2 Extrasolar Planets

A classic 21st Century application of our binary system understanding is in the study of **extrasolar planets**, also called **exoplanets**. This field has become possible with enhanced technologies on two fronts.

[*Reading Assignment: C & O: Section 7.4*]

• The first technical advance was in the ability to measure Doppler shift velocities in spectral lines at the precision of better than 10 metres/sec. If one applies Kepler III to Jupiter, one arrives at a perihelion speed of $v_{\rm J} \sim$ 13 km/sec, i.e. around $2\pi a_{\rm J}/P_{\rm J}$. The center of mass considerations then simply yield the corresponding value for the sun about Jupiter:

$$v_{\odot} \;\approx\; \frac{m_{\rm J}}{M_{\odot}}\, v_{\rm J} \;\sim\; 12.5\,\text{metres/sec} \quad . \tag{13}$$

This sets the Doppler precision scale to infer the presence of Jupiters around nearby stars using state of the art spectrometers.

∗ Sub-Jupiter objects (Neptunes) were beginning to be detected by this Doppler technique by 2003-2004.

Plot: μ Arae Velocity Curves from La Silla and AAT - a Neptune

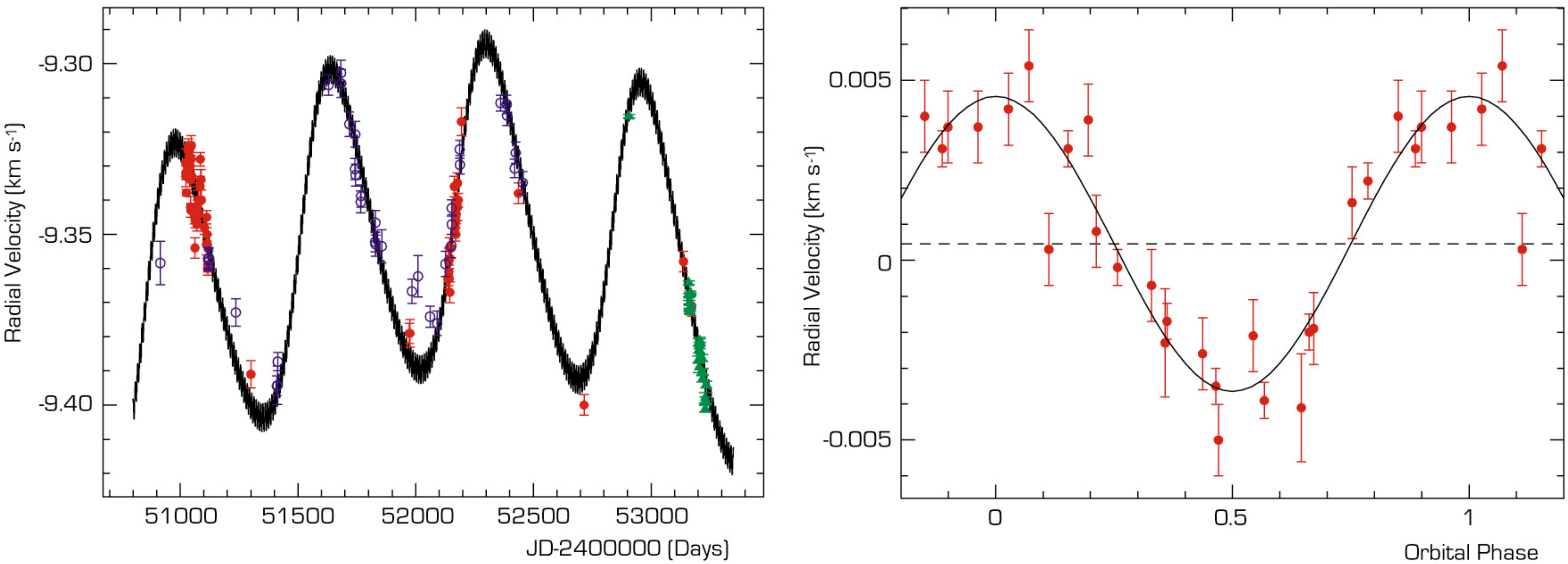
• The main complementary technique ushered in by the **Kepler** mission in 2008 was that of photometric transit determination using eclipsing of stars by planets. Advances were forged by the ability to measure photometric decrements on the order of 1% or less. This can be employed in conjunction with the Doppler approach.

