Accordingly,

$$\mathcal{F}_\odot \;=\; \frac{L_\odot}{4\pi(1\text{AU})^2} \;=\; 1.36\times 10^6 \text{erg sec}^{-1}\text{cm}^{-2} \tag{2}$$

is the solar flux at Earth and is called the **solar constant**.

∗ Note the cgs to SI unit conversion: 1 erg = 1 g cm^2 sec^{-2} $= 10^{-7}$ kg m^2 sec^{-2} $= 10^{-7}$ Joules.

The flux and luminosity both depend on the waveband under consideration.

Plot: Apparent magnitude benchmarks

• Observational equivalents of $\mathcal{F}$ and L are the **apparent magnitude** m and the **absolute magnitude** M. Using the Greek scale as a guideline led astronomers to the *definitions*

$$\mathcal{F} \;\propto\; 10^{-2m/5} \quad \text{and} \quad L \;\propto\; 10^{-2m/5}\, d^2 \;\propto\; 10^{-2M/5} \tag{3}$$

The constants of proportionality were "benchmarked" by setting $m = M$ for sources at a distance of $d = 10\,\text{pc}$, a typical distance scale for nearby stars. Hence,

$$\boxed{m - M \;=\; 5\log_{10}\left(\frac{d}{10\,\text{pc}}\right) \quad ,} \tag{4}$$

and we call this quantity a source's **distance modulus.**

∗ Observe that for the sun, the brightest object in the sky, a complete outlier with $m_\odot = -26.81$, however its absolute magnitude is $M_\odot = 4.76$, typical of other main sequence stars.

• It is appropriate to now define the distance scale of a **parsec**, abbreviated **pc**. It is the distance at which 1 AU on the sky subtends 1 arcsec, i.e. the distance at which the **parallax** of a star is $\pi = 1\,''$:

$$1\text{pc} \;=\; \frac{1.496\times 10^{13}\text{cm}}{\pi/180/3600} \;=\; 3.09\times 10^{18}\text{cm} \;=\; 3.26 \text{ lt yr.} \tag{5}$$

