• **Giant molecular clouds** (GMCs) are enormous complexes of dust and gas, typically with radii $r \sim 50\,\mathrm{pc}$, with $T \sim 20 - 30\,\mathrm{K}$ and number densities $n \sim 100 - 300\ \mathrm{cm}^{-3}$. Residing within such clouds are **cores** (*draw schematic*) of radii $r \sim 0.1 - 1\,\mathrm{pc}$ and $T \sim 100 - 200\,\mathrm{K}$ and $n \sim 10^7 - 10^9\ \mathrm{cm}^{-3}$.

Plot: Orion and Monoceros Molecular Clouds

∗ The existence of **fragmentation** into such cores, with masses typically around $10 - 1000 M_\odot$, indicates that they are the sites of star formation.

∗ Thousands of GMCs are known in our galaxy, mostly in the spiral arms.

1.1 Gravitational Collapse: Jeans Criterion

If molecular clouds are the sites for star formation, what conditions must guarantee collapse? Obviously, gravity must outweigh kinetic motions. In the following, neglect rotation and the influence of magnetic fields. **C & O, Sec. 12.2**

• The gravitational potential of a spherical cloud of uniform density and mass M_c and radius r_c is

$$U \sim -\frac{3}{5}\,\frac{GM_c^2}{r_c} \quad , \tag{1}$$

and this must exceed twice the virial kinetic temperature $K = 3NkT/2$ to seed collapse. Here $N = M_c/\mu m_H$. Hence $2K < |U|$ and the criterion is

$$\frac{3M_c kT}{\mu m_H} < \frac{3}{5}\,\frac{GM_c^2}{r_c} \quad . \tag{2}$$

If we set $r_c = (3M_c/4\pi\rho_c)^{1/3}$, then we arrive at the **Jeans criterion**

$$M_c > M_{\mathrm{J}} = \left(\frac{5kT}{G\mu m_H}\right)^{3/2} \left(\frac{3}{4\pi\rho_c}\right)^{1/2} \tag{3}$$

where M_{J} is called the **Jeans mass**, or equivalently

$$r_c > R_{\mathrm{J}} = \left(\frac{15kT}{4\pi G\mu m_H \rho_c}\right)^{1/2} \tag{4}$$

where R_{J} is called the Jeans length.

$*$ e.g. For diffuse hydrogen clouds of $T \sim 50\,\mathrm{K}$ and $n \sim 500\,\mathrm{cm}^{-3}$, we have $M_{\mathrm{J}} \sim 1500 M_{\odot}$, in excess of the $1 - 100 M_{\odot}$ masses in such clouds; i.e. they are stable against gravitational collapse.

$*$ e.g. Contrast with cores of GMCs, where $T \sim 150\,\mathrm{K}$ but the densities are much higher, $n \sim 10^8\ \mathrm{cm}^{-3}$. These have $M_{\mathrm{J}} \sim 20 M_{\odot}$, implying that *GMC cores are unstable to collapse*, suggesting them as sites of star formation.

Plot: Orion and Monoceros Star Associations

$*$ There is also frequent physical association between GMCs and young O and B main-sequence stars, again indicating a star formation connection.

$*$ Note that the existence of multiple cores in GMCs suggests that stars should commonly form in groups, as is observed.

- Just as in structure formation calculations in the early universe, here there is a power spectrum of density perturbations with collapse seeded once the Jeans criterion is met.

Now we estimate the **collapse timescale** for cores of clouds, assuming that pressure gradients don't influence the infall (i.e. $|dP/dr| \ll GM_c \rho_c / r_c^2$). For optically thin clouds, the temperature remains nearly constant, so that the collapse is *isothermal*. Newton's law for this hydrodynamic system is

