Time limit: 110 minutes.
Instructions: This test contains 25 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Compute the unique positive integer that, when squared, is equal to six more than five times itself.

2. Upon entering a tall building, you are confronted with a problem that has existed since the late nineteenth century—should you take the stairs, or the elevator? If you decide to take the stairs, it will take you 20 seconds to walk up each flight of stairs. If you decide to take the elevator, you will have to wait for 3 minutes for the elevator to arrive, after which it will take 3 seconds to move up each floor. Compute the minimum number of floors for which taking the elevator takes less time than taking the stairs.

3. David flips a fair coin five times. Compute the probability that the fourth coin flip is the first coin flip that lands heads.

4. The analog clock in front of you displays the time 3:40. Compute the number of degrees in the smaller angle formed by the minute hand and the hour hand of the clock.

5. Compute the smallest positive integer $x > 100$ such that every permutation of the digits of x is prime.

6. Charles has a salary of $\$123,456,789$ this year. Next year, his salary will increase by 300%. The year after, his salary will decrease by 75%. Compute his salary in two years.

7. Isosceles trapezoid $ABCD$ has $AB = 10$, $CD = 20$, $BC = AD$, and an area of 180. Compute the length of BC.

8. Ted has 2 red socks, 2 blue socks, and 2 green socks. He grabs three different socks at random. Compute the probability that they are all different colors.

9. Compute the smallest positive integer with exactly 6 distinct factors.

10. The coordinates of three vertices of a parallelogram are $A(1, 1)$, $B(2, 4)$, and $C(-5, 1)$. Compute the area of the parallelogram.

11. Alice and Bob are playing a game in which Alice has a $\frac{1}{3}$ probability of winning, a $\frac{1}{2}$ probability of tying, and a $\frac{1}{6}$ probability of losing. Given that Alice and Bob played a game which did not end in a tie, compute the probability that Alice won.

12. An infinite geometric sequence has a first term of 12, and all terms in the sequence sum to 9. Compute the common ratio between consecutive terms of the geometric sequence.

13. We define a positive integer p to be *almost prime* if it has exactly one divisor other than 1 and p. Compute the sum of the three smallest numbers which are almost prime.

14. Let a 5 digit number be termed a "valley" number if the digits (not necessarily distinct) in the number $\underline{a}\,\underline{b}\,\underline{c}\,\underline{d}\,\underline{e}$ satisfy $a > b > c$ and $c < d < e$. Compute the number of valley numbers that start with 3.

15. Given a regular 2014-gon, we construct 2014 isosceles triangles on the exterior of the polygon such that each isosceles triangle has an edge of the polygon as its base and has legs formed by the extensions of the two adjacent sides. Compute in degrees the largest angle of one such triangle.

16. Including the leap years, compute the average number of days in a year. Express your answer as a mixed number. (Years that are evenly divisible by 4 are leap years. However years that are evenly divisible by 100 are not leap years, unless they are also evenly divisible by 400, in which case they are leap years.)

17. We say that a number is *arithmetically sequenced* if the digits, in order, form an arithmetic sequence. Compute the number of 4-digit positive integers which are arithmetically sequenced.

18. Spencer eats ice cream in a right circular cone with an opening of radius 5 and a height of 10. If Spencer's ice cream scoops are always perfectly spherical, compute the radius of the largest scoop he can get such that at least half of the scoop is contained within the cone.

19. Let x be a two-digit positive integer. Let x' be the number achieved by switching the two digits in x (for example: if $x = 24$, $x' = 42$). Compute the number of x's that exist such that $x + x'$ is a perfect square.

20. For any 4-tuple (a_1, a_2, a_3, a_4) where each entry is either 0 or 1, call it *quadratically satisfiable* if there exist real numbers $x_1, \ldots, x_4$ such that $x_1x_4^2 + x_2x_4 + x_3 = 0$ and for each $i = 1, \ldots, 4$, x_i is positive if $a_i = 1$ and negative if $a_i = 0$. Find the number of *quadratically satisfiable* 4-tuples.

21. The tetrahedral dice printing company has misprogrammed their machines to print either 1, 2, 3, or 4 dots on each face, uniformly at random and independently. Compute the probability that the total number of dots on a random tetrahedral (four-sided) die is greater than 10.

22. For any positive integer $x \geq 2$, define $f(x)$ to be the product of the distinct prime factors of x. For example, $f(12) = 2 \cdot 3 = 6$. Compute the number of integers $2 \leq x < 100$ such that $f(x) < 10$.

23. For a positive integer a, let $f(a)$ be the average of all positive integers b such that $x^2 + ax + b$ has integer solutions. Compute the unique value of a such that $f(a) = a$.

24. Compute the number of ways there are to select three distinct lattice points in three-dimensional space such that the three points are collinear and no point has a coordinate with absolute value exceeding 1.

25. Let ABC be a triangle such that $AB = 3$, $BC = 4$, and $AC = 5$. Let X be a point in the triangle. Compute the minimal possible value of $AX^2 + BX^2 + CX^2$.