Plot: Stellar parallax

APPARENT MAGNITUDE BENCHMARKS

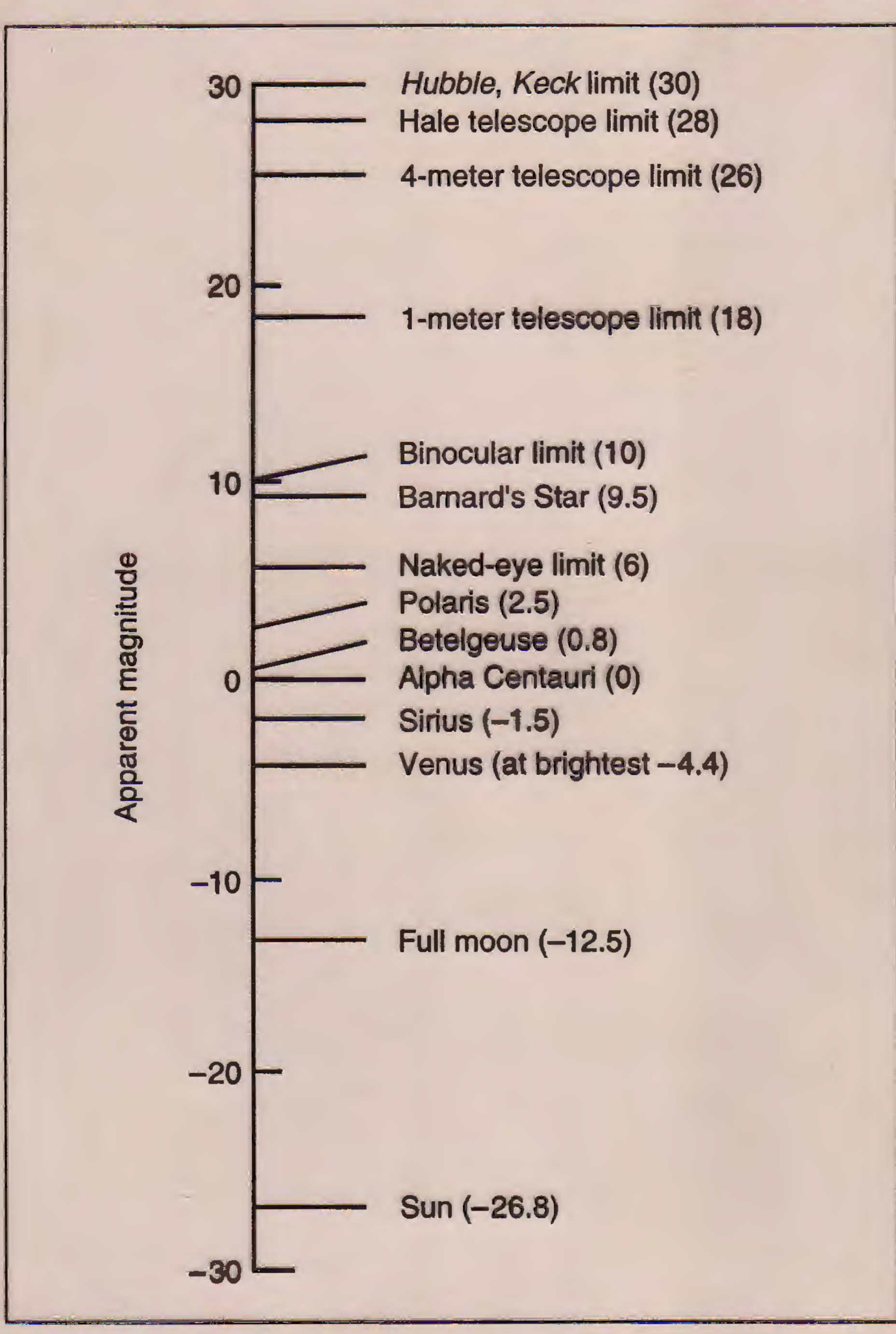

Figure 17.9 Apparent magnitudes of some astronomical objects. The original magnitude scale was defined so that the brightest stars in the night sky had magnitude 1, and the faintest stars visible to the naked eye had magnitude 6. It has since been extended to cover much brighter and much fainter objects. An increase of 1 in apparent magnitude corresponds to a decrease in apparent brightness of a factor of approximately 2.5.

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A brief list if apparent magnitudes as benchmarkers is as follows:

∗ full moon: $m = -12.5$

∗ Jupiter: $m = -2$

∗ naked eye limit: $m = +6$

∗ binocular limit: $m = +9$

∗ 8m telescope: $m = +29$

• The distance modulus is a tool that can be used to estimate the distance to stars that are beyond the parallactic limit (which admittedly has moved further away in the Gaia era). A star of known spectral type can have its absolute magnitude M fairly well known. Its apparent magnitude is measured and its distance d inferred.

• Magnitude depends on color too, so astronomers use magnitude differentials to provide indications of color. **C & O, Sec. 3.6**

∗ e.g. $B-V \equiv M_B - M_V$ is a color marker so that a red star has typically $B - V = 1.0$, whereas a blue star has typically $B - V = -0.5$.

Plot: Color bands and a putative stellar spectrum

• **Bolometric** magnitudes average over all wavebands, i.e. represent the mean magnitude of a source:

$$M_{bol} \;=\; M_V + \text{corr}_V \;\equiv\; M_B + \text{corr}_B \quad , \tag{6}$$

where the difference of correction factors $\text{corr}_V - \text{corr}_B$ represents the spectrum or color of a star.

∗ This difference is more *positive* for bluer (i.e. hotter) stars, and more negative for redder stars.