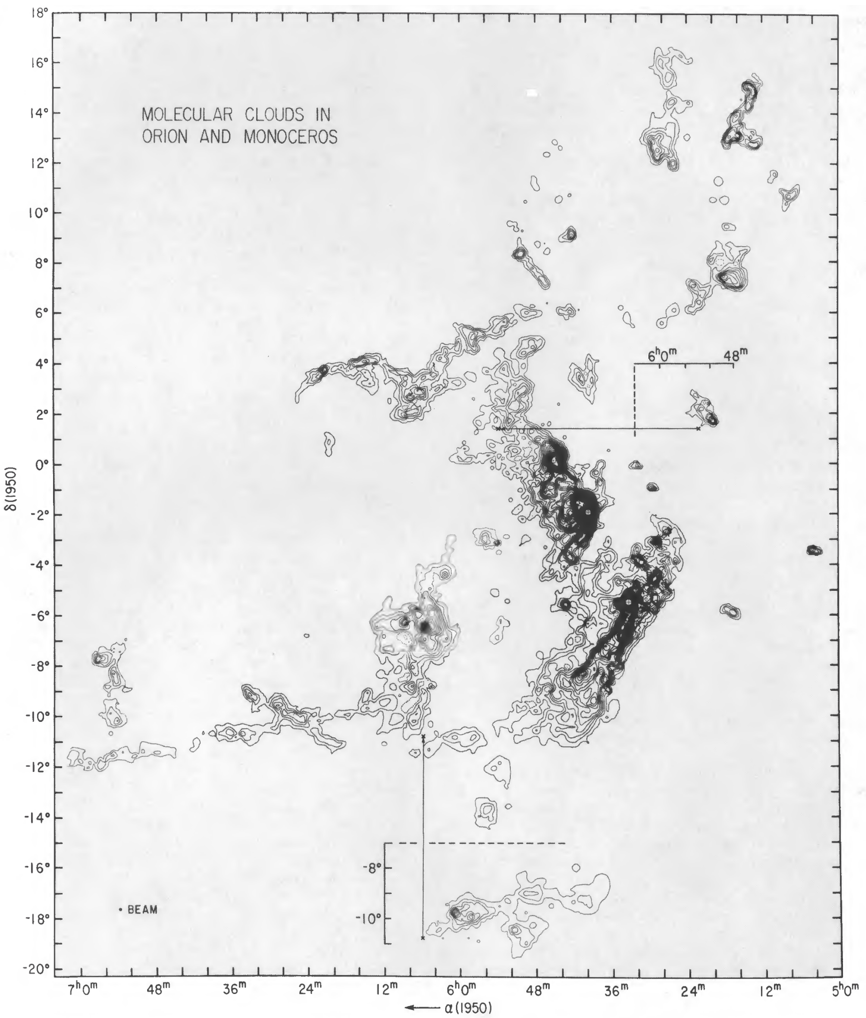
$$\frac{d^2 r}{dt^2} \;=\; -\frac{GM}{r^2} \quad , \tag{5}$$

cancelling out mass/density factors and letting r denote the time-dependent radius of the cloud, initially r_c.

Since mass shells do not cross during collapse in this simplified scenario (i.e., there is hydrodynamic turbulence), the enclosed mass is a constant of the motion, so we can set $M = M_c = 4\pi \rho_c r_c^3 / 3$. Multiplying by dr/dt leads to a perfect derivative, so that the ODE integrates to

$$\frac{dr}{dt} \cdot \frac{d^2 r}{dt^2} \;=\; -\frac{GM}{r^2} \cdot \frac{dr}{dt} \quad \Rightarrow \quad \frac{1}{2} \left(\frac{dr}{dt} \right)^2 \;=\; \frac{4\pi}{3}\, G \rho_c r_c^3 \left(\frac{1}{r} - \frac{1}{r_c} \right) \quad . \tag{6}$$

