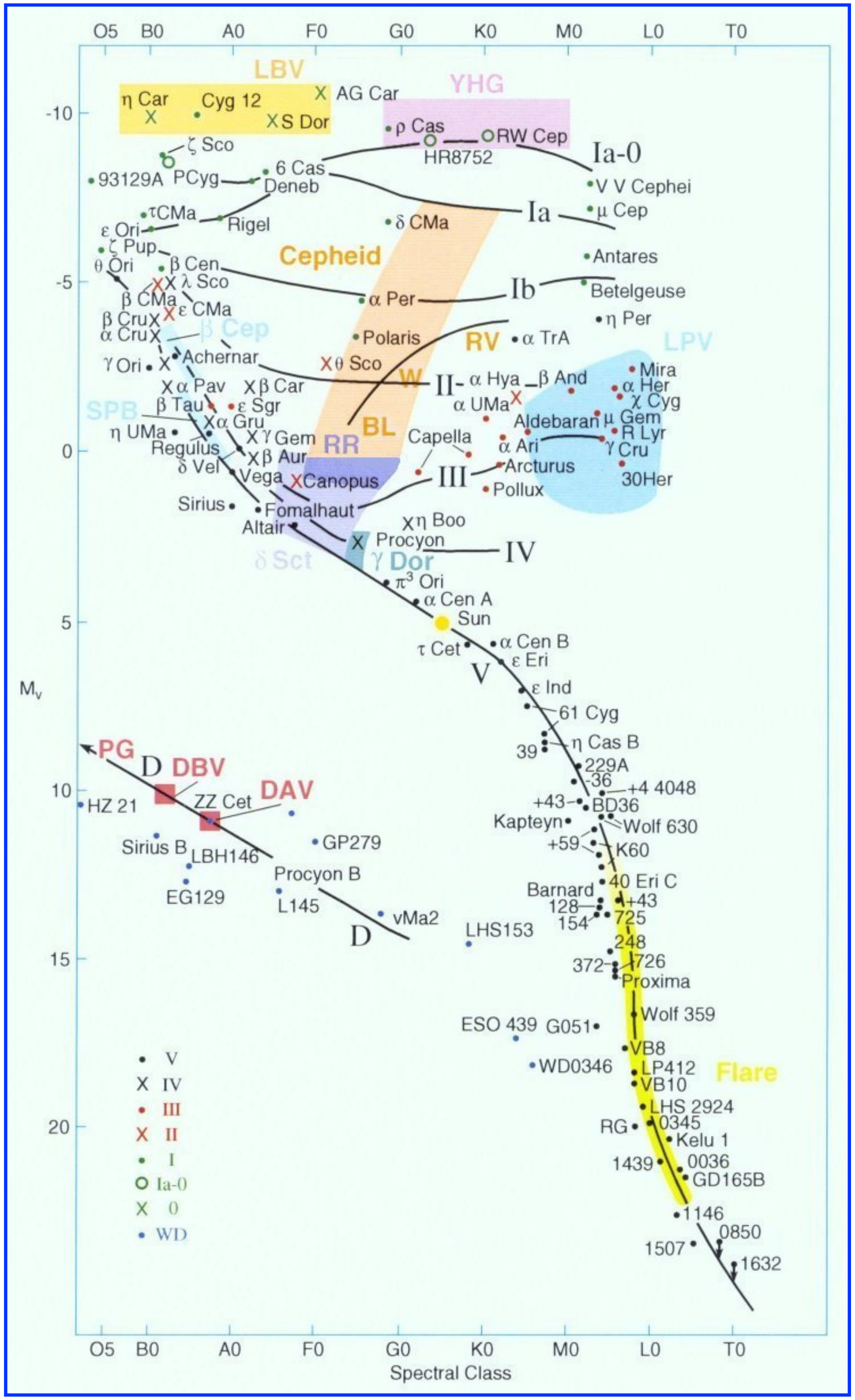
Plot: Pulsating Stars in the H-R Diagram

∗ As one progresses down this strip, the stars become denser, and their periods shorter. This trend is a pointer to the physics of pulsation.

• Early models argued in favor of Keplerian tidal effects from binary companions as the origin of the pulsations.