Plot: Photometric Color Band Wavelengths

COLOR BANDS

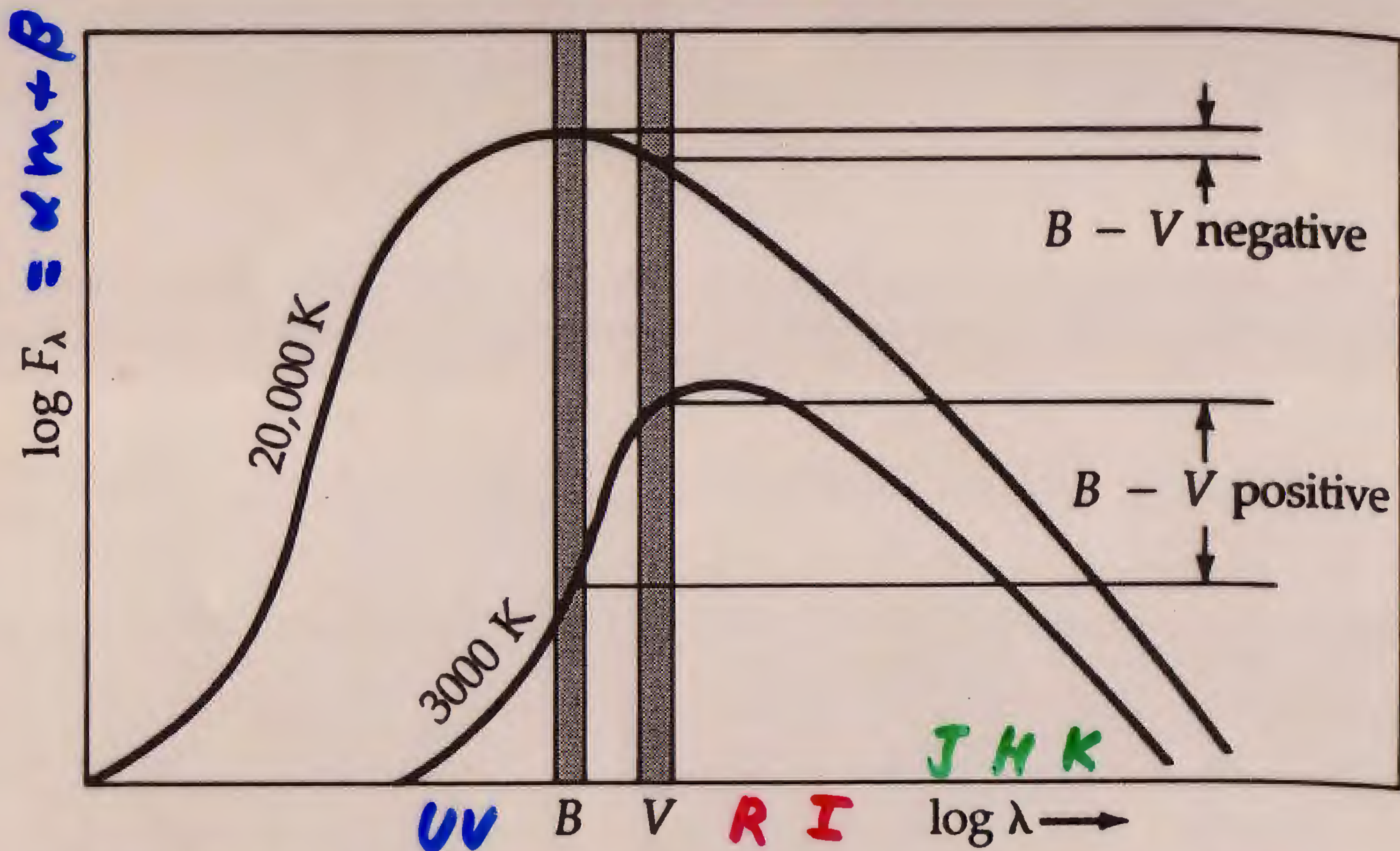

FIGURE 11–4 Color index in the *BV* system. Blackbody curves for 20,000 K and 3000 K, along with their intensities at *B* and *V* wavelengths. Note that *B* − *V* is negative for the hotter star, positive for the cooler one.

