Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Find the volume common to 3 cylinders of radius 1 that lie along the three coordinate axes.

2. Let $y(x)$ be the solution to the differential equation $y'' = yx$. The following limit exists:

$$\lim_{x\to\infty} \frac{\ln y}{x^{3/2}}$$

   - compute it.

3. Find f satisfying $4x(1-x)f'' + 2(1-2x)f' + f = 0$ and $f(1) = 1$, $f'(1) = 1/2$.

4. Compute

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{\tan x}}.$$

5. Compute

$$\int_1^e \frac{\ln(x)}{(1+\ln(x))^2} dx.$$

6. Compute

$$\frac{1}{\pi}\int_0^{\pi} \left(\frac{\sin(10x)}{\sin x}\right)^2 dx.$$

7. Compute

$$\int_0^{1/e} \frac{dx}{\sqrt{-\ln(x)-1}}.$$

8. We have a triangle ABC with $AB = 2$, $BC = 3$, and $AC = 4$. Consider all lines XY such that X lies on AC, Y lies on BC, and triangle XYC has area half of that of ABC. What is the minimum possible length of XY?

9. u is a twice differentiable real-valued function on $[-1,1]$ with $u^2 + 2u'^2 + 2uu'' = 0$, $u(0) = \sqrt{5}$, and $u'(0) = \frac{3}{\sqrt{5}}$. Determine $u\left(\frac{\pi}{4}\right)$.

10. In tennis, players have two chances to hit a serve in. If the first serve is in, the point is played to completion (until either player wins the point). If the first serve is out, the player hits a second serve. If the second serve is in, the point is played to completion; otherwise, the server automatically loses the point. Andy can precisely control the velocity v of his serve up to 100mph. The faster his serve, the higher the probability of him winning the point if the serve goes in, but the higher the probability that the serve goes out. For a given v, the probability that Andy's serve is in is $p(v) = \frac{150-v}{150}$, and the probability that he wins the point after his serve goes in is $q(v) = \frac{v}{100}$. Assuming that he chooses optimal velocities for his first and second serves, compute the probability that Andy wins the point.