3.4 Thermonuclear Chains

The thermonuclear central regions of the sun and stars possess complex networks of nuclear interactions, whose rates depend strongly on the temperature. The network is reduced in possibilities by conservation rules of particle/nuclear physics, including charge and lepton number, and number of nucleons (a quark conservation issue). **C & O, pp. 309–14**

The first of these is the **proton-proton chain**, or PP chain, that basically starts the ball rolling by fusing hydrogen into helium.

Plot: PP Chain

∗ The characteristic energy E_c for the combined PP chain is such that the rates are significant for a central temperature of $T = 1.6 \times 10^7\,\mathrm{K}$.

• In the PP chain, νs are generally produced, and a rarely absorbed. Photons are also created, typically with MeV energies, but are always absorbed due to the high opacities, and reprocessed down to X-ray and UV energies.

∗ Hence νs have a high probability of escape, and so can carry away large amounts of energy. Accordingly, neutrinos are key messengers of nucleosynthesis from the cores of the sun and other stars.

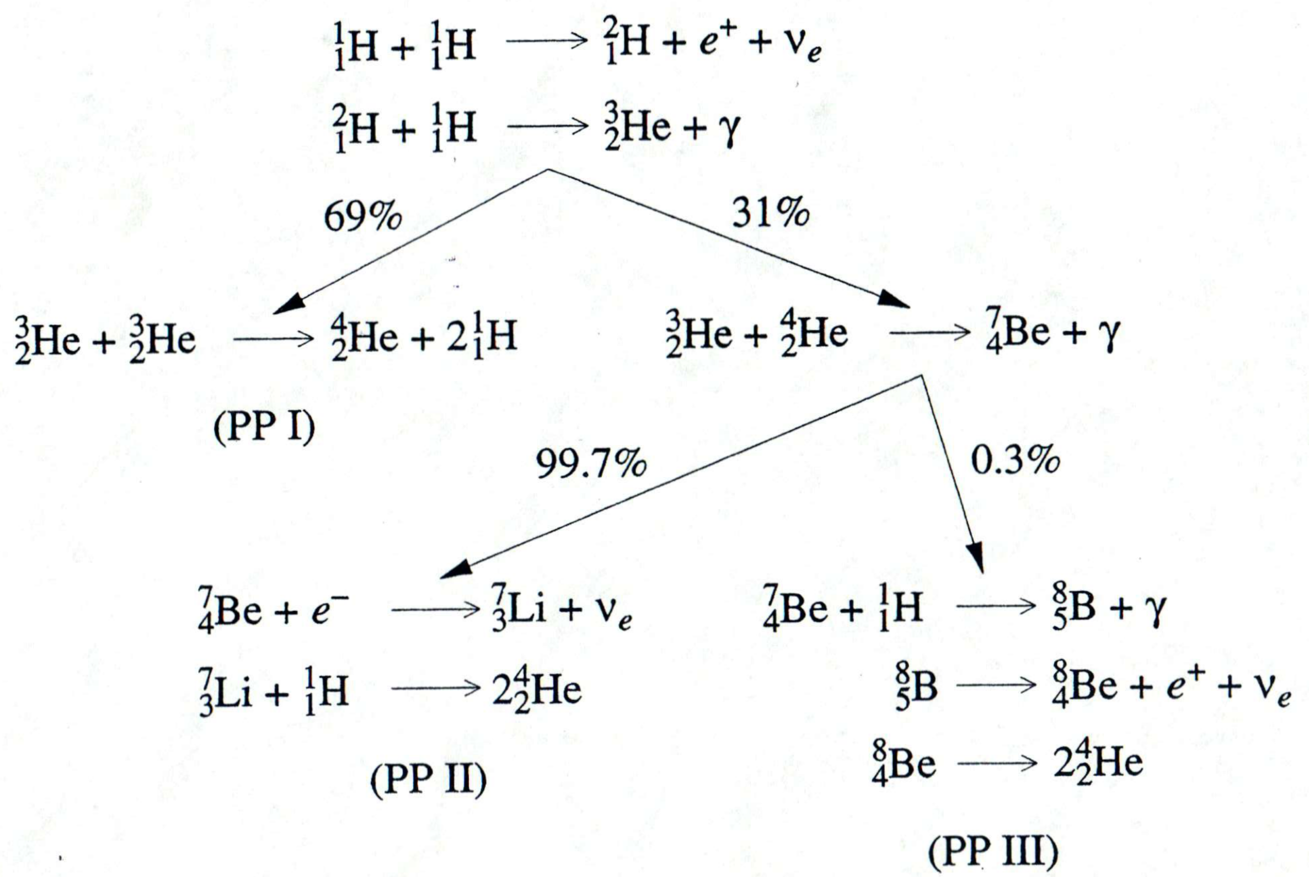
∗ Nuclear notation uses ${}^A_Z X$ to denote a nucleus of **mass number** A, which is the total number of protons and neutrons, and **charge number** Z, which is the total number of protons.