Orion-Monoceros Molecular Cloud Complex



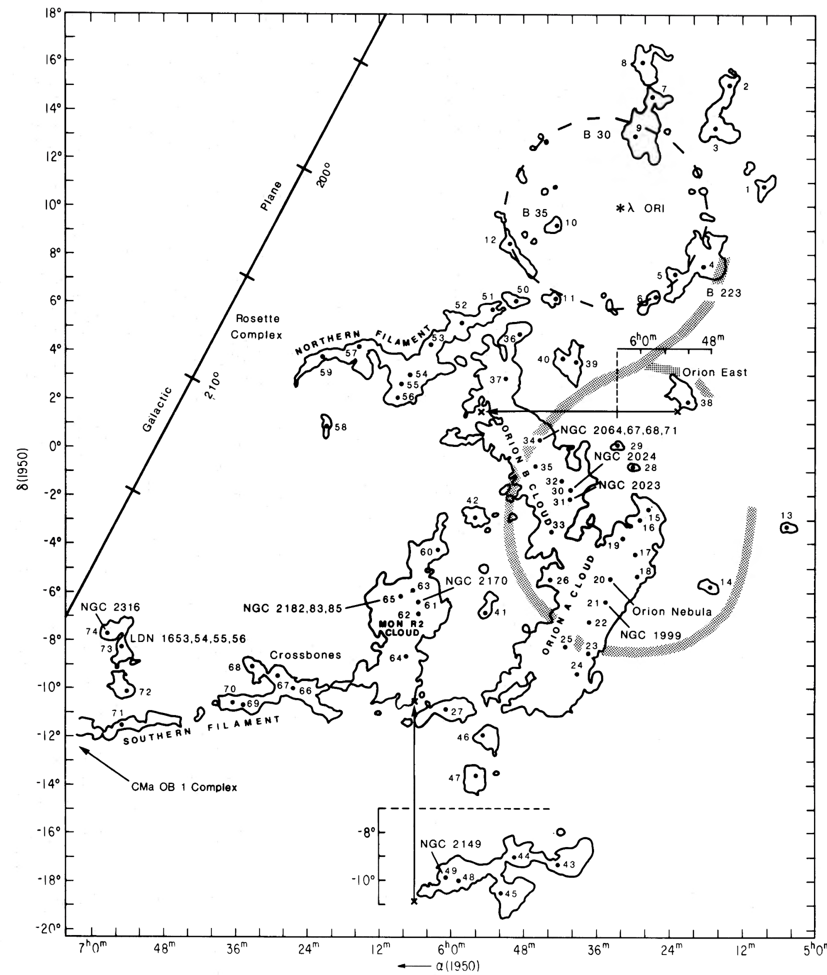


FIG. 3.—Schematic diagram of the molecular clouds: the lowest contour from Fig. 2. Dots with numbers, corresponding to those in Table 1, indicate locations of CO emission peaks. Some NGC numbers indicate the optically prominent objects coincident with CO peaks. The extent of UV emission from Barnard's loop is indicated by the shaded arc (from O'Dell, York, and Henize 1967; Isobe 1973). The dashed line roughly indicates the extent of the λ Ori ring of clouds.

- *Left panel*: CO map. *Right panel*: schematic highlighting cloud cores.
- From: R. Maddalena et al. (1986, ApJ **303**, 375)

Here we set $dr/dt = 0$ initially, i.e. at $r = r_c$. Then setting $\theta = r/r_c$, and

$$t_{\rm ff} \;=\; \sqrt{\frac{3\pi}{32}\,\frac{1}{G\rho_c}} \quad , \tag{7}$$

we solve

$$\frac{d\theta}{dt} \;=\; -\frac{\pi}{2t_{\rm ff}}\,\sqrt{\frac{1}{\theta} - 1} \tag{8}$$

via the substitution $\theta = \cos^2\zeta$ to yield (*plot* $\rho = \rho(t)$)

$$-2\sin\zeta\cos\zeta\,\frac{d\zeta}{dt} \;=\; -\frac{\pi}{2t_{\rm ff}}\,\frac{\sin\zeta}{\cos\zeta} \quad\Rightarrow\quad \frac{\pi}{2t_{\rm ff}}\,dt \;=\; (1+\cos 2\zeta)\,d\zeta \tag{9}$$

which integrates to

$$\zeta + \frac{1}{2}\,\sin 2\zeta \;=\; \frac{\pi t}{2t_{\rm ff}} \quad . \tag{10}$$

This cycloidal solution has an analogy of bouncing closed universes in a closed Newtonian cosmology.

• Since initially $\zeta = 0$ corresponding to $\theta = 1$, the radius of the sphere reaches zero when $\zeta = \pi/2$, so that $t_{\rm ff}$ is the **free-fall timescale**. This could be established using dimensional analysis only under isothermal assumptions.

• e.g. For cores of GMCs at $n \sim 10^8$ cm^{-3} that satisfy the Jeans criterion, $\rho_c \sim 2\times 10^{-16}$ g cm^{-3}, and we arrive at $t_{\rm ff} \sim 5000$ years; *once collapse starts, it is very quick*, i.e. inevitable.

• **More massive clouds collapse faster**, for given radius.

• Now let us explore the impact of temperature evolution. If the collapse is truly adiabatic, then $T \propto \rho^{\gamma-1}$, where $\gamma = C_P/C_V$ is the ratio of specific heats. We thus deduce that as collapse proceeds and the density rises, so does T, providing hydrodynamic pressure support. Then, using Eq. (3),

$$M_{\rm J} \;\propto\; \rho^{(3\gamma-4)/2} \quad , \tag{11}$$

i.e. $M_{\rm J} \propto \rho^{1/2}$ for $\gamma = 5/3$. Hence the Jeans mass increases during collapse to infinity for an ideal gas.

