1. **Answer: 10**

   Solution: Let x be the number of hours spent training during the second week. We have that $3x+x+\frac{x}{2}=45$. Therefore, $x=\boxed{10}$.

2. **Answer: 240**

   While we could set up a system of equations, a slicker solutions notices that Nick takes 5 minutes to type the extra 1200 words, so his speed is $1200/5=\boxed{240}$ words per minute.

3. **Answer: 149**

   Solution: The first few Tribonacci numbers are 0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 149. $\boxed{149}$ is the smallest Tribonacci number greater than 100, and it also turns out to be prime, so that is our answer.

4. **Answer: 96**

   Solution: In the minimum case, Michael can work 8 hours a day for five days, thereby earning no overtime pay and earning exactly \$320. In the maximum case, Michael works 40 hours without a single break. This spans two days; there are 16 hours of work at regular pay and 24 hours of work at overtime pay. Therefore, Michael earns $16\times\$8+24\times\$12=\$416$. Therefore, $x=416$ and $y=320$, so $x-y=\boxed{96}$.

5. **Answer: 10**

   Solution: Since 60 people claimed to be good at math, 40 people must have claimed to be bad at math, of which 30 were telling the truth. Hence, $\boxed{10}$ people were being modest.

6. **Answer: 119**

   Solution: We know that the hour and minute hands cross exactly once. Let m be the number of minutes past one o'clock that this happens. The angle between the minute hand and the 12 must be equal to the angle between the hour hand and the 12. Since 1 minute is $\frac{360^\circ}{60}=6^\circ$ on the clock and 1 hour is $\frac{360^\circ}{12}=30^\circ$, we have $6m=30(1+\frac{m}{60})$, so $m=\frac{60}{11}=5\frac{5}{11}$. Note that the second hand is not at the same position at this time, so we do not have to worry about a triple crossing.

   On the other hand (no pun intended), the second hand crosses the hour hand once every minute, for a total of 60 crossings. Also, the second hand crosses the minute hand once every minute except the first and last, since those crossings take place at 1:00 and 2:00, for a total of 58 crossings. There is a grand total of $1+60+58=\boxed{119}$ crossings.

7. **Answer: 250**

   Solution: The set of the first ten positive integers contains five odd integers and five even integers. There are $\binom{5}{k}$ ways to choose k odd integers from five odd integers, and also there are $\binom{5}{k}$ ways to choose k even integers from five even integers. Thus there are $\binom{5}{k}^2$ ways to pick a balanced subset containing k odd integers and k even integers so the answer is $\sum_{k=1}^{4}\binom{5}{k}^2=5^2+10^2+10^2+5^2=\boxed{250}$.

8. **Answer: 23**

   Solution: The minimum food cost for a team is $6(\$3)=\$18$, and the maximum food cost is $10(\$4)=\$40$. Note that all intermediate values can be achieved. Suppose n dollars can be achieved by purchasing a hamburgers and b hot dogs, where $18\le n<40$. If $b>0$, then $n+1$ dollars can be achieved by purchasing $a+1$ hamburgers and $b-1$ hot dogs. If $b=0$, then $n+1$ dollars can be achieved by purchasing $a-2$ hamburgers and $b+3$ hot dogs. (This increases the number of team members by 1.) Repeating this process until \$40 is reached, the number of team members cannot decrease, and since we end up with 10 team members, the number of team members is always contained within 6 and 10.

   Hence the number of different values is $40-18+1=\boxed{23}$.

9. **Answer: 277**

Solution: Notice that there are 6 sets of 1's and 3's that sum to 16. For a given set suppose there are n 3's we have a total of $(16-3n)+n = 16-2n$ numbers so we want to compute $\binom{16-2n}{n}$. Hence the total number of possible sequences is:

$$\sum_{n=0}^{5} \binom{16-2n}{n} = \boxed{277}$$

10. **Answer: 1242**

Solution: All one-digit numbers have no repeating digits, so that gives us 9 numbers. For a two-digit number to have no repeating digits, the first digit must be between 1 and 9, while the second digit must not be equal to the first (but can be 0), giving us $9 \cdot 9 = 81$ numbers. For a three-digit number the same argument yields $9 \cdot 9 \cdot 8 = 648$ numbers. For a four-digit number between 1000 and 1999, we obtain $1 \cdot 9 \cdot 8 \cdot 7 = 504$ numbers. Finally, there are no numbers between 2000 and 2012 inclusive with no repeating digits, so the total is $9 + 81 + 648 + 504 = \boxed{1242}$.