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Photometric Color Bands

Filter Letter	Effective Wavelength Midpoint λ_{eff} for Standard Filter[2]	Full width at half maximum[2][c] (archetypal Bandwidth) ($\Delta\lambda$)[d]
Ultraviolet		
U	365 nm	66 nm
Visible		
B	445 nm	94 nm
G[3]	464 nm	128 nm
V	551 nm	88 nm
R	658 nm	138 nm
Near-Infrared		
I	806 nm	149 nm
Z	900 nm[4]	
Y	1020 nm	120 nm
J	1220 nm	213 nm
H	1630 nm	307 nm
K	2190 nm	390 nm
L	3450 nm	472 nm
Mid-Infrared		
M	4750 nm	460 nm
N	10500 nm	2500 nm
Q	21000 nm[5]	5800 nm[5]

2 Extinction and Reddening

Observed fluxes or magnitudes cannot be taken at face value as the exact measure of brightness of a star; they must be corrected for **interstellar extinction**, the attenuation of radiation on its path from source to Earth due to scattering or absorption in the interstellar medium (ISM). **C & O, pp. 259-61**

Plot: Absorption column and intensity attenuation

- From the absorption column set-up, the differential change in **intensity** $dI_\lambda = I_\lambda - I_0$ is related to path length ds by

$$\frac{dI_\lambda}{I_\lambda} = -\kappa_\lambda \, ds \quad , \tag{7}$$

where κ_λ is called the **absorption coefficient**. This simply integrates to

$$I_\lambda = I_0 \, e^{-\tau_\lambda} \quad , \quad \tau_\lambda = \kappa_\lambda \, \Delta s \quad . \tag{8}$$

The *intensity is the energy of light passing through unit area in unit solid angle per unit time*. Here τ_λ is called the **optical depth**, and is the key measure of how opaque the ISM is. $\tau_\lambda \ll 1$ corresponds to *optically thin* conditions, whereas $\tau_\lambda \gg 1$ defines *optically thick* environs.

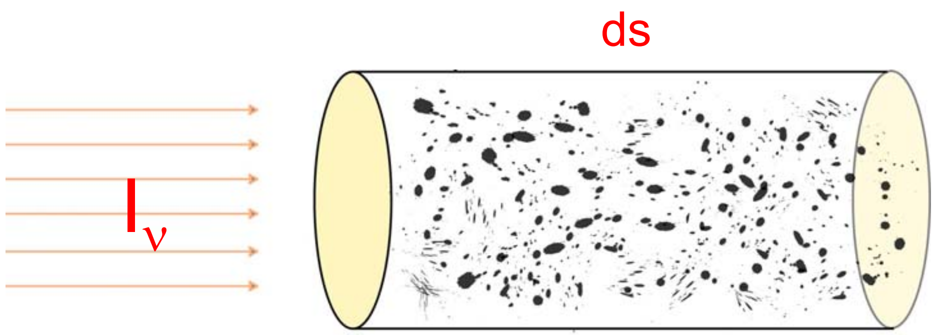
- In general, $\tau_\lambda = \tau(\lambda)$, and blue light is preferentially scattered (Rayleigh scattering) by dust, i.e. lost from the line of sight. This is manifested in the scattering of light by bound electrons that oscillate at some natural frequency ω_0, with a cross section

$$\sigma_{\rm Ray} \propto \left(\frac{\omega}{\omega_0} \right)^4 \quad . \tag{9}$$

∗ This phenomenon is evinced in the blue nature of the sky via the scattering of sunlight. Like most scattering processes, it is polarization dependent, and so the sky can be darkened by a polaroid.

Interactions between photons and matter

absorption of radiation



$$dI_\nu = -\kappa_\nu I_\nu ds$$

κ_ν : absorption coefficient

$$[\kappa_\nu] = \mathrm{cm}^{-1}$$

microscopical view: $\kappa_\nu = n\,\sigma_\nu$

loss of intensity in the beam (true absorption/scattering)

Over a distance *s:*

$I_\nu^o \xrightarrow{\;s\;} I_\nu(s)$

$$I_\nu(s) = I_\nu^o\, e^{-\int_0^s \kappa_\nu\, ds}$$

optical depth (dimensionless)

$$\tau_\nu := \int_0^s \kappa_\nu\, ds$$

or: $d\tau_\nu = \kappa_\nu\, ds$

Convention: τ_ν = 0 at the outer edge of the atmosphere, increasing inwards

Slide Courtesy of R-P Kudritzi, IFA, U. Hawaii

• Since $\tau_{\rm red} < \tau_{\rm blue}$, the ISM generates **interstellar reddening** of starlight as well as extinction. Both need to be corrected for in transit to Earth.