Obviously, this is an oversimplification, since it would imply all collapses *would cease due to pressure support* if they evolve into an adiabatic phase.

The physical resolution of this paradox is that the heating during collapse is curtailed when the gas becomes dense enough to become radiative, so that the effective γ approaches $4/3$, pressure is relieved and *the Jeans mass again becomes independent of density.*

$\boxed{\textbf{Plot:}}$ Jeans Mass Evolution during Free-Fall

• Essentially the fragmented mass corresponds to the minimum mass at the point when the collapse *transitions from isothermal to adiabatically radiative* character, bypassing an adiabatic, but non-radiative phase that would halt collapse. We can estimate this minimum mass as follows.

• The energy liberated in the collapse is clearly $\Delta E \approx 3GM_{\rm J}^2/(10R_{\rm J})$. Averaging this over the collapse time $t_{\rm ff} = \sqrt{3\pi/(32\, G\rho_{\rm J})} \propto R_{\rm J}^{3/2}/M_{\rm J}^{1/2}$ gives a gravitational luminosity (which could be tapped by radiative processes) of

$$L_{\rm ff} \sim \frac{\Delta E}{t_{\rm ff}} \sim \frac{GM_{\rm J}^2}{R_{\rm J}} \frac{M_{\rm J}^{1/2}}{R_{\rm J}^{3/2}} = G^{3/2} \left(\frac{M_{\rm J}}{R_{\rm J}} \right)^{5/2} \quad . \tag{12}$$

This can be set equal to a radiative luminosity of $4\pi R_{\rm J}^2 \sigma T^4$ times a radiative efficiency factor ϵ, signalling the epoch when adiabatic evolution starts, i.e. the minimum mass is achieved.

$$G^{3/2} \left(\frac{M_{\rm J}}{R_{\rm J}} \right)^{5/2} \sim L_{\rm ff} \sim L_{\rm rad} \approx 4\pi\epsilon M_{\rm J}^2 \sigma T^4 \left(\frac{M_{\rm J}}{R_{\rm J}} \right)^{-2} \quad . \tag{13}$$

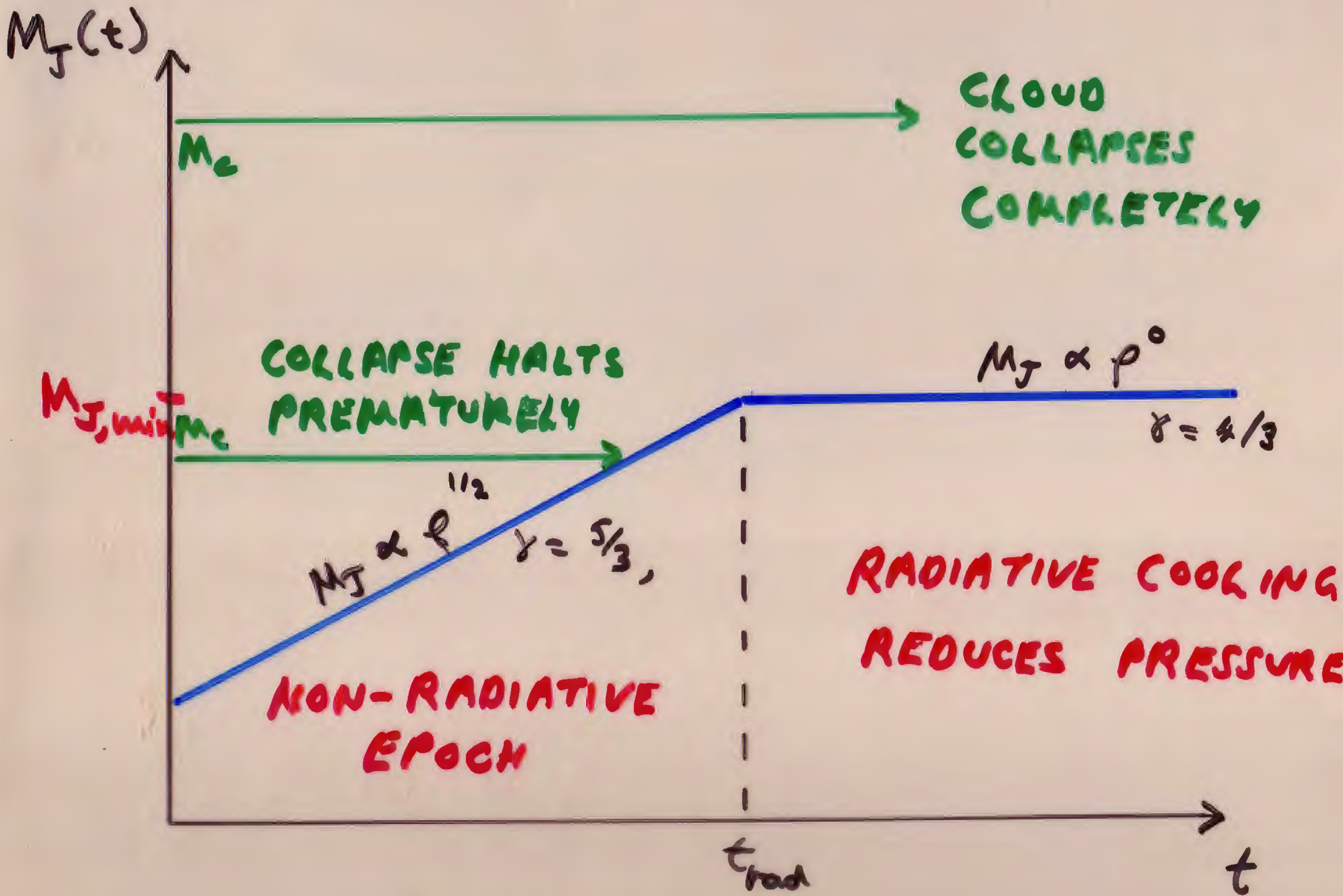
Eqs. (3) and (4) can effect elimination of $R_{\rm J}$ via $M_{\rm J}/R_{\rm J} = 5kT/(G\mu m_H)$, yielding an estimate to the minimum Jeans mass realized in collapses:

$$M_{\rm J}\vert_{\rm min} = 0.03\, T^{1/4}\, M_\odot \quad , \tag{14}$$

for temperatures in Kelvin and $\epsilon = 1$ and $\mu = 1$.

∗ This sets the rough lower bound to the mass scale for **protostar** formation: $\underline{\text{with } T \sim 10\,\text{K we get } M_{\rm J}\vert_{\rm min} \sim 0.05 M_\odot}$. No main sequence stars are observed with lower masses!

JEANS MASS EVOLUTION DURING FREE-FALL



1.2 Pre-Main Sequence Stars

• Collapse starts slowly with a rising temperature and luminosity and then enters a phase where it accelerates at virtually constant luminosity; i.e., $T_e \propto R^{-1/2}$ approximately. Gravitational potential energy seeds the heating. **C & O, Sec. 12.3**

$\boxed{\textbf{Plot:}}$ Cloud Collapse Evolution and Timelines

• After $\sim 10^5$ years (i.e. roughly a Kelvin-Helmholtz timescale) for a solar mass protostar, the **Hayashi** track on the H-R diagram is followed, when the effective T is constant, L and R decline. The path of the track is influenced by cloud rotation and magnetic field pressure buoyancy.

• A well-defined and *highly-convective* core develops in this epoch, and the protostar has almost reached the ZAMS.

$\boxed{\textbf{Plot:}}$ Hayashi Pre-Main Sequence Tracks and Timelines

• Note: **More massive clouds collapse faster**.

