1. **Answer: $4\ln(x-2) - 4\ln(x-1) + 3/(x-1)$ or equivalent forms**

   Expand by partial fractions:

$$\frac{x+2}{(x-1)^2(x-2)} = \frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}.$$

   Then $x+2 = A(x-1)^2 + B(x-2)(x-1) + C(x-2)$. If $x=2$, then $4 = A$. If $x=1$, then $C=-3$. If $x=0$, then $2 = 4+2B+6 \Rightarrow B=-4$. So

$$\begin{aligned}\int \frac{x+2}{(x-1)^2(x-2)}dx &= \int \left[\frac{A}{x-2} + \frac{B}{x-1} + \frac{C}{(x-1)^2}\right] dx \\ &= 4\ln(x-2) - 4\ln(x-1) + 3/(x-1).\end{aligned}$$

2. **Answer: $\boldsymbol{\frac{\pi^2}{4}}$**

   By taking the second derivative, we see that the points of inflection occur when $-\cos x = 0$, so $x = \frac{\pi}{2}, \frac{3\pi}{2}$. At $\frac{\pi}{2}$, the slope of the tangent line is the derivative evaluated at this point, which is $-\sin\left(\frac{\pi}{2}\right) = -1$. So the tangent line has equation $y = -x+b$, and since $\cos\left(\frac{\pi}{2}\right) = 0$, $0 = -\frac{\pi}{2} + b$ and so the tangent line is given by $y = -x + \frac{\pi}{2}$. Similarly, we can find that the tangent line at $\frac{3\pi}{2}$ is $y = x - \frac{3\pi}{2}$. The intersection of the two tangent lines forms the vertex of the triangle, which occurs when $-x + \frac{\pi}{2} = x - \frac{3\pi}{2}$, or at $x = \pi$. At this point, both tangent lines equal $-\frac{\pi}{2}$. Hence the triangle formed has height $\frac{\pi}{2}$. The tangent lines cross the x-axis when $y=0$, giving $\frac{\pi}{2}$ and $\frac{3\pi}{2}$, which are separated by π. Hence the area of the triangle is $\frac{1}{2}\cdot\frac{\pi}{2}\cdot\pi = \frac{\pi^2}{4}$.

3. **Answer: $(1, 2/3, 1/2, 4/15)$**

   Just compute the Taylor expansion of $f''(x) - 2xf'(x) - 2f(x)$ to the third term, which is

$$(2a + 6bx + 12cx^2 + 20dx^3 + \cdots) - (2x + 4ax^2 + 6bx^3 + \cdots) - (2 + 2x + 2ax^2 + 2bx^3 + \cdots).$$

   All coefficients should be zero, so $2a-2=0$, $6b-4=0$, $12c-6a=0$ and $20d-8b=0$. Solving these equations gives the answer.

4. **Answer: $\ln(2)$**

   Note that $\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$ and that $\ln(x+1) = \int \frac{1}{1+x}dx = C + x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^3}{4} + \ldots$. Since $\ln(1) = 0 = C$, we set $C=0$. Evaluating at $x=1$, we get that $\ln(2) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$.

5. **Answer: $\boldsymbol{f(x) = (2x-1)e^x}$**

   Differentiate both sides to get

$$f'(x) = f'(x) + \int_0^x e^{x-y} f'(y)dy - (x^2+x)e^x.$$

   But

$$\int_0^x e^{x-y} f'(y)dy = f(x) + (x^2 - x + 1)e^x$$

   so by substituting it we get

$$f(x) + (x^2-x+1)e^x - (x^2+x)e^x = 0,$$

   and $f(x) = (2x-1)e^x$.

6. **Answer: $\boldsymbol{\frac{5\sqrt{5}+4}{6}}$**

   First, we eliminate the absolute value by finding the intervals on which the expression inside is positive and negative. Sketching $\sin(2x)$ and $\sin(3x)$ suggests that there are two points $0 < x < \pi$ such that $\sin(2x) = \sin(3x)$. The identity $\sin(x) = \sin(\pi - x)$ gives us the solution $x = \frac{\pi}{5}$, and $\sin(x) = \sin(3\pi - x)$

gives us $x = \frac{3\pi}{5}$. Then we have $\sin(3x) > \sin(2x)$ on the intervals $(0, \frac{\pi}{5})$ and $(\frac{3\pi}{5}, \pi)$ but $\sin(2x) > \sin(3x)$ on $(\frac{\pi}{5}, \frac{3\pi}{5})$. So the integral becomes:

$$\int_0^{\pi} |\sin(2x) - \sin(3x)|\,\mathrm{d}x = \int_0^{\frac{\pi}{5}} [\sin(3x) - \sin(2x)]\mathrm{d}x + \int_{\frac{\pi}{5}}^{\frac{3\pi}{5}} [\sin(2x) - \sin(3x)]\mathrm{d}x + \int_{\frac{3\pi}{5}}^{\pi} [\sin(3x) - \sin(2x)]\mathrm{d}x$$