• In nuclear interactions, *both mass number and charge are conserved.*

• In nuclear interactions, **lepton number** is also conserved.

∗ Electrons and their neutrino are of lepton number $+1$, while positrons and the antineutrino are of lepton number -1.

Proton-proton (PP) Thermonuclear Chain



From: Carroll & Ostlie

- The three main branches of the **pp chain** of thermonuclear reactions for temperatures and densities in the core of the sun.

∗ Sample weak reactions take the form:

$$\begin{aligned} \nu + X &\rightarrow Y + e \\ \bar{\nu} + A &\rightarrow B + e^{+} \\ X &\rightarrow Y + e + \bar{\nu} \\ A &\rightarrow B + e^{+} + \nu \end{aligned} \tag{34}$$

and photons have zero lepton number, yielding annihilation reactions:

$$e + e^{+} \rightarrow \gamma\gamma \quad , \quad \nu + \bar{\nu} \rightarrow \gamma\gamma \quad . \tag{35}$$

• After the PP chain, the nature of fusion reactions is controlled by nuclear stability, which peaks with the Fe nucleus:

Plot: Nuclear Binding Energies

It was a mystery for a long time, how to generate carbon from helium. Both were extremely stable. Moreover, helium was more stable than its neighboring more massive isotopes, so that **endothermic** reactions (energy-absorbing) appeared necessary along the path to carbon.

• Fred Hoyle came up with a prediction, that the mechanism for carbon production from helium must be along the lines of

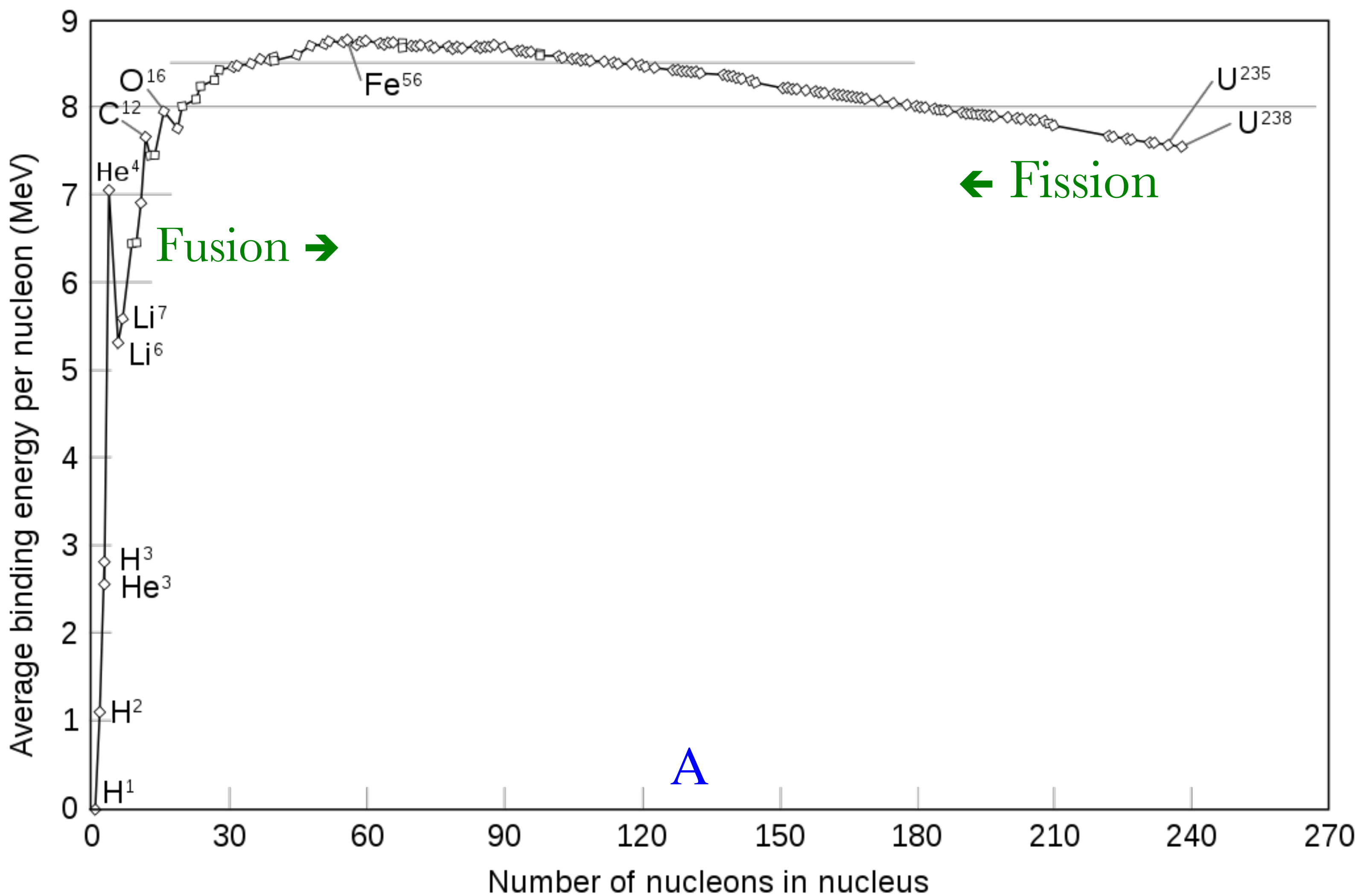
$$\begin{aligned} {}^{4}_{2}He + {}^{4}_{2}He &\leftrightarrow {}^{8}_{4}Be^{*} \\ {}^{8}_{4}Be^{*} + {}^{4}_{2}He &\rightarrow {}^{12}_{6}C + \gamma \quad , \end{aligned} \tag{36}$$