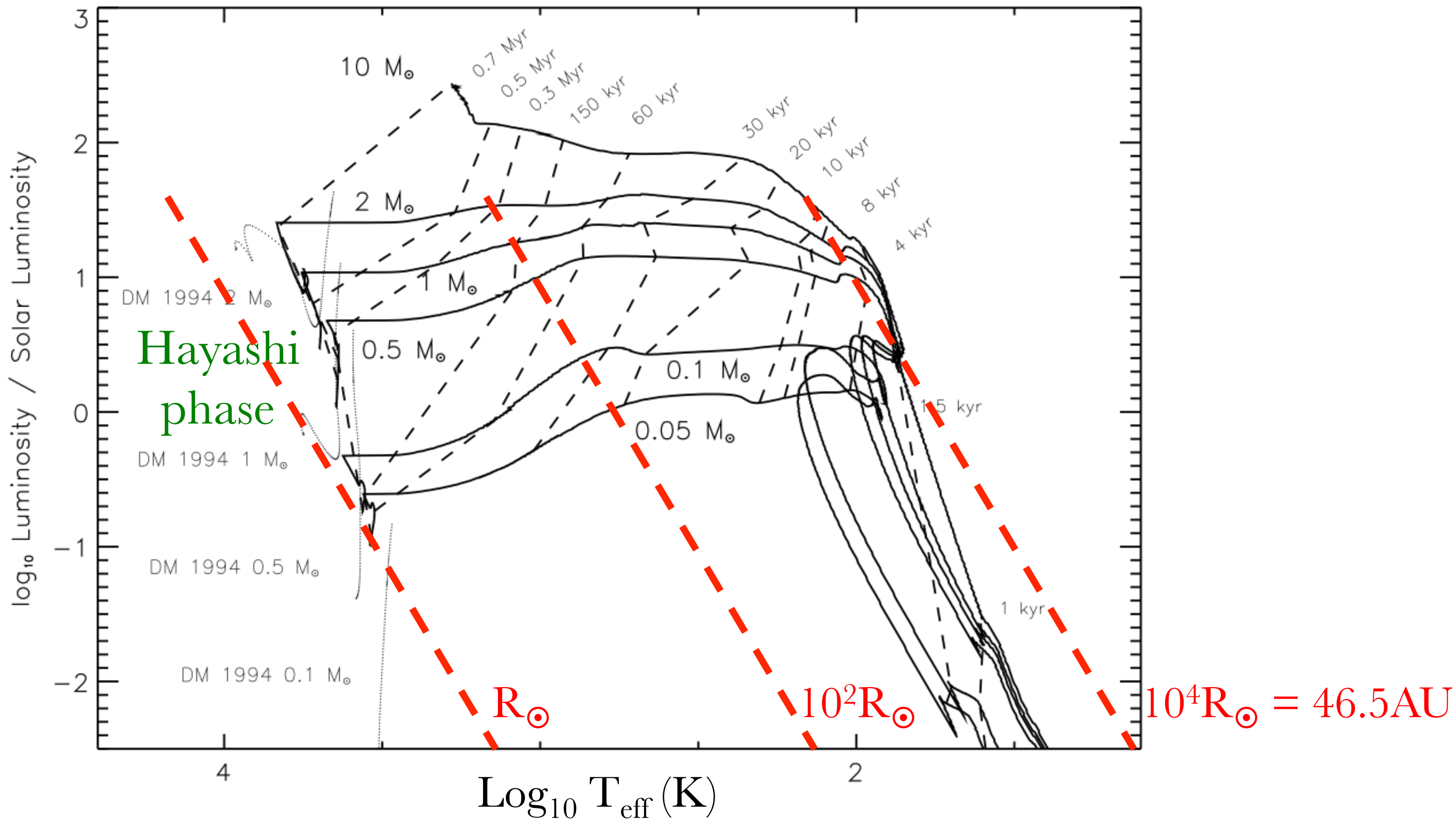
[*Reading Assignment: Hayashi and pre-main sequence tracks, Sec. 12.3*]

• The power spectrum of GMC masses in a turbulent ISM indicates a predominance of lower masses, dictating that most stars form as dwarfs.