Next, we let $F(x)$ equal $\frac{\cos(2x)}{2} - \frac{\cos(3x)}{3}$, so that $F'(x) = \sin(3x) - \sin(2x)$. Then the integral simplifies to:

$$\begin{aligned}
&\left[F\left(\frac{\pi}{5}\right) - F(0)\right] - \left[F\left(\frac{3\pi}{5}\right) - F\left(\frac{\pi}{5}\right)\right] + \left[F(\pi) - F\left(\frac{3\pi}{5}\right)\right] \\
&= F(\pi) + 2F\left(\frac{\pi}{5}\right) - 2F\left(\frac{3\pi}{5}\right) - F(0) \\
&= \frac{1}{2} + \frac{1}{3} + \cos\frac{2\pi}{5} - \frac{2}{3}\cos\frac{3\pi}{5} - \cos\frac{6\pi}{5} + \frac{2}{3}\cos\frac{9\pi}{5} - \frac{1}{2} + \frac{1}{3} \\
&= \frac{2}{3} + \frac{5}{3}\cos\frac{2\pi}{5} + \frac{5}{3}\cos\frac{\pi}{5}.
\end{aligned}$$

Now we use the identities $\cos\frac{\pi}{5} = \frac{\sqrt{5}+1}{4}$ and $\cos\frac{2\pi}{5} = \frac{\sqrt{5}-1}{4}$ to simplify this further:

$$\begin{aligned}
\frac{2}{3} + \frac{5}{3}\cos\frac{2\pi}{5} + \frac{5}{3}\cos\frac{\pi}{5} &= \frac{2}{3} + \frac{5}{3}\left(\frac{\sqrt{5}-1}{4} + \frac{\sqrt{5}+1}{4}\right) \\
&= \frac{2}{3} + \frac{5\sqrt{5}}{6} \\
&= \frac{5\sqrt{5}+4}{6}.
\end{aligned}$$

7. **Answer: 12600**

Since $f^{(n)}(0) = a_n n!$, where a_n is the nth Taylor series coefficient, we just need to find the Taylor series of f and read off the appropriate coefficient. The Taylor series is given by

$$f(x) = x^3\left(1 + \frac{x^2}{1!} + \frac{x^4}{2!} + \cdots\right)\left(1 + x^2 + x^4 + \cdots\right).$$

The coefficient of x^7 is $\frac{1}{2!} + \frac{1}{1!} + 1 = \frac{5}{2}$, so $f^{(7)}(0) = 7! \cdot \frac{5}{2} = 12600$.

8. **Answer: $\mathbf{\frac{3}{2}}$**

Differentiate the equation to get

$$\cos(x) + \frac{dy}{dx}\cos(y) = 0$$

and again

$$-\sin(x) + \frac{d^2y}{dx^2}\cos(y) - \left(\frac{dy}{dx}\right)^2\sin(y) = 0.$$

By solving these we have

$$\frac{dy}{dx} = -\frac{\cos(x)}{\cos(y)}$$

and

$$\frac{d^2y}{dx^2} = \frac{\sin(x)\cos^2(y) + \sin(y)\cos^2(x)}{\cos^3(y)}.$$

Let $\sin(x) = t$, then $\sin(y) = 1 - t$. Also $\cos(x) = \sqrt{1-t^2}$ and $\cos(y) = \sqrt{1-(1-t)^2} = \sqrt{t(2-t)}$. Substituting it gives

$$\frac{d^2y}{dx^2} = \frac{t^2(2-t) + (1-t)(1-t^2)}{t^{3/2}(1-2t)^{3/2}} = t^{-3/2}\frac{t^2-t+1}{(1-2t)^{3/2}}.$$

Since $\lim_{x\to 0}\frac{x}{t} = 1$, $\alpha = \frac{3}{2}$ should give the limit $\lim_{x\to 0} x^\alpha \frac{d^2y}{dx^2} = \frac{\sqrt{2}}{4}$. All other values of α will make this limit undefined or zero.

9. **Answer:** $\boldsymbol{\frac{\pi}{4}}$

We make the substitution, $x = \frac{\pi}{2} - y$ (note that the actual variable of integration is irrelevant so we leave it as x). Then we have:

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+(\tan x)^\pi} = \int_{\frac{\pi}{2}}^0 \frac{-dx}{1+\tan\left(\frac{\pi}{2}-x\right)^\pi} = \int_0^{\frac{\pi}{2}} \frac{dx}{1+(\cot x)^\pi} = \int_0^{\frac{\pi}{2}} \frac{(\tan x)^\pi\, dx}{(\tan x)^\pi + 1}.$$

Then we add the original integral to both sides:

$$2\int_0^{\frac{\pi}{2}} \frac{dx}{1+(\tan x)^\pi} = \int_0^{\frac{\pi}{2}} \frac{1}{1+(\tan x)^\pi} + \frac{(\tan x)^\pi}{1+(\tan x)^\pi}dx = \int_0^{\frac{\pi}{2}} \frac{1+(\tan x)^\pi}{1+(\tan x)^\pi}dx = \int_0^{\frac{\pi}{2}} dx = \frac{\pi}{2}.$$

So the integral we want is $\frac{\pi}{4}$.

10. **Answer:** $\boldsymbol{\frac{1}{2010(\ln 2)^{2010}}}$

Make the substitution $\frac{1}{x} = e^t - 1$, so that $-\frac{e^t\ dt}{(e^t-1)^2} = dx$. This transforms the original integral into

$$\int_0^1 \frac{dx}{x(x+1)\left(\ln\left(1+\frac{1}{x}\right)\right)^{2011}} = \int_{\ln 2}^\infty \frac{e^t(e^t-1)^2\ dt}{e^t(e^t-1)^2 t^{2011}}\ dt = \int_{\ln 2}^\infty \frac{dt}{t^{2011}}\ dt$$
$$= -\frac{1}{2010t^{2010}}\bigg|_{\ln 2}^\infty = \frac{1}{2010\left(\ln 2\right)^{2010}}.$$