11. **Answer: 48**

Solution: Suppose that there are a total of x problems on the problems set. Then Stephen's rate is $\frac{x}{6}$ problems per hour, while Jim's is $\frac{x}{8}$. their combined rate (including the efficiency bonus) times 3 hours is the number of problems, $3(\frac{x}{6} + \frac{x}{8} + 2) = x$. Solving for x, we obtain $x = \boxed{48}$.

12. **Answer: $\mathbf{21 - 12\sqrt{3}}$**

Solution: Let x be the length of a side of square $DEFG$. Then $DE = EF = x$. Note that $\triangle ADE$ is equilateral since $\overline{DE} \parallel \overline{BC}$ and hence $\triangle ADE \sim \triangle ABC$, so $AE = DE = x$, and consequently $EC = 1-x$. Since $\triangle ECF$ is a $30° - 60° - 90°$ triangle, we have the proportion

$$\frac{EF}{EC} = \frac{x}{1-x} = \frac{\sqrt{3}}{2},$$

so $x = \frac{\sqrt{3}}{2+\sqrt{3}} = 2\sqrt{3} - 3$. Hence the area of $DEFG$ is $x^2 = \boxed{21 - 12\sqrt{3}}$.

13. **Answer: $\mathbf{18\sqrt{5}}$**

Solution: Since $\angle ADB = \angle ABC = 90°$, $\triangle ABC \sim \triangle ADB$. In particular, $\dfrac{AB}{AD} = \dfrac{AC}{AB}$, so $AC = \dfrac{AB^2}{AD}$. Therefore, $AC = \dfrac{36^2}{24} = 54$. Since $AD = 24$, $DC = 30$. By Power of a Point, $BC = \sqrt{30 \times 54} = \boxed{18\sqrt{5}}$.

14. **Answer: 7**

Solution: Using brute force, we note that 3, 4, 5, and 6 are invalid, but $7 = 21_3$. Thus, the answer is $\boxed{7}$.

15. **Answer: $\mathbf{72\sqrt{3}}$**

Solution: Since opposite sides of a parallelogram are equal, $AB = BC = CD = DA = 12$. Since adjacent angles of a parallelogram are supplementary, $\angle BCD = \angle CDA = 60°$. Therefore, when we draw diagonal BD, we get two equilateral triangles, both with side length 12. The area of an equilateral triangle with side length s is $\dfrac{s^2\sqrt{3}}{4}$, so therefore the area of the parallelogram is $\dfrac{12^2\sqrt{3}}{2} = \boxed{72\sqrt{3}}$.

16. **Answer: 9**

Solution: Clearly the largest value of x would be obtained if $a = \sum_{i=0}^{1961} 9(10)^i$, hence $x = 1962(9) = 17658$. It follows that, $y = 1 + 7 + 6 + 5 + 8 = 27$ and finally that $z = 2 + 7 = \boxed{9}$.

17. **Answer: $\mathbf{\frac{3\pi}{10}}$**

Solution: Let O be the center of the circle. Note that CO bisects AB, so the areas of $\triangle ACO$ and $\triangle BCO$ are equal. Hence, the desired difference in segment areas is equal to the difference in the areas of the corresponding sectors. The sector corresponding to $\overset{\frown}{AC}$ has area $\frac{2\pi}{5}$, and the sector corresponding to $\overset{\frown}{BC}$ has area $\frac{\pi}{10}$, so the desired difference is $\boxed{\frac{3\pi}{10}}$.

18. **Answer: 6**

Solution: Let a be the expected number of minutes necessary to move from a corner to the center square. Let b be the expected number of minutes necessary to move from one of the middle edge squares to the center. We have that $a = b + 1$ and $b = \frac{2(a+1)}{3} + \frac{1}{3}$. Solving yields $b = 5$ and $a = \boxed{6}$.