Plot: Kepler candidate exoplanet census, circa 2011

• Outline experimental sequence of ground-based, Kepler and SIM using handouts. Eclipse studies and interferometry.

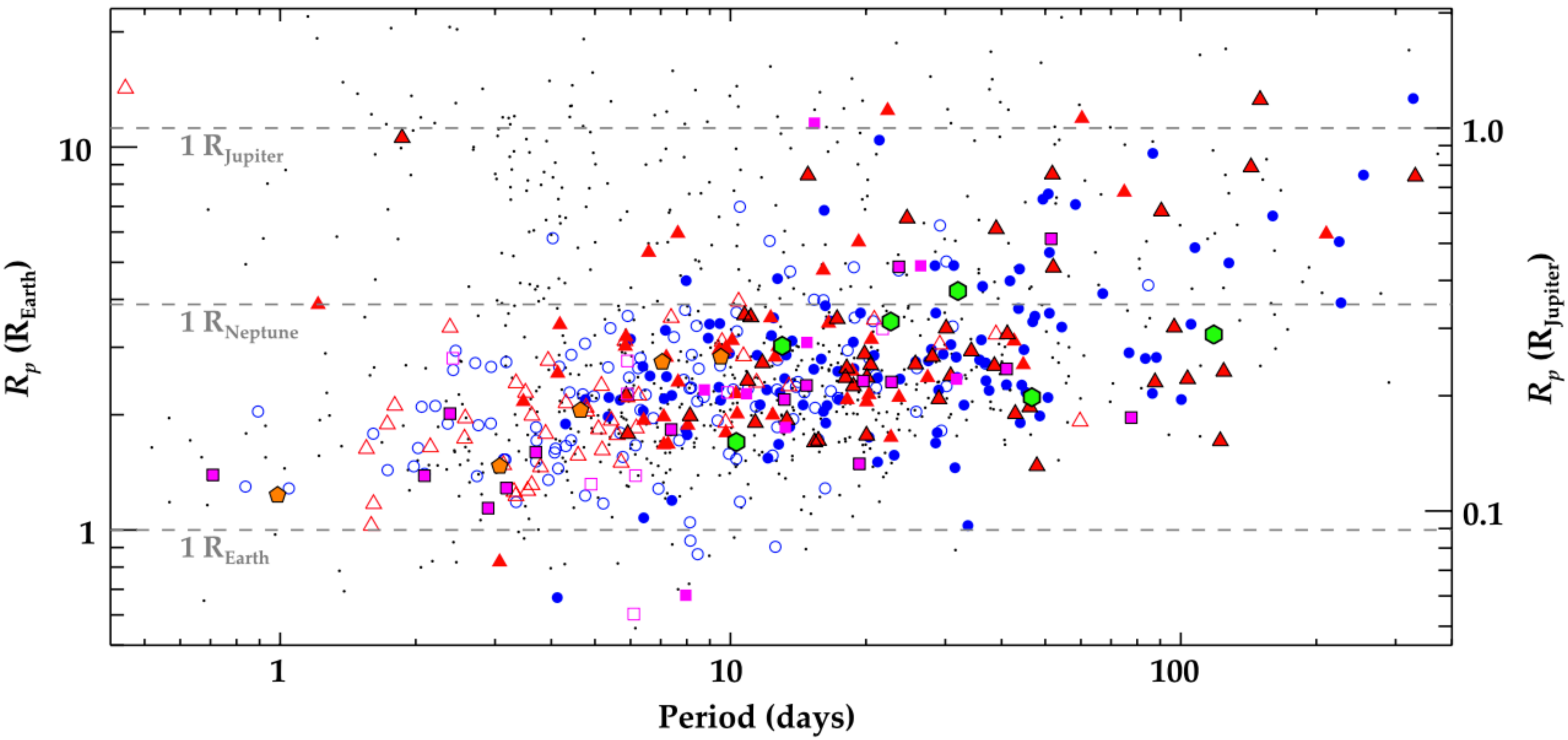
μ Arae Velocity Variations

Courtesy: ESO Public Image Archives



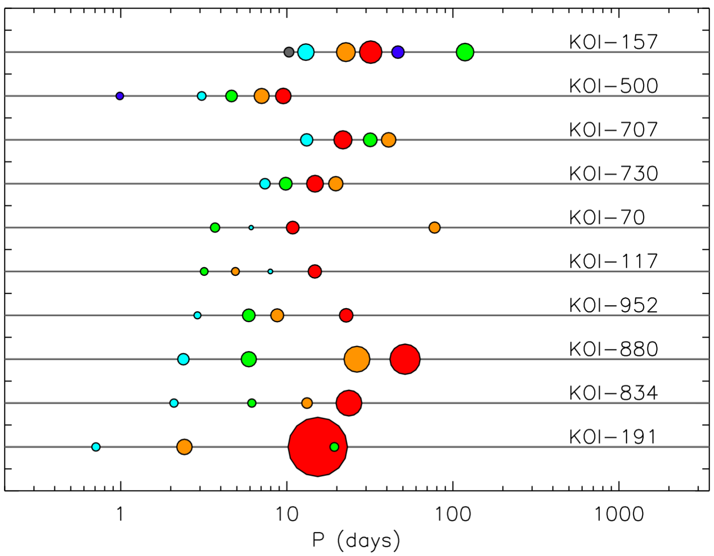
- *Left panel*: Measurements of the radial velocity of the star μ Arae obtained by HARPS (ESO 3.6m telescope at La Silla - green triangles), CORALIE (Euler 1.2m telescope, La Silla - red dots) and UCLES (Anglo-Australian Telescope - blue circles). Best fit solid line assumes the existence of two planets and a long-period companion. The width of the line implies the existence of a short period planet. Data shown span July 1998 to August 2004.
- *Right Panel*: The HARPS radial velocity measurements phase-folded with the orbital period of the short period (9.5 days) exoplanet. The semi-amplitude of the curve is less than 5 metre/sec, which at this 9.5 day period implies a minimum mass for the sub-Jupiter planet of 14 times that of the Earth.