where the $*$ denotes an excited state of beryllium that is unstable. Hoyle indicated the approximate energy of the state, and proposed that the collision rate must be high enough for the third alpha particle to interact before the beryllium state decayed. This is known as the **triple-alpha** reaction,

∗ Such three-body reactions are rare, so that high temperatures (typically over $10^{8}\,\mathrm{K}$) become necessary to raise the collision rates.

∗ It was a triumph for Hoyle's prediction, that nuclear physicists discovered the posited excited nuclear state of beryllium several years later.

Nuclear Binding Energy per Nucleon



- Data from Atomic Mass Data Center, Berkeley National Laboratory

• Once carbon has been produced, the requisite high temperatures can quickly generate stable oxygen and neon by the successive absorption of alpha particles:

$$\begin{aligned} {}^{12}_{6}C + {}^{4}_{2}He &\rightarrow {}^{16}_{8}O + \gamma \\ {}^{16}_{8}O + {}^{4}_{2}He &\rightarrow {}^{20}_{10}Ne + \gamma \quad , \end{aligned} \tag{37}$$

• As the nuclei become more massive, the Coulomb barrier becomes more prohibitive, so the requisite temperatures for fusion become more imposing. They therefore generally require more massive stars. When temperatures reach between $6 \times 10^8\,\mathrm{K}$ and $10^9\,\mathrm{K}$, carbon and oxygen nuclei can begin fusing.

Plot: Carbon and Oxygen Fusion

• The abundance of heavy elements on Earth exceeds solar abundances, arguing in favor of the solar system having formed from products of more massive stars earlier in the universe.

• Hans Bethe (1938) proposed an independent cycle for the production of helium using carbon, nitrogen and oxygen as catalysts. This is called the **CNO cycle**.

Plot: CNO Cycle

The activation of such a cycle requires a considerably higher temperature in general, so that at the solar center, it is somewhat weak and extremely temperature-sensitive.

Carbon and Oxygen Fusion

From: Carroll & Ostlie

$$^{12}_{6}\mathrm{C} + ^{12}_{6}\mathrm{C} \rightarrow \begin{cases} ^{16}_{8}\mathrm{O} + 2\,^{4}_{2}\mathrm{He} & *** \\ ^{20}_{10}\mathrm{Ne} + ^{4}_{2}\mathrm{He} & \\ ^{23}_{11}\mathrm{Na} + p^{+} & \\ ^{23}_{12}\mathrm{Mg} + n & *** \\ ^{24}_{12}\mathrm{Mg} + \gamma & \end{cases}$$

$$^{16}_{8}\mathrm{O} + ^{16}_{8}\mathrm{O} \rightarrow \begin{cases} ^{24}_{12}\mathrm{Mg} + 2\,^{4}_{2}\mathrm{He} & *** \\ ^{28}_{14}\mathrm{Si} + ^{4}_{2}\mathrm{He} & \\ ^{31}_{15}\mathrm{P} + p^{+} & \\ ^{31}_{16}\mathrm{S} + n & \\ ^{32}_{16}\mathrm{S} + \gamma & \end{cases}$$

- The main branches of carbon fusion (left) and oxygen fusion (right), processes that proceed in stars more than 10 times as massive as the sun.

