6 Stellar Opacity

Collisional concepts that control stellar opacity can be paralleled to those pertinent to atomic excitation and ionization just explored. **C & O, Sec. 9.2**

The **cross section**, or effective target area for atom or ion collisions is scaled by the Bohr radius: $\sigma = 2\pi a_0^2$. Atoms of speed v sweep out a volume $V = \sigma v t$ in time t, and so encounter $nV = n\sigma v t$ target atoms for collisional excitation or ionization. Here n is the *number* density of atoms.

- Thus the average distance traveled between collisions is

$$l = \frac{vt}{nV} = \frac{1}{n\sigma} \quad ; \tag{20}$$

The distance l is called the **mean free path**. Assuming the Bohr radius of $a_0 = 5.29 \times 10^{-9}$ cm, we arrive at $l = 1.9 \times 10^{-2}$ cm for a solar photospheric density of $n = \rho/m_H = 1.5 \times 10^{17}\,\mathrm{cm}^{-3}$.

∗ This scale is much shorter than the temperature gradient scalelength in the sun, so excitation/ionization considerations can assume isothermal scenarios, i.e. or *local thermodynamic equilibrium.*

Now turn to radiation. At any given position, we expect the diminution of intensity to be proportional to the density ρ of atoms along a path ds. This coupling leads to the differential form

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda \, ds \quad , \tag{21}$$

where κ_λ is called the (mass) **absorption coefficient** or **opacity**.

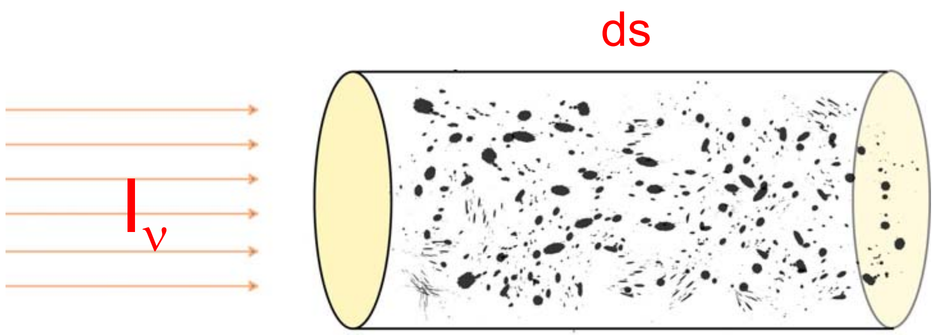
- The attenuation of a light beam can then be determined:

$$I_\lambda = I_{\lambda,0} \exp\left\{ -\int_0^s \kappa_\lambda \rho \, ds' \right\} \quad . \tag{22}$$

For a uniform opacity and density, this becomes a simple exponential dependence on path length s, as we have already explored when considering ISM and atmospheric extinction.

Interactions between photons and matter

absorption of radiation



$$dI_\nu = -\kappa_\nu I_\nu ds$$

κ_ν : absorption coefficient

$$[\kappa_\nu] = \text{cm}^{-1}$$

microscopical view: $\kappa_\nu = n\, \sigma_\nu$

loss of intensity in the beam (true absorption/scattering)

Over a distance *s:*

I_ν^o —— s ——→ $I_\nu(s)$

$$I_\nu(s) = I_\nu^o \, e^{-\int_0^s \kappa_\nu \, ds}$$

$$\tau_\nu := \int_0^s \kappa_\nu \, ds$$

optical depth (dimensionless)

or: $d\tau_\nu = \kappa_\nu \, ds$

Slide Courtesy of R-P Kudritzi, IFA, U. Hawaii

<u>Convention</u>: $\tau_\nu = 0$ at the outer edge of the atmosphere, increasing inwards

• The **optical depth** is then defined to be the measure of exponential attenuation:

$$\tau_\lambda \;=\; \int_0^s \kappa_\lambda \rho \, ds' \quad . \tag{22}$$

The optical depth can be viewed as the number of mean free paths along a given path. Clearly, $\tau > 1$ conditions are quickly achieved as the solar atmosphere is penetrated.

∗ The narrow band of surface layers where $\tau \lesssim 1$ define the region that we probe spectroscopically: it is called the stellar **photosphere**.

• By analogy with the case of atomic collisions,

$$l \;=\; \frac{1}{\kappa_\lambda \rho} \;\equiv\; \frac{1}{n\sigma} \tag{23}$$

is the absorption mean free path.

