1. Define a number to be *boring* if all the digits of the number are the same. How many positive integers less than 10000 are both prime and boring?

2. Two different squares are randomly chosen from an 8×8 chessboard. What is the probability that two queens placed on the two squares can attack each other? Recall that queens in chess can attack any square in a straight line vertically, horizontally, or diagonally from their current position.

3. Given that $\log_{10} 2 \approx 0.30103$, find the smallest positive integer n such that the decimal representation of 2^{10n} does not begin with the digit 1.

4. Find the sum of all integers x, $x \geq 3$, such that 201020112012_x (that is, 201020112012 in base x) is divisible by $x - 1$.

5. A short rectangular table has four legs, each 8 inches long. For each leg Bill picks a random integer x, $0 \leq x < 8$ and cuts x inches off the bottom of that leg. After he's cut all four legs, compute the probability that the table won't wobble (i.e. that the ends of the legs are coplanar).

6. Two ants are on opposite vertices of a regular octahedron (an 8-sized polyhedron with 6 vertices, each of which is adjacent to 4 others), and make moves simultaneously and continuously until they meet. At every move, each ant randomly chooses one of the four adjacent vertices to move to. Eventually, they will meet either at a vertex (that is, at the completion of a move) or on an edge (that is, in the middle of a move). Find the probability that they meet on an edge.

7. Determine the greatest common divisor of the elements of the set $\{n^{13} - n \mid n \in \mathbb{Z}\}$.

8. Let a set of positive integers S, all greater than 1, *cover* an integer x if for every pair of integers k and l such that $2 \leq k < l \leq x$, $\left\lceil \frac{k}{y} \right\rceil \neq \left\lceil \frac{l}{y} \right\rceil$ for at least one integer y in S. How many numbers are in the smallest set S which covers 30?

9. Evaluate $\sum_{n=1}^{\infty} \frac{1}{n^3(n+1)^3}$. You can use Euler's result $\sum_{n=1}^{\infty} \frac{1}{n^2} = \pi^2/6$.

10. How many integer polynomials mod 5 of degree at most 3 do not have any integer roots mod 5?

    Note that two polynomials $F(x)$ and $G(x)$ are equivalent mod 5 if and only if $F(x) - G(x) = 5 \cdot H(x)$ for some integer polynomial $H(x)$, and $F(x)$ having n as a root mod 5 simply means $5 \mid F(n)$.