The CNO Cycle

$$^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N + \gamma$$

$$^{13}_{7}N \rightarrow ^{13}_{6}C + e^{+} + \nu_e$$

$$^{13}_{6}C + ^{1}_{1}H \rightarrow ^{14}_{7}N + \gamma$$

$$^{14}_{7}N + ^{1}_{1}H \rightarrow ^{15}_{8}O + \gamma$$

$$^{15}_{8}O \rightarrow ^{15}_{7}N + e^{+} + \nu_e$$

$$^{15}_{7}N + ^{1}_{1}H \rightarrow ^{12}_{6}C + ^{4}_{2}He.$$

From:
Carroll &
Ostlie

- How C, N and O can act as catalysts in massive stars to seed fusion of hydrogen into helium. This often arises in shells surrounding the core.

4 Stellar Structure

The nucleosynthetic contribution to stellar structure is the preserve of the central regions. The rest of the stellar interior provides a means of, and a barrier to, the transport of energy outwards. This transport can take place in three main ways:

C & O, Secs. 10.4 & 10.5

(i) radiative transport, mediated by the absorption and scattering opacities of the interior:

$$\frac{d}{dr}\left(\frac{a}{3}T^4\right) \equiv \frac{dP_{rad}}{dr} = -\frac{\bar{\kappa}\rho}{c}\,F_{rad} = -\frac{\bar{\kappa}\rho}{c}\,\frac{L_r}{4\pi r^2} \tag{38}$$

which rearranges to a temperature gradient equation:

$$\frac{dT}{dr} = -\frac{3}{4ac}\,\frac{\bar{\kappa}\rho}{T^3}\,\frac{L_r}{4\pi r^2} \tag{39}$$

(ii) convection, which is the chief mode of heat transport in the outer third of the sun, where pressure-driven buoyancy of hot bubbles is much faster than diffusive transport,

$$\frac{3k}{2}\,\frac{dT}{dr} = -\mu m_H\,\frac{GM(r)}{r^2} \quad , \tag{40}$$

and (iii) heat conduction in collisions between particles.

• Discussion of these is outlined in Sec. 10.4 of Carroll and Ostlie, and will not be explored here.

• There are essentially, 5 key differential equations that describe the structure and must be solved numerically. In addition, a constituent equation of state must be specified, as well as boundary conditions. Historically, it has been easiest to study only radial variations, the CPU speed and memory now permit multi-dimensional solution.

Plot: Stellar Structure Equations

Numerical solutions are constrained by the mass, radius and luminosity appropriate for a particular type of star, which all couple to the internal temperatures and density gradients.

Stellar Structure Equations

hydrostatic equilibrium: $$\frac{dP}{dr} = -\frac{GM(r)\rho}{r^2}$$

mass conservation: $$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

nucleosynthetic emissivity: $$\frac{dL(r)}{dr} = 4\pi r^2 \rho \frac{dQ}{dt}$$

radiative transport: $$\frac{dT}{dr} = -\frac{3}{4ac} \frac{\bar{\kappa}\rho}{T^3} \frac{L(r)}{4\pi r^2}$$

adiabatic convection: $$\frac{dT}{dr} = -\frac{2\mu m_H}{3k} \frac{GM(r)}{r^2}$$

• These constraints define the **Vogt-Russell Theorem**:

The mass and composition of a star uniquely determine its radius, luminosity and internal structure, as well as its subsequent evolution.

• Generally, the evolution is slow, since most stars are predominantly composed of hydrogen, and hydrogen burns slowly. The theorem indicates that eventually, the stellar appearance (i.e. radius, surface temperature and luminosity) will change *in response to altered nuclear reactions* in the interior.

• Our analysis of hydrostatic equilibrium indicates that higher mass stars have hotter interiors, so that their thermonuclear reactions can access higher masses, perhaps the CNO cycle.

∗ The burning rates are much faster, so the lifetimes should be shorter.

∗ Also, the total energy generation is greater, i.e. luminosity is greater, so that the stars should have higher surface temperatures, i.e be bluer.