In a stellar context, atmospheric opacity receives contributions from *bound-bound* transitions, *bound-free* absorption (photo-ionization), *free-free* emission (bremsstrahlung), and electron (Compton) scattering. We can write

$$\kappa \;=\; \kappa_{bb} + \kappa_{bf} + \kappa_{ff} + \kappa_{es} \tag{24}$$

These depend sensitively on the temperature, density and composition of a stellar atmosphere.

Plot: Opacity in ρ-T Space for White Dwarfs

• Hence usually we employ an average opacity called the **Rosseland Mean Opacity** to describe the effective absorption. **C & O, pp. 249–50**

[*Handout: Rosseland Mean Opacity*]

This mean is a strong function of temperature and density.

Plot: Rosseland Mean Opacity

OPACITY IN ρ-T SPACE

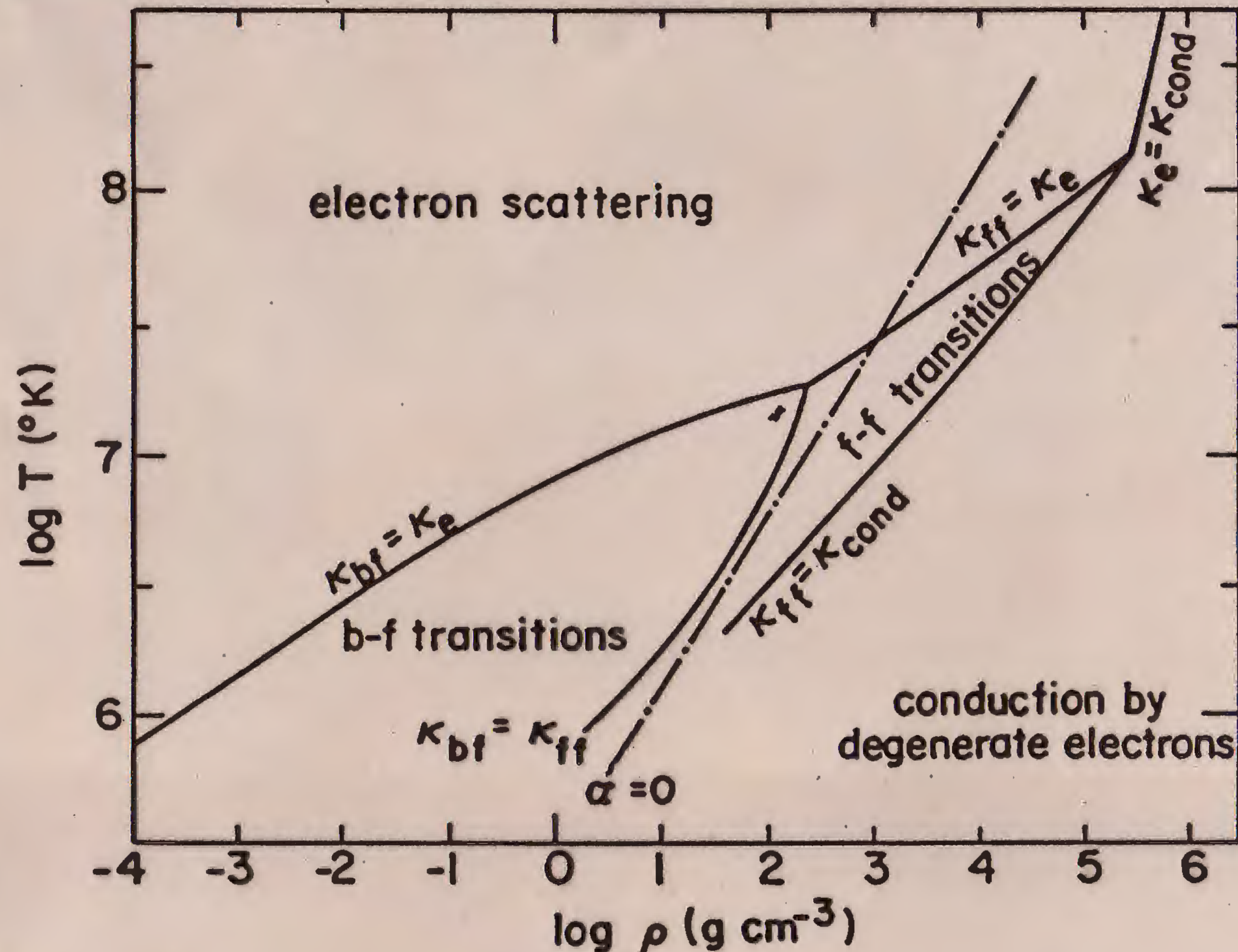


Fig. 8.3 Opacity as a function of density and temperature in a star of population I composition. The diagram is divided into four regions characterized by different mechanism of energy transport. The sources of opacity that dominate these mechanisms are electron scattering, bound-free transitions, free-free transitions, and the effective opacity that would describe the energy transport by degenerate electrons. The dashed line shows where the degeneracy *parameter* α (see equation 4-93) equals zero (after Hayashi, Hoshi, and Sugimoto, Ha62c).