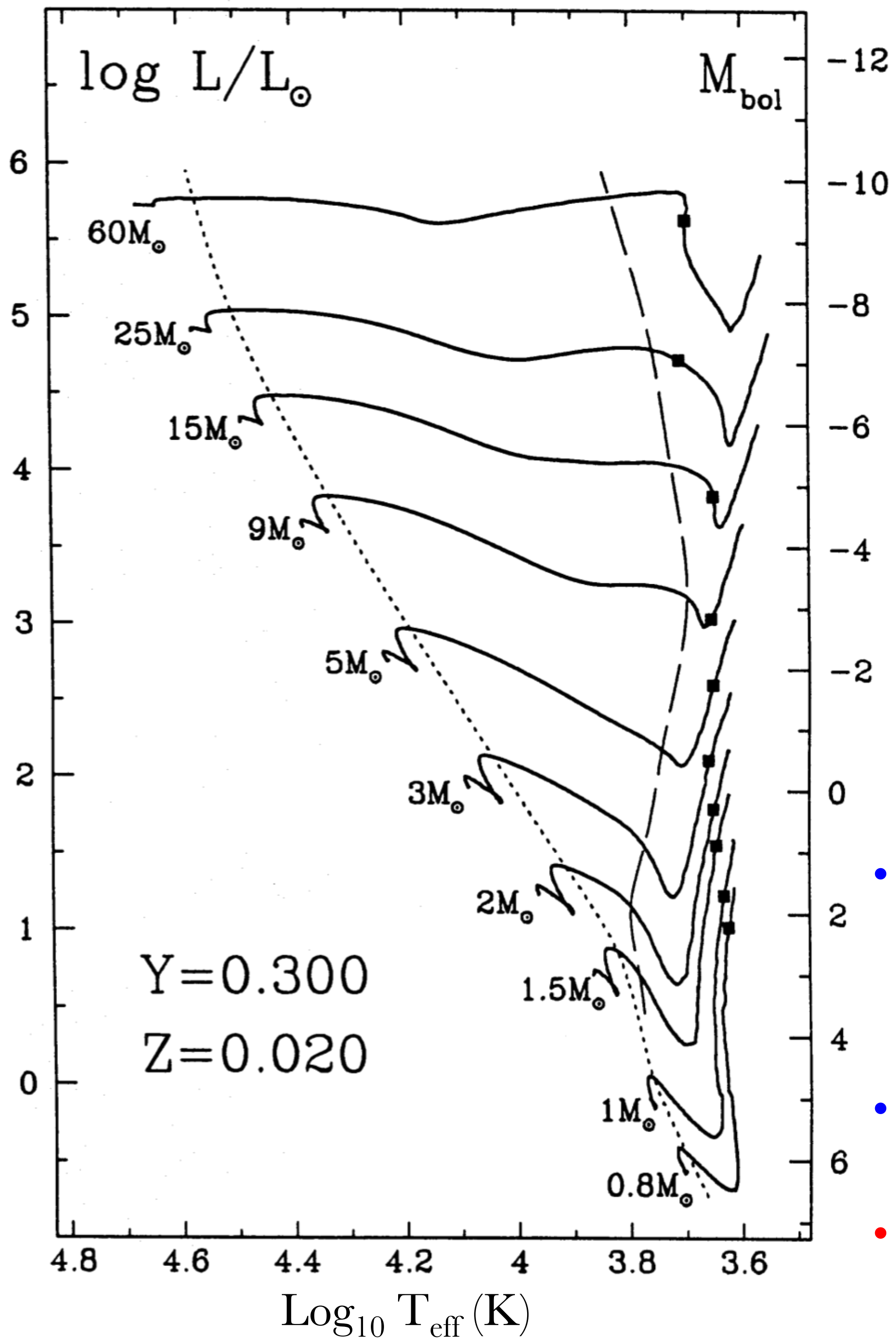
$\boxed{\textbf{Plot:}}$ Initial Mass Function (IMF)

• Few stars are massive, generally O and B spectral types. As they are hot, they possess plenty of ionizing UV radiation.

Cloud Collapse Evolution



- Pre-Hayashi phase evolutionary tracks in the Hertzsprung-Russell diagram for the collapse and early pre-main sequence evolution of 0.05, 0.1, 0.5,1, 2 and 10 $M_\odot$ cloud fragments (full lines). Dashed lines indicate isochrones for the collapse tracks, labeled with the respective ages. Zero age is defined here as the moment when the respective cloud fragment becomes optically thick and the interior is thermally locked as the first photosphere forms. From Wuchterl & Tscharnuter (A&A **398**, 1081, 2013).