In magnitudes, the wavelength-dependent extinction is expressed as an attenuation magnitude:

$$A_\lambda \;=\; m_\lambda - m_\lambda(\text{no dust}) \;=\; 2.5\log_{10}\frac{I_0}{I} \;=\; \tau_\lambda(2.5\log_{10}e) \;=\; 1.08\tau_\lambda \quad . \tag{10}$$

The observed magnitude is then corrected by A_λ to find the true magnitude.

• One can then define **color excesses:**

$$E_{B-V} \;=\; \underbrace{(B-V)}_{\text{observed}} - \underbrace{(B_0-V_0)}_{\text{intrinsic}} \quad . \tag{11}$$

The intrinsic values are inferred from a star's spectral class and comparison with other stars.

• Interstellar extinction is not the whole story: **atmospheric extinction** complicates the picture. It can be extracted by performing a zenith comparison, a check that can be performed on a star during a night's observation.

Plot: Atmospheric extinction geometry and zenith variation

The ratio of optical depths for zenith angle θ relative to the zenith value is simply

$$\frac{\tau(\theta)}{\tau(0)} \;=\; \frac{s}{h} \;=\; \sec\theta \tag{12}$$

to first order, neglecting the altitudinal distribution of attenuation. Converting this to magnitudes then results in

$$m_\theta \;=\; m_0 + 1.08\tau_0 \sec\theta \quad , \tag{13}$$

so that the intersection of the m_θ - $\sec\theta$ plot with the ordinate axis yields the upper atmosphere magnitude (incident) m_0, and the slope $k_\lambda = 1.08\tau_0$ is the extinction coefficient per unit airmass.

[*Reading Assignment: C & O Sec. 3.3: the wave nature of light*]

