1. In triangle ABC, $AC = 7$. D lies on AB such that $AD = BD = CD = 5$. Find BC.
2. What is the perimeter of a rectangle of area 32 inscribed in a circle of radius 4?
3. Robin has obtained a circular pizza with radius 2. However, being rebellious, instead of slicing the pizza radially, he decides to slice the pizza into 4 strips of equal width both vertically and horizontally. What is the area of the smallest piece of pizza?
4. $ABCD$ is a regular tetrahedron with side length 1. Find the area of the cross section of $ABCD$ cut by the plane that passes through the midpoints of AB, AC, and CD.
5. In square $ABCD$ with side length 2, let P and Q both be on side AB such that $AP = BQ = \frac{1}{2}$. Let E be a point on the edge of the square that maximizes the angle PEQ. Find the area of triangle PEQ.
6. $ABCD$ is a rectangle with $AB = CD = 2$. A circle centered at O is tangent to BC, CD, and AD (and hence has radius 1). Another circle, centered at P, is tangent to circle O at point T and is also tangent to AB and BC. If line AT is tangent to both circles at T, find the radius of circle P.
7. $ABCD$ is a square such that $\overline{AB}$ lies on the line $y = x + 4$ and points C and D lie on the graph of parabola $y^2 = x$. Compute the sum of all possible areas of $ABCD$.
8. Let equilateral triangle ABC with side length 6 be inscribed in a circle and let P be on arc AC such that $AP \cdot PC = 10$. Find the length of BP.
9. In tetrahedron $ABCD$, $AB = 4$, $CD = 7$, and $AC = AD = BC = BD = 5$. Let I_A, I_B, I_C, and I_D denote the incenters of the faces opposite vertices A, B, C, and D, respecitvely. It is provable that AI_A intersects BI_B at a point X, and CI_C intersects DI_D at a point Y. Compute XY.
10. Let triangle ABC have side lengths $AB = 16$, $BC = 20$, $AC = 26$. Let $ACDE$, $ABFG$, and $BCHI$ be squares that are entirely outside of triangle ABC. Let J be the midpoint of EH, K be the midpoint of DG, and L the midpoint of AC. Find the area of triangle JKL.