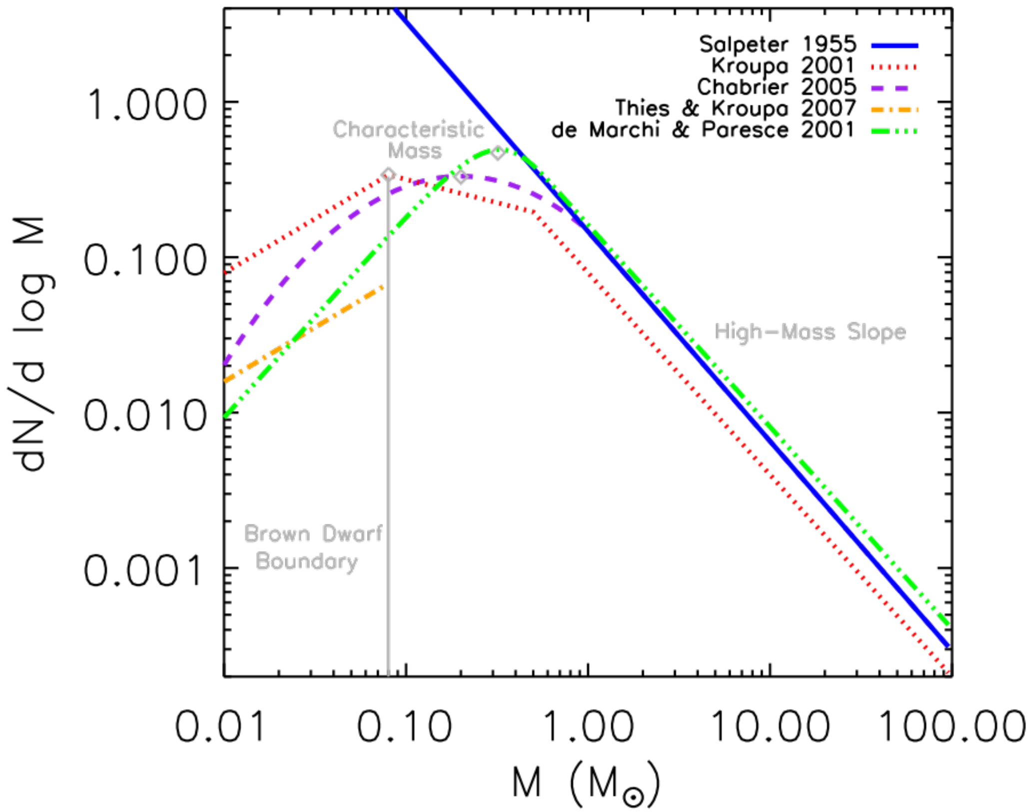


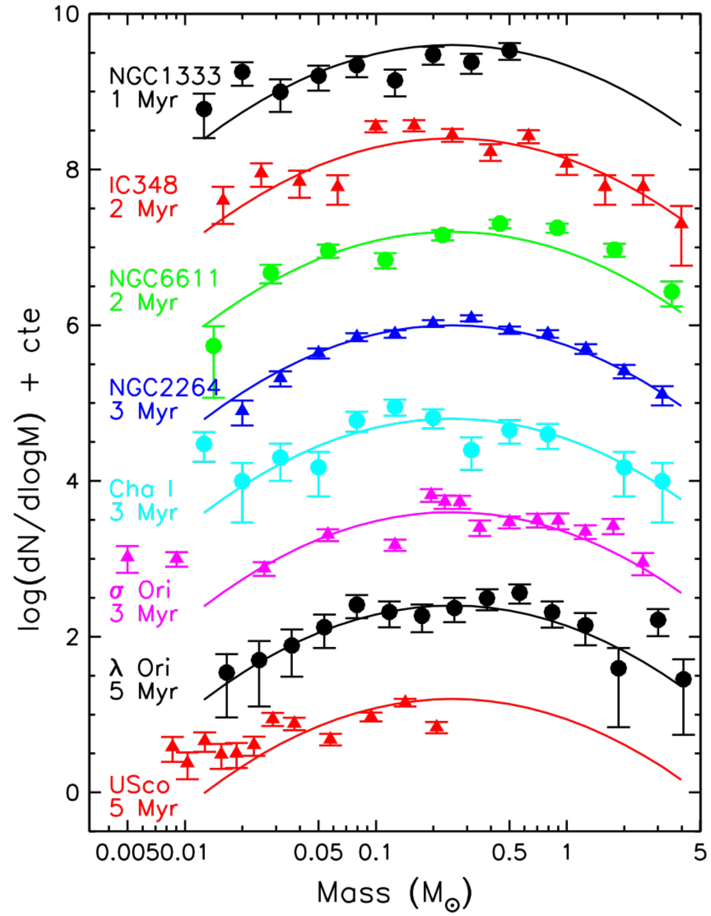
Hayashi Pre-Main Sequence Tracks

Initial mass	Contraction time (Myr)
60M$_\odot$	0.0282
25	0.0708
15	0.117
9	0.288
5	1.15
3	7.24
2	23.4
1.5	35.4
1	38.9
0.8	68.4

- *Left*: Theoretical Hayashi evolution tracks for proto-stars of different masses. Convection is prolific during the horizontal branch "isoluminosity" $T_{eff} \sim R^{-1/2}$ condensing phase.
- *Right*: Table of contraction times (in Myr) for different mass stars.
- Bernasconi & Maeder (A&A **307**, 829, 1996).

Stellar Initial Mass Function





- *Left*: Model IMFs including the original power-law one due to Salpeter (1955). Others address the turn down near the brown dwarf boundary (BDB), and are normalized to unit area.
- *Right*: observational IMFs near and slightly below the BDB from several star forming regions.
- From S. Offner et al., Protostars and Planets VI (2014, **914**, 53).