ROSSELAND MEAN OPACITY

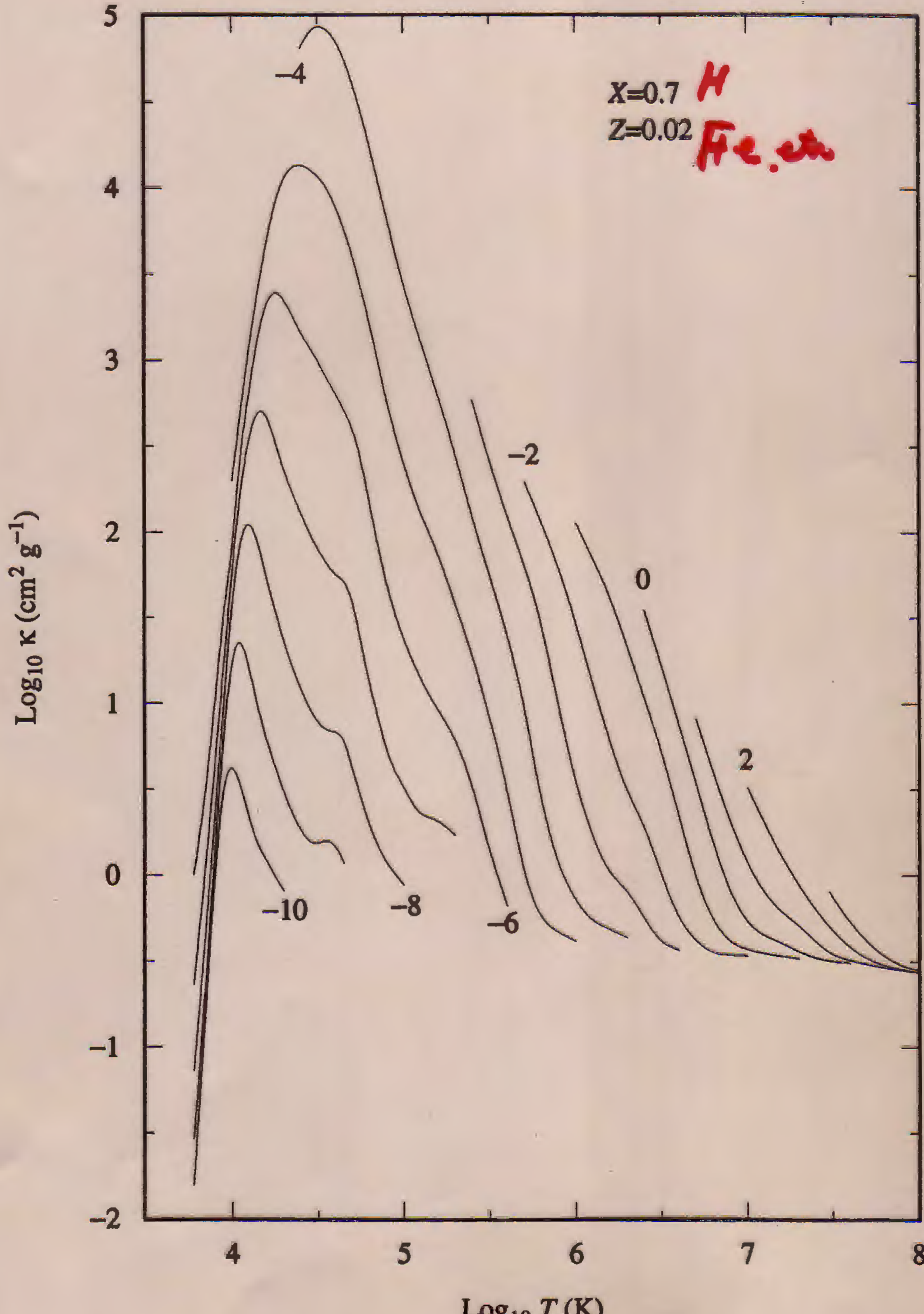


Figure 9.10 Rosseland mean opacity. The curves are labeled by the value of the density ($\log_{10}\rho$ in g cm^{-3}). (Data from Rogers and Iglesias, *Ap. J. Suppl.*, *79*, 507, 1992.)

