

1. A mathematician is playing an epic game of dice throwing with a physicist. They take turns throwing two fair 6-sided dice, each numbered 1 through 6, with the numbers displayed on the top face of the two dice summed up. The first one who rolls a sum greater than 6 wins. The physicist, being less awesome than the mathematician, thinks the game is fair, that is, it does not matter who goes first. So he lets the mathematician go first. What is the probability that the mathematician wins the game?
2. Five students at a meeting remove their name tags and put them in a hat; the five students then each randomly choose one of the name tags from the bag. What is the probability that exactly one person gets their own name tag?
3. Two ants begin on opposite corners of a cube. On each move, they can travel along an edge to an adjacent vertex. Find the probability they both return to their starting position after 4 moves.
4. Let T_n denote the number of terms in $(x + y + z)^n$ when simplified, i.e. expanded and like terms collected, for non-negative integers $n \geq 0$. Find

$$\sum_{k=0}^{2010} (-1)^k T_k = T_0 - T_1 + T_2 - \cdots - T_{2009} + T_{2010}.$$

5. Let $\{a_i\}_{i=1,2,3,4}$, $\{b_i\}_{i=1,2,3,4}$, $\{c_i\}_{i=1,2,3,4}$ be permutations of $\{1, 2, 3, 4\}$. Find the minimum of $a_1 b_1 c_1 + a_2 b_2 c_2 + a_3 b_3 c_3 + a_4 b_4 c_4$.
6. Compute the summation

$$\sum_{k=0}^{2011} k \binom{2011}{k} \frac{1}{3}^k \left(\frac{2}{3}\right)^{2011-k}.$$

7. Compute the sum of all n for which the equation $2x + 3y = n$ has exactly 2011 nonnegative $(x, y \geq 0)$ integer solutions.
8. An unfair coin has a $2/3$ probability of landing on heads. If the coin is flipped 50 times, what is the probability that the total number of heads is even?
9. How many functions f that take $\{1, 2, 3, 4, 5\}$ to $\{1, 2, 3, 4, 5\}$, not necessarily injective or surjective (i.e. one-to-one or onto), satisfy $f(f(f(x))) = f(f(x))$ for all x in $\{1, 2, 3, 4, 5\}$?
10. Find the number of ways of filling a $2 \times 2 \times 8$ box with 16 $1 \times 1 \times 2$ boxes (rotations and reflections of the $2 \times 2 \times 8$ box are considered distinct).