19. **Answer: -125**

Solution: If the three roots of f are r_1, r_2, r_3, we have $f(x) = x^3 - (r_1+r_2+r_3)x^2 + (r_1r_2+r_1r_3+r_2r_3)x - r_1r_2r_3$, so $f(-1) = -1 - (r_1 + r_2 + r_3) - (r_1r_2 + r_1r_3 + r_2r_3) - r_1r_2r_3$. Since $r_1r_2r_3 = 64$, the arithmetic mean-geometric mean inequality reveals that $r_1 + r_2 + r_3 \geq 3(r_1r_2r_3)^{1/3} = 12$ and $r_1r_2 + r_1r_3 + r_2r_3 \geq 3(r_1r_2r_3)^{2/3} = 48$. It follows that $f(-1)$ is at most $-1 - 12 - 48 - 64 = \boxed{-125}$. We have equality when all roots are equal, i.e. $f(x) = (x - 4)^3$.

20. **Answer: $\frac{13}{36}$**

Solution: All squares that are on the edge of the chess board can hit 21 squares; there are 28 such squares. The 20 squares on the edge of the inner 6×6 chessboard can hit 23 squares, the 12 squares on the boundary of the 4×4 chessboard can hit 25 squares, and the remaining 4 can hit 27 squares. The probability then follows as $\frac{21 \times 28 + 23 \times 20 + 25 \times 12 + 27 \times 4}{64 \times 63} = \boxed{\frac{13}{36}}$.

21. **Answer: 252**

Solution: We can use the standard method of setting up a two-variable system and solving for the height of the trapezoid. However, since one base is half the length of the other, we may take a shortcut. Extend AB and CD until they meet at E. Clearly, BC is a midline of triangle EAD, so we have $EA = 2BA = 26$ and $ED = 2CD = 28$. The area of EAD is therefore four times that of a standard 13-14-15 triangle, which we know is $\frac{1}{2} \times 14 \times 12 = 84$ (since the altitude to the side of length 14 splits the triangle into 9-12-15 and 5-12-13 right triangles). The area of the trapezoid is $\frac{3}{4}$ the area of EAD by similar triangles, and is therefore $3 \times 84 = \boxed{252}$.

A similar solution notices that after dropping perpendiculars from B and C to AD, we are left with a rectangle and two triangles that sum to a 13-14-15 triangle.

22. **Answer: 1020**

Solution: Clearly, either 1 or 10 must be in the a^{th} or $(a+1)^{th}$ slot of the permutation. Since we can construct an equivalent permutation with 1 in the middle by replacing each number i with $11 - i$, assume that 10 is in one of these slots, and we will multiply the final result by 2. We can pick any nonempty strict subset of the first 9 positive integers, sort it, place it at the beginning of the permutation, then place 10, then place the unchosen numbers in decreasing order. There are $2^9 - 2 = 510$ ways to do this. Therefore, there are $2 \times 510 = \boxed{1020}$ quadratic permutations of the first 10 positive integers.

23. **Answer: 5**

Solution: The one-digit boring primes are 2, 3, 5, and 7. The only two-digit boring prime is 11, since 11 divides all other two-digit boring numbers. No three-digit boring numbers are prime, since 111 divides all of them and $111 = 3 \times 37$. No four-digit boring numbers are prime since they are all divisible by 11. Therefore, there are $\boxed{5}$ positive integers less than 10000 which are both prime and boring.

24. **Answer: -24**

Solution: Consider the polynomial $Q(x) = P(x) - 3$. Q has roots at $x = 2$ and $x = 3$. Moreover, since these roots are maxima, they both have multiplicity 2. Hence, Q is of the form $a(x-2)^2(x-3)^2$, and so $P(x) = a(x-2)^2(x-3)^2 + 3$. $P(1) = 0 \implies a = -\frac{3}{4}$, allowing us to compute $P(5) = -\frac{3}{4}(9)(4) + 3 = \boxed{-24}$.

25. **Answer: $(-2, 2)$**

Solution: Substitute $y = x^3$, so now we want to find the values of k such that $y + \frac{1}{y} = k$ has no real solutions in y. In particular, since $y = x^3$ is an invertible function, $x^3 + \frac{1}{x^3} = k$ does not have a real solution in x if and only if $y + \frac{1}{y} = k$ has no real solutions in y. Clearing the denominators, $y^2 + 1 = ky$ or equivalently $y^2 - ky + 1 = 0$. This quadratic equation has no solutions when the discriminant $k^2 - 4$ is negative, which occurs when $-2 < k < 2$. Therefore the ordered pair is $\boxed{(-2, 2)}$.