1. **Answer: 5**

   Solution: The one-digit boring primes are 2, 3, 5, and 7. The only two-digit boring prime is 11, since 11 divides all other two-digit boring numbers. No three-digit boring numbers are prime, since 111 divides all of them and $111 = 3 \times 37$. No four-digit boring numbers are prime since they are all divisible by 11. Therefore, there are $\boxed{5}$ positive integers less than 10000 which are both prime and boring.

2. **Answer: $\frac{13}{36}$**

   Solution: All squares that are on the edge of the chess board can hit 21 squares; there are 28 such squares. The 20 squares on the edge of the inner 6×6 chessboard can hit 23 squares, the 12 squares on the boundary of the 4×4 chessboard can hit 25 squares, and the remaining 4 can hit 27 squares. The probability then follows as $\dfrac{21 \times 28 + 23 \times 20 + 25 \times 12 + 27 \times 4}{64 \times 63} = \boxed{\dfrac{13}{36}}$.

3. **Answer: 30**

   Solution: Observe that 2^{10n} begins with the digit 1 when the fractional part of $\log_{10} 2^{10n} = 10n \log_{10} 2 \approx 3.0103n$ is $< \log_{10} 2$. Therefore, we want $0.0103n > \log_{10} 2 \approx 0.30103 \Rightarrow n \geq \boxed{30}$.

4. **Answer: 32**

   Solution: Note that $x \equiv 1 \pmod{x-1}$, and so $x^n \equiv 1 \pmod{x-1}$ for all positive integers n. Hence, the number 201020112012 in base x is congruent to the sum of its digits $= 12 \pmod{x-1}$. Therefore, we simply need to find all $x \geq 3$ such that $12 \equiv 0 \pmod{x-1} \iff (x-1) \mid 12$, so $x - 1 = 1, 2, 3, 4, 6, 12 \implies x = 3, 4, 5, 7, 13$ (since $x \geq 3$). Hence, our answer is $\boxed{32}$.

5. **Answer: $\frac{43}{512}$**

   Solution: We can describe a table by a, b, c, d $(1 \leq a, b, c, d \leq 8)$ the final lengths of each of the four legs in clockwise order. How much a table is tipped north to south will be determined by the difference between the lengths a, b and c, d, and east to west by the difference between the lengths a, c and b, d. Hence, for the table to not wobble we must have $a - c = b - d \iff a - b = c - d \iff a + d = b + c$.

   We can therefore split into cases based on $S = a + d = b + c$. The number of ordered pairs (x, y) such that $x + y = S$ and $1 \leq x, y \leq 8$ is $T_S = 8 - |S - 9|$ (similar to adding the values on two 8-sided dice). The number of choices for (a, d) is therefore T_S and the number of choices for (b, c) is T_S, so the number of choices for (a, b, c, d) is T_S^2.

   Summing over all possible values of S this is

$$\begin{aligned} T_2^2 + \ldots T_{16}^2 &= (8 - |2 - 9|)^2 + \ldots (8 - |16 - 9|)^2 \\ &= 1^2 + 2^2 + \cdots + 7^2 + 8^2 + 7^2 + \cdots + 2^2 + 1^2 \\ &= 2\left(1^2 + \cdots + 7^2\right) + 8^2 \\ &= 2 \cdot \frac{7 \cdot 8 \cdot 15}{6} + 8^2 \\ &= 7 \cdot 8 \cdot 5 + 8^2 \\ &= 8\,(7 \cdot 5 + 8) \end{aligned}$$

   Hence, the probability is

$$\frac{8\,(7 \cdot 5 + 8)}{8^4} = \frac{7 \cdot 5 + 8}{8^3} = \boxed{\frac{43}{512}}$$

6. **Answer: $\dfrac{2}{11}$**

   Solution: If the two ants are not on the same vertex, they can either be on opposite vertices or on adjacent vertices. Let x and y be the probabilities that the ants will eventually meet on an edge when starting out from opposite vertices and from adjacent vertices, respectively. From opposite vertices, one of the ants must move to one of the remaining four vertices, which are all equivalent with respect to the other ant. That ant can either meet the first ant at a vertex, become adjacent to it (two ways to do this), or again become opposite from it. So

$$x = \frac{1}{4}x + \frac{1}{2}y.$$

If the two ants are adjacent, the cases become slightly more complicated. If the first ant moves towards the second ant, the second ant can move towards it (meeting on an edge); otherwise they will be adjacent. If the first ant moves away from the second ant, they will become adjacent no matter what the second ant does. If the first ant moves to the side (two ways to do this), they will be opposite if the second ant chooses the other direction, and will meet at a vertex if it chooses the same direction. Otherwise they will be adjacent. So

$$y = \frac{1}{8}x + \frac{11}{16}y + \frac{1}{16}.$$

This system of equations is easily solved to obtain $x = \boxed{\frac{2}{11}}$.

7. **Answer: 2730**

   Solution: Let D be the desired greatest common divisor. By Fermat's Little Theorem we have: $n^{13} \equiv (n^6)^2(n) \equiv (n^3)^2(n) \equiv n^4 \equiv n^2 \equiv n \mod 2$.

