Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. David flips a fair coin five times. Compute the probability that the fourth coin flip is the first coin flip that lands heads.

2. Find the largest integer that divides $p^2 - 1$ for all primes $p > 3$.

3. We say that a number is *arithmetically sequenced* if the digits, in order, form an arithmetic sequence. Compute the number of 4-digit positive integers which are arithmetically sequenced.

4. For any positive integer $x \geq 2$, define $f(x)$ to be the product of the distinct prime factors of x. For example, $f(12) = 2 \cdot 3 = 6$. Compute the number of integers $2 \leq x < 100$ such that $f(x) < 10$.

5. Compute the number of ways there are to select three distinct lattice points in three-dimensional space such that the three points are collinear and no point has a coordinate with absolute value exceeding 1.

6. Fred lives on one of 10 islands sitting in a vast lake. One day a package drops uniformly at random on one of the ten islands. Fred can't swim and has no boat, but luckily there is a teleporter on each island. Each teleporter teleports to only one of the other ten teleporters (note that it may teleport to *itself*), and no two teleporters teleport to the same teleporter. If the configuration of the teleporters is chosen uniformly at random from all configurations that satisfy these constraints, compute the probability that Fred can get to the package using the teleporters.

7. A one-player card game is played by placing 13 cards (Ace through King in that order) in a circle. Initially all the cards are face-up and the objective of the game is to flip them face-down. However, a card can only be flipped face-down if another card that is '3 cards away' is face-up. For example, one can only flip the Queen face-down if either the 9 or the 2 (or both) are face-up.

   A player wins the game if they can flip all but one of the cards face-down. Given that the cards are distinguishable, compute the number of ways it is possible to win the game.

8. $a_0, a_1, \ldots$ is a sequence of positive integers where $a_n = n!$ for all $n \leq 3$. Moreover, for all $n \geq 4$, a_n is the smallest positive integer such that

   $$\frac{a_n}{a_i a_{n-i}}$$

   is an integer for all integers i, $0 \leq i \leq n$. Find a_{2014}.

9. Compute the smallest positive integer n such that the leftmost digit of 2^n (in base 10) is 9.

10. Compute the number of permutations of $1, 2, 3, \ldots, 50$ such that if m divides n the mth number in the permutation divides the nth number.