Extrasolar Planets: Kepler Census

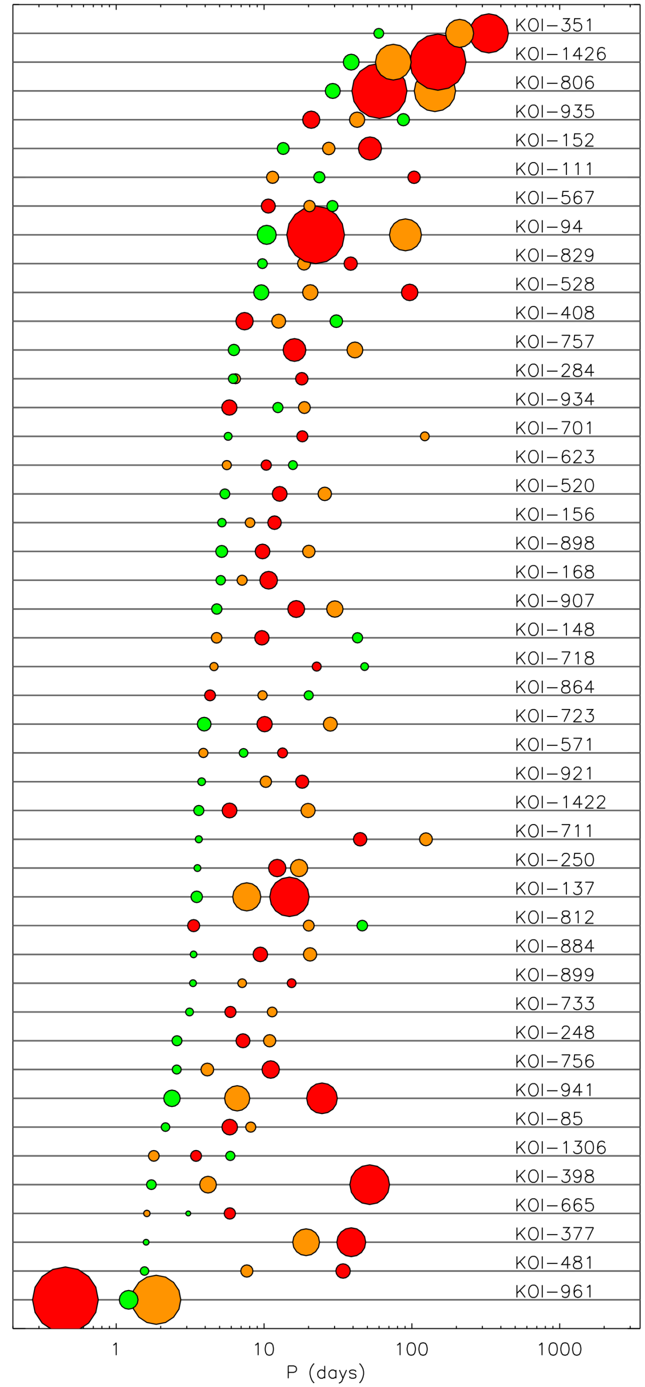


- Lissauer et al. (2011, ApJS **197**, 8) lists over 1200 candidate planets from Kepler mission transits, emphasizing the prevalence of multi-planet systems.
- Period range is a few months down to a fraction of a day;
- Radius range is from Jupiters down to sub-Earth scale!

Multi-Planet Systems According to Kepler



- Lissauer et al. (2011, ApJS **197**, 8): multi-planet systems are common.
- 4-6 planets (above); 3-4 planets (RHS)
- Planet size α dot size; colors correlate with size and temperature.



3 Virial Theorem

For larger ensembles of stars, global averages can be described using an energy theorem, called the **virial theorem**. It is applicable to solar systems, open and globular clusters, galaxies, clusters of galaxies and gas therein.

C & O, Sec. 2.4

• It provides a means for estimating total mass content using determinations of the kinetic energy content. A key example is *the inference of dark matter from galactic rotation curves.*

• First, in two body systems, we have

$$E \;=\; \frac{1}{2}\mu v_p^2 - G\frac{M\mu}{r_p} \quad , \quad r_p \;=\; a(1-e) \;\; , \quad v_p^2 \;=\; \frac{GM}{a}\left(\frac{1+e}{1-e}\right) \;\; , \tag{14}$$

which leads to

$$E \;=\; \frac{GM\mu}{a}\left\{\frac{1}{2}\,\frac{1+e}{1-e} - \frac{1}{1-e}\right\} \;=\; -\frac{GM\mu}{2a} \;=\; -\frac{Gm_1m_2}{2a} \quad . \tag{15}$$

Therefore,

$$\boxed{E \;=\; \frac{1}{2}\,\langle U\rangle \quad , \quad \langle U\rangle \;=\; -\frac{Gm_1m_2}{a} \quad .} \tag{16}$$