∗ Shapley (1914) pointed out that the short (~ 3 day) period variables would swallow up their companions if they were in Keplerian orbits:

$$\frac{a^3}{P^2} \;=\; \frac{(1\mathrm{AU})^3}{(1\mathrm{yr})^2} \quad\Rightarrow\quad a \;\approx\; \left(\frac{3}{365}\right)^{2/3}\mathrm{AU} \;\approx\; 6.9\times 10^{11}\mathrm{cm} \;\approx\; 8.75\,R_\odot \quad . \tag{8}$$



HR Diagram: Instability Strip

- HR diagram with many nearby stars identified, Morgan-Keenan luminosity classes labelled, and the instability strip highlighted. [Credit: J. B. Kaler, *The Cambridge Encyclopedia of Stars*]

4 The Physics of Stellar Pulsation

The pulsation of a star must correspond to a change in density, and therefore also pressure in the hydrostatic coupling. Fluctuations of these quantities are **sound waves**, and so the Cepheid period-luminosity relation should be governed by sound wave physics.

C & O, Sec. 14.3

• Eddington (1918) came up with a firm theoretical framework for stellar pulsation based on stellar structure and radial sound waves, eliminating the binary hypothesis from discussion.

• The speed of sound in an adiabatically compressible gas is given by $c_s = \sqrt{\partial P/\partial\rho} = \sqrt{\gamma P/\rho}$ for $P \propto \rho^\gamma$. The equation of hydrostatic equilibrium can be written, for uniform density ρ, as

$$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2} = -\frac{4\pi}{3}\,G\rho^2 r \quad . \tag{9}$$

Using the boundary condition $P = 0$ at the surface, this integrates to a pressure profile of

$$P(r) = \frac{2\pi}{3}\,G\rho^2(R^2 - r^2) \quad . \tag{10}$$

• For fluctuations across the diameter of the star, standing sound waves (in a sort of waveguide or cavity) will possess a fundamental period of

$$\Pi \approx 2\int_0^R \frac{dr}{c_s} = \frac{2}{\sqrt{2\pi\gamma G\rho/3}}\int_0^R \frac{dr}{\sqrt{R^2 - r^2}} = \sqrt{\frac{3\pi}{2\gamma G\rho}} \quad . \tag{11}$$

If we take $M = 5M_\odot$ and $R = 50R_\odot$ for typical Cepheid, then $\Pi \sim 10\,$days, as is observed.

∗ This roughly defines the **radial oscillation** fundamental, and there are higher harmonics that contribute to light curve irregularity.

∗ Most variables are driven by radial oscillations.

∗ Yet there are also non-radial modes of oscillation, and these connect to sound waves propagating in shell or surface layers. Harmonic frequencies then connect to spherical harmonic "quantum numbers."

$*$ The study of such oscillations is the field of **asteroseismology**, wherein pulsation signals in Fourier space provide probes of density stratification. Even more interestingly, rotation broadens the pulsation period to a QPO so that rotation periods can be gleaned from seismic studies. An example is provided by Kepler's view of host stars for exoplanets.

$*$ A similar diagnostic is afforded by **quasi-periodic oscillations** (QPOs) in the X-ray light curves of neutron stars.

[*Reading Assignment: Non-radial Stellar Pulsations: Sec. 14.4*]

• Notice that $\Pi \sim t_{\rm ff}$, a consequence of dimensional analysis for self-gravitating systems of uniform density.

• Now consider the instability strip in the H-R diagram. If we assume that temperature adjustments during oscillations are small, then $L \propto R^2 \propto \rho^{-2/3}$ for fixed stellar mass.

• Since $\rho \propto \Pi^{-2}$ for sound waves, one can then set $L \propto \Pi^{4/3}$, or

$$\log_{10} \frac{\langle L \rangle}{L_\odot} \; = \; \frac{4}{3} \log_{10} \Pi^d + \text{const} \quad , \tag{12}$$

Modest adjustments in temperature during the oscillations reconcile the form in Eq. (12) with the observed correlation in Eq. (6).

• Eddington proposed a theory of cyclic heating in a valve mechanism, that Kippenhahn and others connected to regions of partial ionization in the outer layers of stars, as the origin of the instability that drives the sound waves.

$*$ the instability is an interplay between ionization, opacity and heating that is known as the κ**-mechanism**.