Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Given a rectangle with area 6 and perimeter 10, compute the length of the shorter side of the rectangle.

2. A garden contains 210 daisies and some number of roses. Each daisy contains two petals and two leaves, while each rose contains four petals and one leaf. There are 672 petals in the garden. Compute the number of leaves in the garden.

3. $ABCD$ is a square with side length 12. There is a point X on AB such that $AX = 4$, and a point Y on BC such that $CY = 5$. Compute the area of $\triangle BXY$.

4. A triangle with side lengths 6, 10, and 14 has area x. A triangle with side lengths 9, 15, and 21 has area y. Compute $\frac{x}{y}$.

5. In a Super Smash Brothers tournament, $\frac{1}{2}$ of the contestants play as Fox, $\frac{1}{3}$ of the contestants play as Falco, and $\frac{1}{6}$ of the contestants play as Peach. Given that there were 40 more people who played either Fox or Falco than who played Peach, how many contestants attended the tournament?

6. Lynnelle really loves peanut butter, but unfortunately she cannot afford to buy her own. Her roommate Jane also likes peanut butter, and Jane just bought a 100mL jar. Lynnelle has decided to steal some peanut butter from Jane's jar every day immediately after Jane eats, but to make sure Jane doesn't notice Lynnelle never steals more than 20mL and never steals so much that the amount remaining in the jar is more than halved. For example, if 50mL of peanut butter remains in the jar then Lynnelle will steal 20mL that day (since half of 50mL is 25mL, and Lynelle will steal at most 20mL in one day), and if 8mL remains then Lynnelle will steal 4mL that day (leaving 4mL, half of 8mL). If Jane eats a constant 10mL of peanut butter each day (or the rest of the jar, if the jar has less than 10mL in it) until the jar is empty, compute the amount Lynnelle steals (in mL).

7. Compute the number of ways 6 girls and 5 boys can line up if all 11 people are distinguishable and no two girls stand next to each other.

8. The line $y = x + 2015$ intersects the parabola $y = x^2$ at two points, (a, b) and (c, d). Compute $a + c$.

9. Ted wants to plot a sad face on his graphing calculator. He decides to represent the mouth with a downwards-opening parabola of the form $y = -x^2 + bx + c$. It should be centered directly below the eyes, which are drawn at $(0, 5)$ and $(6, 5)$, and should pass through the point $(2, 0)$ because that is Ted's favorite point. Compute the equation Ted should use for the parabola.

10. Consider a unit square $ABCD$. Let E be the midpoint of BC and F the intersection of AC and DE. Compute the area of triangle ADF.

11. Tyrant Tal, a super genius, wants to create an army of Tals. He and his clones can clone themselves, but the process takes an entire hour. Additionally, once the clone is created, it must

wait 2 hours before creating its own clones. Hence, at the end of the first hour, there could be 2 Tals (the original and 1 clone), and a clone created during the fifth hour can clone itself during the eighth hour. Compute the maximum possible size of his army after 10 hours (Tyrant Tal starts by himself and is a part of his own army).

12. An integer n is *almost square* if there exists a perfect square k^2 such that $|n - k^2| = 1$ and k is a positive integer. How many positive integers less than or equal to 2015 are almost square?

13. Two perpendicular lines have slopes that add up to 1. What are their slopes?

14. A certain high school has exactly 1000 lockers, numbered from 1 to 1000, all initially closed. Mark first opens every locker whose number has exactly 3 factors, starting with locker 4. Matt then opens every locker whose number is a power of 2, starting with locker 1. If Matt encounters a locker that Mark has already opened, he leaves it open. Compute the number of lockers that will be open when both Mark and Matt finish.

15. Find the unique $x > 0$ such that $\sqrt{x} + \sqrt{x + \sqrt{x}} = 1$.

16. A 4 inch by 3 inch rectangular sandwich is cut in half diagonally. A circular tomato slice can be placed on one of the sandwich halves so that it is tangent to each of the three edges of the sandwich. Compute the tomato's radius in inches.

17. Jordan is throwing darts at a 10 inch radius dart board. Unfortunately, Jordan's aim is very bad. He only hits the board one third of the time and the distribution of his darts is uniformly random. Assuming that the dart is extremely small and the radius of the bullseye is 5 inches, compute the probability Jordan will make at least one bullseye in 2 shots.

18. In a certain right triangle, dropping an altitude to the hypotenuse divides the hypotenuse into two segments of length 2 and 3 respectively. What is the area of the triangle?

19. Suppose you have 15 circles of radius 1. Compute the side length of the smallest equilateral triangle that could possibly contain all the circles, if you are free to arrange them in any shape, provided they don't overlap.

20. Andy has two identical cups, the first one is full of water and the second one is empty. He pours half the water from the first cup into the second, then a third of the water in the second into the first, then a fourth of the water from the first into the second and so on. Compute the fraction of the water in the first cup right before the 2015th transfer.

21. Let $f(x) = ax^2 + bx + c$ where $a \neq 0$. Find d, where $0 < d < 1$, such that $f(0) = 2014$, $f(d^2) = 2015$, $f(d) = 2016$, and the sum of the roots of f is 0.

22. A positive integer $n > 1$ is called *multiplicatively perfect* if the product of its proper divisors (divisors excluding the number itself) is n. For example, 6 is multiplicatively perfect since $6 = 1 \times 2 \times 3$. Compute the number of multiplicatively perfect integers less than 100.

23. Consider a unit cube and a plane that slices through it. The plane passes through the mid points of two adjacent edges on the top face, two on the bottom face, and the center of the cube. Compute the area of the cross section.

24. There are four seats arranged in a circle and a person is sitting on one of the seats. He rolls a standard six-sided die 6 times. For each roll of the die, if it lands on 4, he moves one seat

clockwise. Otherwise, he moves k seats counterclockwise where k is the number he rolled. Compute the probability that he ends up on the same seat he originally started on.

25. Let the sequence a be defined as $a_0 = 2, a_n = 1 + a_0 \cdot a_1 \cdot \ldots \cdot a_{n-1}$.
    Calculate $\sum_{i=0}^{2015} \frac{1}{a_i}$. Give your answer in terms of a_{2016}.