This is the 2-body virial theorem, and $\langle U\rangle$ is the time-averaged potential energy of the system (proof left to the reader).

$*$ An example of this pertains to near-Earth orbit satellites, where the factor of 2 defines the added kinetic energy required for rockets to escape Earth's gravity relative to those that place satellites in orbit.

To prove the **general virial theorem**, we form the following momentum moment of particle positions:

$$Q \;=\; \sum_i \mathbf{p}_i \cdot \mathbf{r}_i \quad . \tag{17}$$

The time derivative of this can simply be recast as

$$\frac{dQ}{dt} \;=\; \frac{d}{dt} \sum_i m_i \,\frac{d\mathbf{r}_i}{dt} \cdot \mathbf{r}_i \;=\; \frac{1}{2}\,\frac{d^2 I}{dt^2} \quad , \quad I \;=\; \sum_i m_i r_i^2 \quad . \tag{18}$$

Here I is the **moment of inertia** of the multi-body system. This differential equation can be formed in an alternative way by retaining the differentiation of momentum so as to capture the force $\mathbf{F}_i = d\mathbf{p}_i/dt$ explicitly:

$$\frac{dQ}{dt} \;\equiv\; \frac{1}{2}\,\frac{d^2 I}{dt^2} \;=\; \sum_i \mathbf{p}_i \cdot \frac{d\mathbf{r}_i}{dt} + \sum_i \frac{d\mathbf{p}_i}{dt} \cdot \mathbf{r}_i \quad . \tag{19}$$

The first term on the right is just twice **the kinetic energy** K:

$$\sum_i \mathbf{p}_i \cdot \frac{d\mathbf{r}_i}{dt} \;=\; \sum_i m_i \,\mathbf{v}_i \cdot \mathbf{v}_i \;=\; 2K \quad . \tag{20}$$

Accordingly, we have formulated an energy equation, where the force term connects to the gravitational potential energy of the system, and the moment of inertia second derivative term describes the redistribution of rotational kinetic energy in the ensemble of bodies.