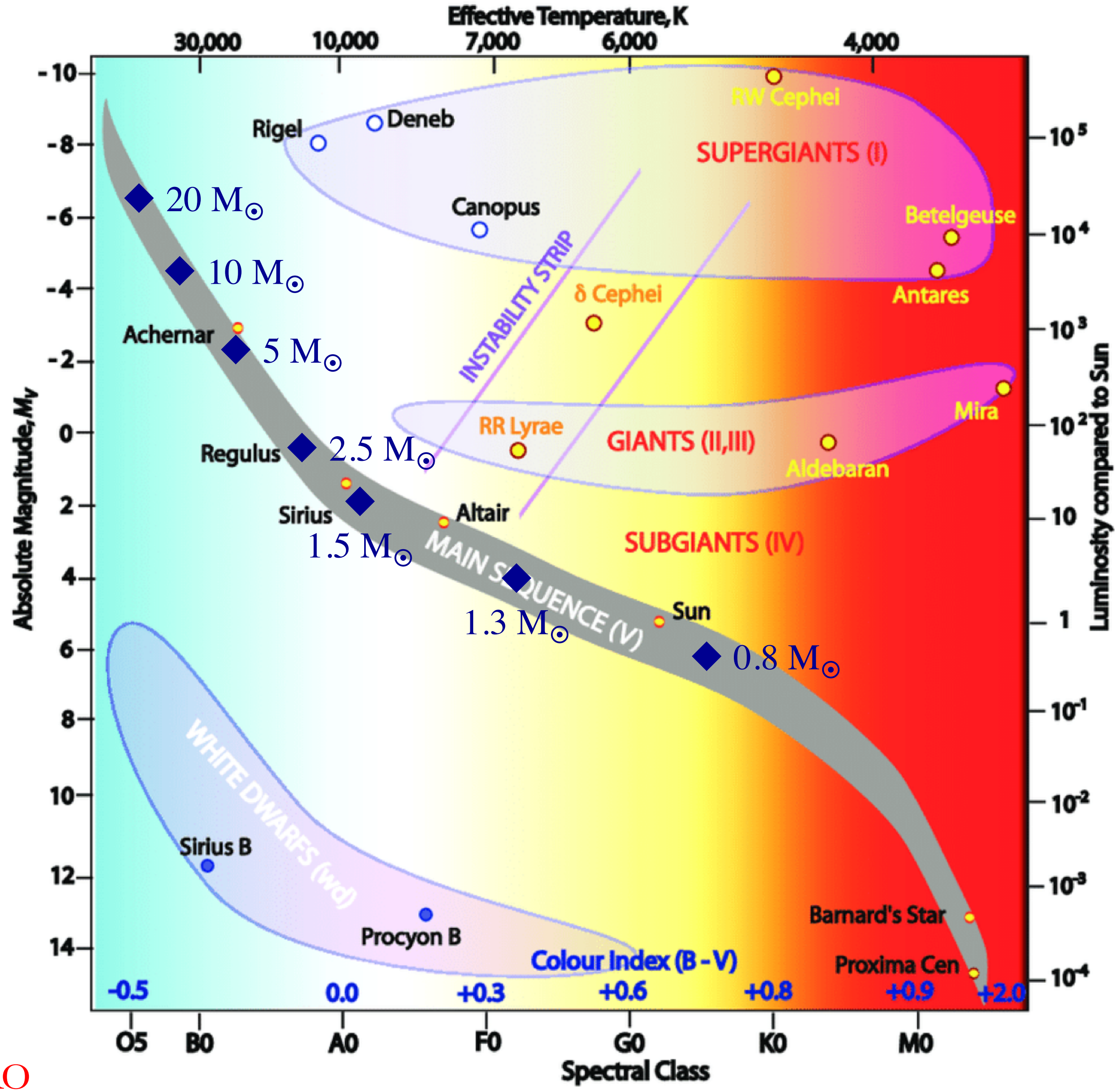
∗ This establishes a *mass mapping* along the main sequence portion of the HR diagram.

Plot: HR diagram and Mass Discrimination

∗ The PP chain persists as the most dominant source of energy generation up to about $1.2M_\odot$, after which the CNO cycle emerges as a player.

• Note that since a $10M_\odot$ star has $L \sim 10^4 L_\odot$, and it burns at somewhat lower efficiency than the PP chain, then it is clear that its lifetime can be of the order of 1000 times shorter than the sun. This ratio is an overestimate due to the evolutionary history of the burning.

Mass Discrimination in the HR Diagram



Graphic credit: R. Hollow, CSIRO

ZAMS Masses from G. Bertelli et al., A&A (2008, **484**, 815; 2009, **508**, 355)