7 Radiative Transfer

Since only the outermost layers of the sun and stars in general are optically thin, radiation transport in stellar interiors is generally diffusive on timescales much longer than the free-streaming time, $R_{\odot}/c$. **C & O, pp. 252–4**

Diffusion can be modeled by a **random walk** of photons out from the stellar interior. The net displacement $\mathbf{d}$ in a large number N of randomly directed steps $\mathbf{l}_i$ is

$$\mathbf{d} \;=\; \sum_{i=1,N} \mathbf{l}_i \tag{25}$$

so that the magnitude can be obtained from

$$d^2 \;=\; \mathbf{d}.\mathbf{d} \;=\; \sum_{i=1,N}\sum_{j=1,N} \mathbf{l}_i.\mathbf{l}_j \;\equiv\; Nl^2 + l^2 \sum_{i\neq j} \cos\theta_{ij} \quad , \tag{26}$$

if all the steps are of equal length $|\mathbf{l}_i| = l$. Here $\cos\theta_{ij} = \mathbf{l}_i.\mathbf{l}_j/l^2$ is the angle between two vector steps. The sum over the angle cosines tends to zero as $N \to \infty$, essentially an integral over angles of the cosine. Hence it follows that the number N of steps to the surface and the optical depth $\tau_\lambda = d/l$ satisfy

$$d \;=\; l\sqrt{N} \quad , \quad N \;=\; \tau_\lambda^2 \quad . \tag{27}$$

• Diffusion is an inefficient means of radiative transfer; convection plays an important role in transporting nuclear energy out of the solar interior.

• From the phenomenon of **limb darkening**, namely that the solar extremeties appear darker than the center to an observer, one can deduce that the temperature declines with distance from the center of the sun.

We can see typically about $\tau_\lambda = 2/3$ deep into the sun. When this is line of sight is tangential to the solar surface, i.e. at the solar **limb**, we can only probe into comparatively shallow layers.

∗ The darker appearance implies a cooler temperature, and therefore *we infer that there is a negative temperature gradient with photospheric radius.*

8 Structure of Spectral Lines

C & O, Sec. 9.5

The shape of a spectral line contains a wealth of information. It consists of a *core* and two *wings*. If F_c is the surrounding continuum flux level, then the quantity $1 - F_\lambda/F_c$ is referred to as the **depth** of the line. The effective strength of the line is called the **equivalent width** W of the line:

$$W \;=\; \int \frac{F_c - F_\lambda}{F_c}\, d\lambda \quad . \tag{28}$$

This is to be distinguished from the **full width at half-maximum** (FWHM), which is the width where it is at least half as deep as the maximum depth, i.e. side-to-side width, denoted by $(\Delta\lambda)_{1/2}$.

Plot: Typical Spectral Line Shape

There are three main mechanisms for broadening spectral lines; natural broadening, Doppler broadening and pressure broadening.

• **Natural broadening**: spectral lines are not infinitely sharp due to the quantum nature of energy states. Heisenberg's uncertainty principle yields an energy uncertainty $\Delta E \approx \hbar/\Delta t$ in an atomic state that lives for time Δt. *Energy levels are fuzzy.*

Hence, if an atomic electron transitions between states $i \to f$, which have respective decay times of Δt_i and Δt_f, then the natural width of the line is

$$\Delta\lambda \;\approx\; \frac{\lambda^2}{2\pi c}\left(\frac{1}{\Delta t_i} + \frac{1}{\Delta t_f}\right) \quad , \tag{29}$$

which can be derived using $dE/d\lambda \propto \lambda^{-2}$.

$*$ e.g. The lifetime of the first and second excited states of hydrogen is less than about $\Delta t = 10^{-8}$ sec. Hence the natural broadening of the H_α Balmer line at $\lambda = 6563$ Å is around $\Delta\lambda \approx 4.57 \times 10^{-4}$ Å.