Defining $\mathbf{F}_{ij} = -\mathbf{F}_{ji}$ as the gravitational force on body i due to body j, then one can manipulate the force contribution using a double summation trick as follows:

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i \;=\; \sum_i \Biggl(\sum_{j\neq i} \mathbf{F}_{ij} \Biggr) \cdot \mathbf{r}_i \;\equiv\; \frac{1}{2} \sum_i \Biggl(\sum_{j\neq i} \bigl[\mathbf{F}_{ij} - \mathbf{F}_{ji} \bigr] \Biggr) \cdot \mathbf{r}_i \tag{21}$$

Performing a simple index relabeling $i \leftrightarrow j$ on the second term on the RHS establishes

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i \;=\; \frac{1}{2} \sum_i \sum_{j\neq i} \mathbf{F}_{ij} \cdot \bigl[\mathbf{r}_i - \mathbf{r}_j \bigr] \quad . \tag{22}$$

Now, the gravitational force law can be inserted,

$$\mathbf{F}_{ij} \;=\; -\frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|^3}\,\big[\mathbf{r}_i - \mathbf{r}_j\big] \quad , \tag{23}$$

so that

$$\sum_i \mathbf{F}_i \cdot \mathbf{r}_i \;=\; -\frac{1}{2}\sum_i \sum_{j\neq i} \frac{Gm_i m_j}{|\mathbf{r}_i - \mathbf{r}_j|} \;=\; U \quad , \tag{24}$$

the **total potential energy**. Observe that the factor of $1/2$ disappears here since each i, j pair is counted twice.

Collecting results, we have determined that

$$\frac{1}{2}\,\frac{d^2 I}{dt^2} \;=\; U + 2K \quad . \tag{25}$$

For ensembles that are *in kinetic equilibrium on long times (static)*, such as globular clusters, stars themselves, or gas at the centers of clusters of galaxies, the time derivative on the LHS is extremely small and may be neglected. The same can be applied for periodic systems if the equation is integrated over a multiple of the system period. The upshot is then that long term ensemble averages obey

$$\boxed{\langle U\rangle \;=\; -2\langle K\rangle \quad\Rightarrow\quad \langle E\rangle \;\equiv\; \langle K\rangle + \langle U\rangle \;=\; \frac{1}{2}\,\langle U\rangle \quad ,} \tag{26}$$

which is the generalized **virial theorem** for self-gravitating ensembles of bodies or gas particles.

• Its principal use is to forge a direct correlation between the kinetic "temperature" of such a system, i.e. $\langle K\rangle$, an observable, and the total gravitating mass of the ensemble, assuming that its mean radial extent is measurable.

4. PHOTOMETRIC CONCEPTS AND RADIATION

Matthew Baring – Lecture Notes for ASTR 350, Fall 2021

1 Magnitudes

The awareness that some stars are brighter than others dates back to the Greeks, who devised an **apparent magnitude** scale: **C & O, Sec. 3.2**

* $m = 1$ (bright) $\Rightarrow 6$ (faint)

• In the 19th century, astronomical developments enabled determination that such a range corresponded to a factor of 100 in brightness or **flux** $\mathcal{F}$.

* $\mathcal{F} =$ energy in photons/area/time at detector.

• The flux differs from **luminosity** L, which is the total radiated power of a star or source (i.e. the wattage of the light bulb!):

* $L =$ energy in photons/time.

Hence, the inverse square law of photon dilution establishes

$$\boxed{\mathcal{F} \; = \; \frac{L}{4\pi d^2}} \tag{1}$$

for a source at distance d from Earth.

Accordingly,

$$\mathcal{F}_{\odot} \;=\; \frac{L_{\odot}}{4\pi(1\mathrm{AU})^2} \;=\; 1.36\times 10^6 \mathrm{erg\ sec^{-1}cm^{-2}} \tag{2}$$

is the solar flux at Earth and is called the **solar constant**.

$*$ Note the cgs to SI unit conversion: 1 erg = 1 g cm^2 sec^{-2} $= 10^{-7}$ kg m^2 sec^{-2} $= 10^{-7}$ Joules.

