1. AB is a diameter of a circle with radius 1. C lies on this circle such that $\widehat{AC} \,/\, \widehat{BC} = 4$. Find the (positive) difference in area between $\overline{AC} \cup \widehat{CA}$, the segment of the circle cut off by $\overline{AC}$, and $\overline{BC} \cup \widehat{CB}$, the segment cut off by $\overline{BC}$.

2. In trapezoid $ABCD$, $BC \parallel AD$, $AB = 13$, $BC = 15$, $CD = 14$, and $DA = 30$. Find the area of $ABCD$.

3. Let ABC be equilateral triangle of side 1. Draw three circles O_a, O_b, O_c with diameters BC, CA, and AB, respectively. Let S_a denote the area of the region inside O_a and outside of O_b and O_c. Define S_b and S_c similarly, and let S be the area of the intersection between the three circles. Find $S_a + S_b + S_c - S$.

4. Let $ABCD$ be a rectangle with area 2012. There exist points E on AB and F on CD such that $DE = EF = FB$. Diagonal AC intersects DE at X and EF at Y. Compute the area of triangle EXY.

5. What is the radius of the largest sphere that fits inside an octahedron of side length 1?

6. A red unit cube $ABCDEFGH$ (with E below A, F below B, etc.) is pushed into the corner of a room with vertex E not visible, so that faces $ABFE$ and $ADHE$ are adjacent to the wall and face $EFGH$ is adjacent to the floor. A string of length 2 is dipped in black paint, and one of its endpoints is attached to vertex A. How much surface area on the three visible faces of the cube can be painted black by sweeping the string over them?

7. Let ABC be a triangle with side lengths 5, 8, and 9 and incircle O. Consider the other tangent line to O parallel to BC, which intersects AB at B_a and AC at C_a. Let r_a be the inradius of triangle AB_aC_a, and define r_b and r_c similarly. Find $r_a + r_b + r_c$.

8. In triangle ABC, we have $a = 3$, $b = 5$, and $c = 6$. For some radius r, draw the three circles with radius r such that the centers O_a and O_b are outside the triangle while the center O_c is inside the triangle. If circles O_a and O_b intersect at C, O_b and O_c intersect at A, O_c and O_a intersect at B, and all three circles intersect at a fourth distinct point, find r.

9. In quadrilateral $ABCD$, $\angle ABD \cong \angle BCD$ and $\angle ADB = \angle ABD + \angle BDC$. If $AB = 8$, and $AD = 5$, find BC.

10. A large flat plate of glass is suspended $\sqrt{2/3}$ units above a large flat plate of wood. (The glass is infinitely thin and causes no funny refractive effects.) A point source of light is suspended $\sqrt{6}$ units above the glass plate. An object rests on the glass plate of the following description. Its base is an isosceles trapezoid $ABCD$ with $AB \| DC$, $AB = AD = BC = 1$, and $DC = 2$. The point source of light is directly above the midpoint of CD. The object's upper face is a triangle EFG with $EF = 2$, $EG = FG = \sqrt{3}$. G and AB lie on opposite sides of the rectangle $EFCD$. The other sides of the object are $EA = ED = 1$, $FB = FC = 1$, and $GD = GC = 2$. Compute the area of the shadow that the object casts on the wood plate.