   Hence $2 \mid (n^{13} - n)$ for all n, so $2 \mid D$. Similarly we can show that $p \mid D$ for $p \in \{3, 5, 7, 13\}$. Since these are all prime, their product, 2730, divides D.

   $2^{13} - 2 = 8190 = 3(2730)$, so D is either 2730 or 3(2730). As $3^{13} - 3 = 3(3^{12} - 1)$ is not divisible by 9, $D = \boxed{2730}$.

8. **Answer: 10**

   Solution: We claim that the set of all primes less than 30 is the smallest set S which covers 30. We first prove that no smaller set can exist; assume that one does exist. This smaller set cannot contain some prime $p < 30$. Note that, therefore, p and $p+1$ are indistinguishable. This is a contradiction. We must now show that the set of all primes less than 30 is valid. Since two consecutive integers are relatively prime, if some prime p divides k, it does not divide $k+1$. Therefore, $\left\lceil \frac{k+1}{p} \right\rceil = \frac{k}{p} + 1$. Therefore, the set of all primes less than k will always cover k. It remains to compute the number of primes less than 30. There are $\boxed{10}$ primes less than 30.

9. **Answer: $10 - \pi^2$**

   Solution: We use partial fractions repeatedly to obtain that

$$\begin{aligned}
\frac{1}{n^3(n+1)^3} &= \left(\frac{1}{n} - \frac{1}{n+1}\right)^3 \\
&= \frac{1}{n^3} - \frac{1}{(n+1)^3} - 3\frac{1}{n(n+1)}\left(\frac{1}{n} - \frac{1}{n+1}\right) \\
&= \frac{1}{n^3} - \frac{1}{(n+1)^3} - 3\left(\frac{1}{n} - \frac{1}{n+1}\right)^2 \\
&= \frac{1}{n^3} - \frac{1}{(n+1)^3} - \frac{3}{n^2} - \frac{3}{(n+1)^2} + \frac{6}{n(n+1)} \\
&= \frac{1}{n^3} - \frac{1}{(n+1)^3} - \frac{3}{n^2} - \frac{3}{(n+1)^2} + \frac{6}{n} - \frac{6}{n+1}.
\end{aligned}$$

Then by taking sums and using the property of telescoping sums we have

$$\sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3} = 1 - 3\frac{\pi^2}{6} - 3\left(\frac{\pi^2}{6} - 1\right) + 6 = \boxed{10 - \pi^2}.$$

10. **Answer: 204**

Solution: Observe that a polynomial

$$I_a(x) = 1 - (x-a)^{p-1}$$

takes value 1 at $x = a$ and zero elsewhere in mod p, by Fermat's little theorem. Thus for any polynomial F mod p, we have

$$F(x) - \sum_{a=0}^{p-1} F(a) I_a(x) = 0 \pmod p$$

for all integers x. (It is interesting to note that the left hand side is a polynomial of degree $\leq p-1$ with roots $0, 1, \cdots, (p-1)$, and hence it must equal zero for all x mod p.) This means that all polynomials mod p of degree less than p have one-to-one correspondance to p-tuples of $(F(0), F(1), \cdots, F(p-1))$ mod p. Since F not having any roots is equivalent to $F(a) \neq 0$ for all integers a, there are $(p-1)^p$ ways to choose $(F(0), F(1), \cdots, F(p-1))$. This equals the number of integer polynomials mod 5 of degree at most 4 that do not have any integer roots mod 5.

We now investigate when such polynomials are of order at most 3. Note that coefficient of x^{p-1} in $\sum_{a=0}^{p-1} F(a) I_a(x)$ is $-\sum F(a)$, so we want $\sum F(a) = 0 \pmod p$. We define

$$\begin{aligned} A_n &= \text{the number of } n \text{ tuples } (a_1, \cdots, a_n) \text{ satisfying} \\ &1 \leq a_i \leq p-1, \quad p \mid a_1 + \cdots + a_n \end{aligned}$$

where we want to find A_p. We establish the recurrence relation on A_n. For the initial condition we have $A_1 = 0$ and $A_2 = p-1$. For $n > 2$, we note that $(a_1, a_2, \cdots, a_n)$ is in A_n for any combination of $(a_1, a_2, \cdots, a_{n-1})$ by setting $a_n \equiv -(a_1 + \cdots + a_{n-1}) \pmod p$, unless $a_1 + \cdots + a_{n-1}$ is divisible by p. This gives the recurrence relation

$$A_n = (p-1)^{n-1} - A_{n-1}$$

which in expanded form yields

$$A_n = (p-1)^{n-1} - (p-1)^{n-2} + (p-1)^{n-3} - \cdots + (-1)^{n-2}(p-1).$$

So the answer is $A_p = \frac{(-1)^{p-2}(p-1) + (p-1)^p}{1 + (p-1)}$. Evaluating at $p = 5$, we get $A_p = \frac{-4+4^5}{5} = \frac{1020}{5} = \boxed{204}$.