ATMOSPHERIC EXTINCTION

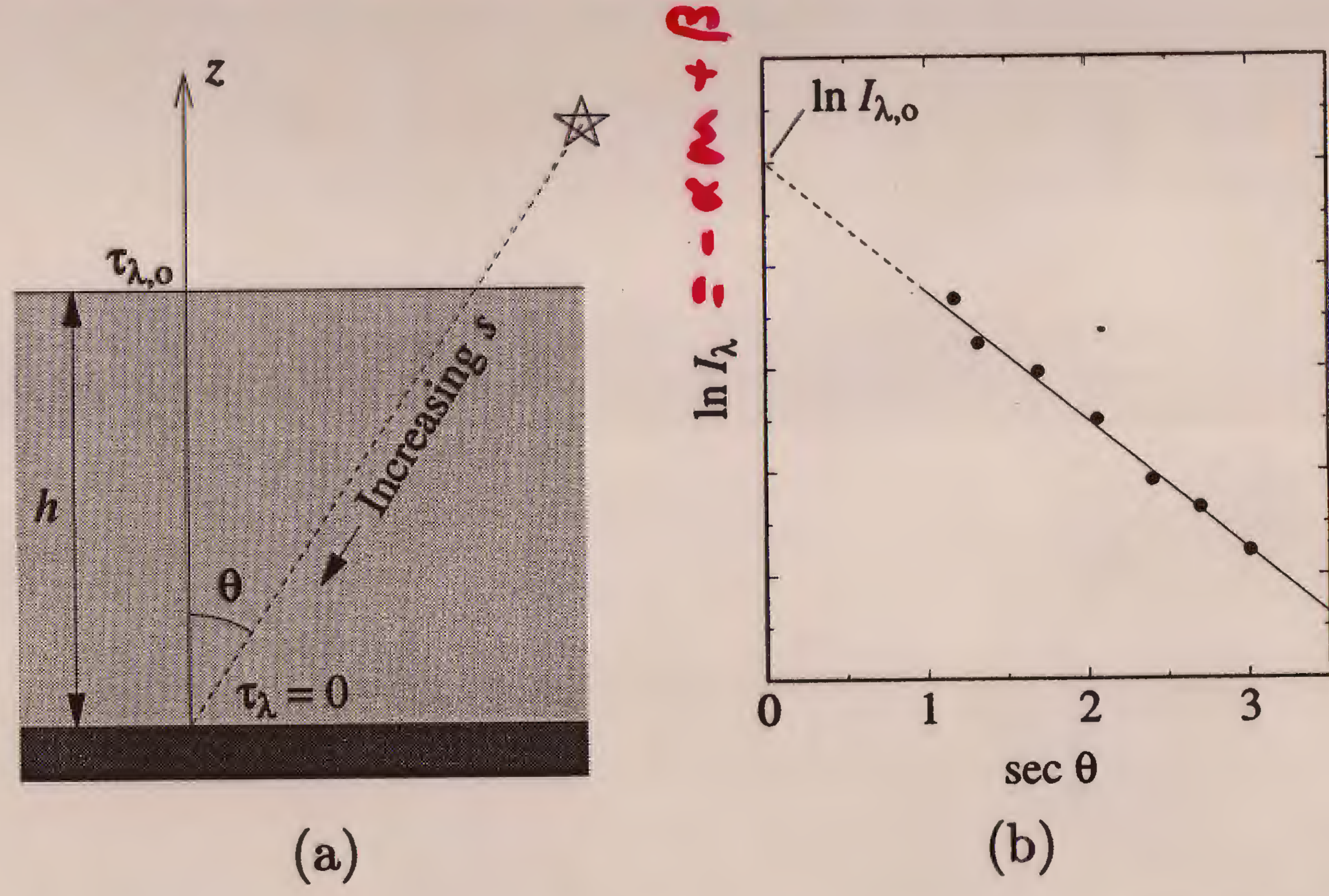


Figure 9.8 (a) A light ray entering Earth's atmosphere at an angle θ. (b) $\ln I_\lambda$ vs. $\sec\theta$.

3 Blackbody Radiation

Blackbody radiation is the principal continuum spectrum that is inherently thermal in nature, an idealized form that is never a perfect reality.

• A blackbody absorbs all light incident upon it and re-radiates it with a characteristic spectrum that is determined by quantum mechanics.

3.1 Wien and Stefan-Boltzmann Laws

• The blackbody spectrum peaks at a wavelength $\lambda_{\rm max}$ that is given by **Wien's displacement law**: **C & O, Sec. 3.4**

$$\lambda_{\rm max} T \;=\; 0.290 \text{ cm } K \quad . \tag{13}$$

e.g. for $T_\odot = 5.77 \times 10^3$ as the surface solar temperature, Wien's law gives $\lambda_{\rm max} = 0.290/5.77 \times 10^3 \equiv 5030$ Å(Angstroms).

∗ Hence, hot stars are blue, and cool stars are red.

The total luminosity of a blackbody was shown by Stefan in 1879 to obey

$$\boxed{L \;=\; A\sigma T^4 \quad ,} \tag{14}$$

for A being the surface area of the blackbody. Eq. (14) is known as the **Stefan-Boltzmann Law**, and

$$\sigma \;=\; 5.67 \times 10^{-5} \text{erg sec}^{-1}\text{cm}^{-2} K^{-4} \tag{15}$$

is the Stefan-Boltzmann constant.

• Boltzmann explained Stefan's experimental determination in terms of thermodynamics and electromagnetism.

e.g. For a sphere like the sun, with a radius $R_\odot = 6.96 \times 10^{10}$ cm,

$$L_\odot \;=\; 4\pi R_\odot^2 \sigma T_\odot^4 \;=\; 3.826 \times 10^{33} \text{erg/sec} \;\Rightarrow\; T_\odot \;=\; 5770K \tag{16}$$

is the effective temperature of the solar surface.