EQUIVALENT WIDTH W

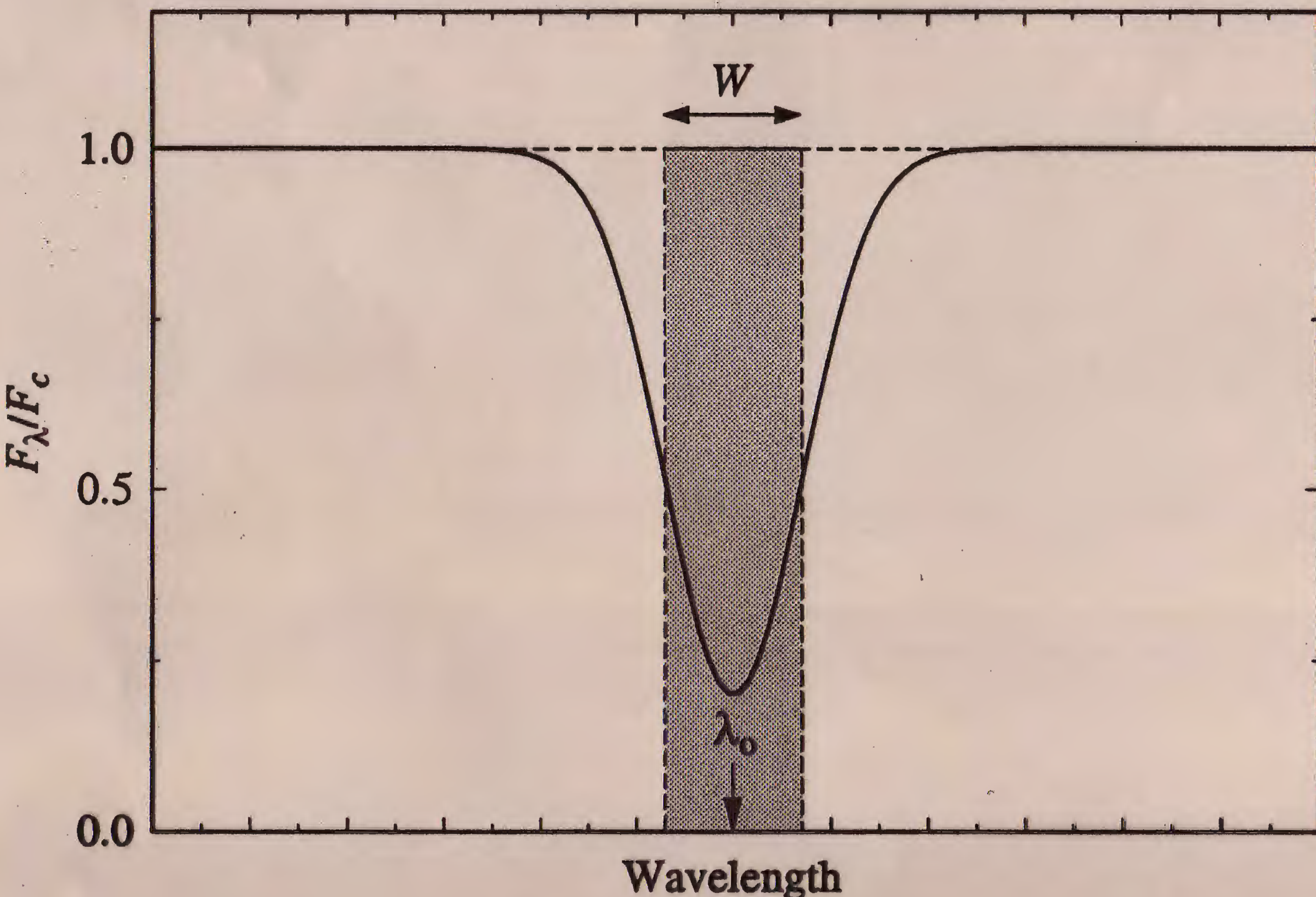


Figure 9.18 The shape of a typical spectral line.

• The frequency profile for natural broadening can be calculated using a quantum harmonic oscillator model and by taking the Fourier transform of the force equation; the result is a **Lorentz profile**: **Lang, pp. 200–1**

$$F_\nu \;\propto\; \frac{1}{2\pi}\,\frac{\Delta\nu}{(\nu-\nu_0)^2+(\Delta\nu/2)^2} \tag{30}$$

for a line of width $\Delta\nu$ at frequency ν_0. This profile is normalized to unity, and the radiative lifetime $t = 1/\Gamma$ of the states is given by

$$t \;=\; \frac{1}{\Gamma} \;=\; \frac{1}{2\pi\Delta\nu} \quad . \tag{31}$$

Γ is called the *natural line width.*