The flux and luminosity both depend on the waveband under consideration.

• Observational equivalents of $\mathcal{F}$ and L are the **apparent magnitude** m and the **absolute magnitude** M. Using the Greek scale as a guideline led astronomers to the *definitions*

$$\mathcal{F} \;\propto\; 10^{-2m/5} \quad \text{and} \quad L \;\propto\; 10^{-2m/5}\, d^2 \;\propto\; 10^{-2M/5} \tag{3}$$

The constants of proportionality were "benchmarked" by setting $m = M$ for sources at a distance of $d = 10\,\mathrm{pc}$, a typical distance scale for nearby stars. Hence,

$$\boxed{m - M \;=\; 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right) \quad ,} \tag{4}$$

and we call this quantity a source's **distance modulus.**

$*$ Observe that for the sun, the brightest object in the sky, a complete outlier with $m_{\odot} = -26.81$, however its absolute magnitude is $M_{\odot} = 4.76$, typical of other main sequence stars.

• It is appropriate to now define the distance scale of a **parsec**, abbreviated **pc**. It is the distance at which 1 AU on the sky subtends 1 arcsec, i.e. the distance at which the **parallax** of a star is $\pi = 1\,''$:

$$1\mathrm{pc} \;=\; \frac{1.496\times 10^{13}\mathrm{cm}}{\pi/180/3600} \;=\; 3.09\times 10^{18}\mathrm{cm} \;=\; 3.26\ \mathrm{lt\ yr.} \tag{5}$$

Plot: Stellar parallax

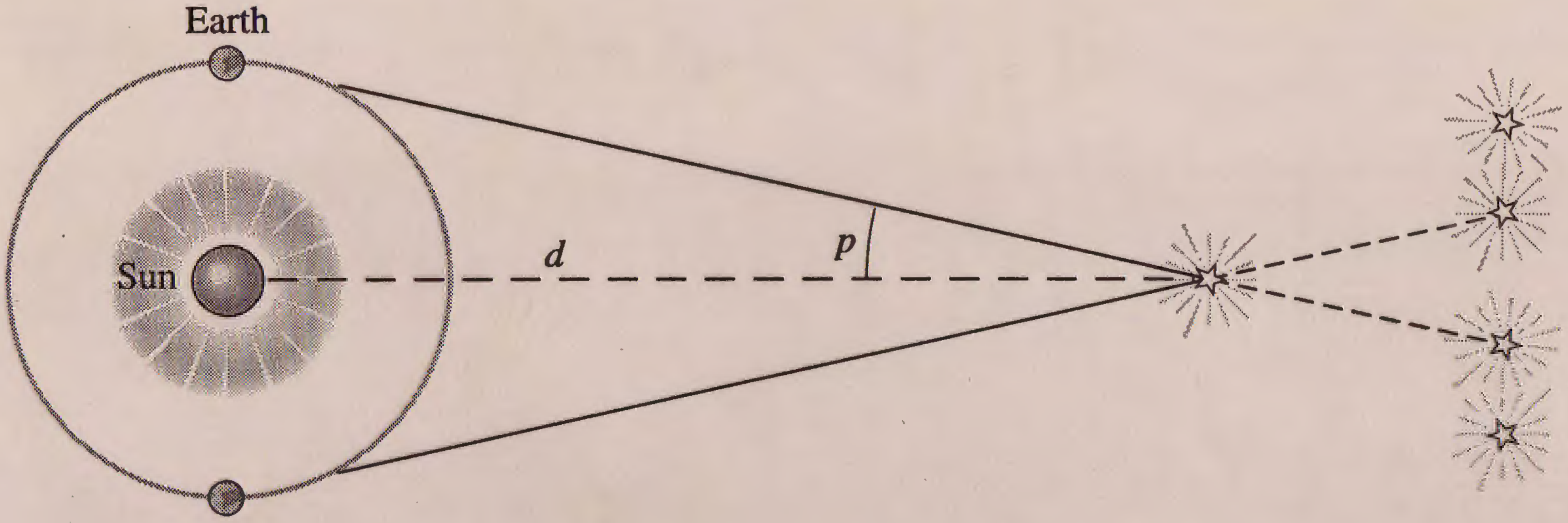


Figure 3.2 Stellar parallax: $d = 1/p''$ pc.