Note: Figures may not be drawn to scale.

1. **Answer:** $\mathbf{(-13, -16, -18)}$

   The normal to the plane is in the direction $< 3, 4, 5 >$ and so the line going through the point perpendicular to the plane is $(11 - 3t, 16 - 4t, 22 - 5t)$ which intersects the plane at $t = 4$ and hence the reflection of the point occurs at $t = 8$, since the original point is at $t = 0$.

2. **Answer:** $\mathbf{107736\pi}$ **or** $\mathbf{(134)^2 6\pi}$

   Translate by -2010 to get $(-402, 0, 0), (0, 402, 0), (0, 0, 402)$, then scale by $1/402$: $(-1, 0, 0), (0, 1, 0), (0, 0, 1)$. Notice that these three points define an equilateral triangle so the center of the circle defined by the 3 points is the circumcenter, which is also the incenter. The incenter of this triangle is $(\frac{-1}{3}, \frac{1}{3}, \frac{1}{3})$, so the radius of the scaled down circle is

$$\sqrt{\left(\frac{-1}{3} - 0\right)^2 + \left(\frac{1}{3} - 1\right)^2 + \left(\frac{1}{3} - 0\right)^2} = \frac{\sqrt{6}}{3}.$$

   The radius of the original circle is $402\frac{\sqrt{6}}{3} = 134\sqrt{6}$. The area is then $\pi(134\sqrt{6})^2 = 107736\pi$.

3. **Answer:** $\mathbf{\frac{\sqrt{7}}{2}}$

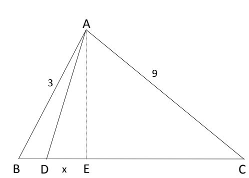
   First, use Heron's Formula to find the area. The semiperimeter is $s = \frac{15}{2}$, so the area is $\sqrt{\frac{15}{2} * \frac{7}{2} * \frac{5}{2} * \frac{3}{2}} = \frac{15\sqrt{7}}{4}$. Now the area is equal to the inradius times the semiperimeter, so $r = \frac{A}{s} = \frac{\sqrt{7}}{2}$.

4. **Answer:** $\mathbf{\frac{5\sqrt{2}}{3}}$

   The lengths of the sides of the large cube containing the cubeoctahedron are $\sqrt{2}$, so the volume of the containing cube is $2\sqrt{2}$. The volumes of the removed pyramids are $\frac{1}{3}BH = \frac{1}{3}\left(\frac{1}{2}\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}\right)\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{24}$. Because there are 8 pyramids removed, the total volume removed is $8\frac{\sqrt{2}}{24} = \frac{\sqrt{2}}{3}$. Thus, the total volume of the cubeoctahedron is $2\sqrt{2} - \frac{\sqrt{2}}{3} = \frac{5\sqrt{2}}{3}$.

5. **Answer:** $\mathbf{\sqrt{15}}$

   By angle bisector theorem, $\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow \frac{3}{9} = \frac{BD}{8-BD} \Rightarrow 24 - 3BD = 9BD$. This implies $BD = 2$ and $DC = 6$. Now draw altitude AE and let $x = DE$ and $h = AE$. Then by Pythagorean theorem, $(BD - DE)^2 + AE^2 = AB^2 \Rightarrow (2 - x)^2 + h^2 = 9$. Similarly, $AE^2 + (DE + DC)^2 = AC^2 \Rightarrow h^2 + (x + 6)^2 = 81$. Expanding, $9 = (2-x)^2 + h^2 = 4 - 4x + x^2 + h^2$ and $81 = h^2 + x^2 + 12x + 36$. Subtracting the two equations, we get $16x + 32 = 72$, or $x = 5/2$. Then $h^2 = 35/4$, and $AD = \sqrt{x^2 + h^2} = \sqrt{25/4 + 35/4} = \sqrt{15}$.



6. **Answer:** $\mathbf{\frac{25\sqrt{13}}{3}}$

   Let N be the opposite point of M in the circle. Then $MN = 50$ and $NB = \sqrt{50^2 - 30^2} = 40$ from that $\triangle MBN$ is right triangle. Let C be the midpoint of AB, then $\triangle MCB$ and $\triangle MBN$ are similar, so $BC = NB \cdot \frac{MB}{MN} = 24$, $MC = MB \cdot \frac{MB}{MN} = 18$. Let L be the intersection of AC and the tangent. Since we have AB and OT parallel, $CL = OT = 25$, so $BL = 1$. Since $\triangle MCB \sim \triangle BLD$, we have $BD = MB \cdot \frac{BL}{MC} = \frac{5}{3}$, so $MD = \sqrt{MB^2 + BD^2} = \frac{25\sqrt{13}}{3}$.

7. **Answer: 24**

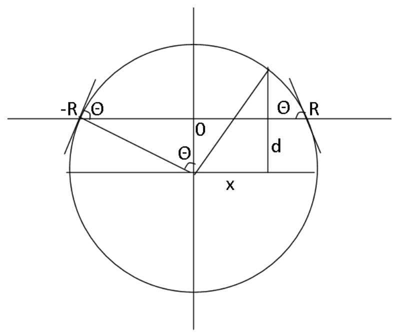
   Let v, e, t, q be the number of vertices, edges, triangular faces, and quadrilateral faces respectively. Note that each vertex is shared by exactly one quadrilateral, and a quadrilateral provides four vertices. By simple counting we get $v = 4q$. Apply the same thing to triangular face, then we have $4v = 3t$. Meanwhile from each vertex we have 5 edges coming out, so $5v = 2e$. Thus we have

   $$q = 1/4v, t = 4/3v, e = 5/2v.$$

   And from the Euler's formula $v - e + (t + q) = 2$, we have $(1 - 5/2 + 1/4 + 4/3)v = 1/12v = 2$, $v = 24$.

8. **Answer: $\sqrt{R^2 \csc^2\theta - x^2} - R\cot\theta$**

   Consider the following diagram, where the sphere has radius r:

   Note that $d = \frac{R}{\tan\theta}$, $r = \frac{R}{\sin\theta}$, and $h = \sqrt{r^2 - x^2} - d$. Plug in r and d gives the above answer.

9. **Answer: $8\sqrt{3}$**

   The center of the sphere is located at the centroid of the tetrahedron, which is located $\frac{1}{4}$ of the way up the altitude from a face to the opposite vertex. In other words, the tetrahedron has height 4. Let its edge length be s. Then the altitude of a face is $s\frac{\sqrt{3}}{2}$, and the distance from the centroid of a face to a vertex is $\frac{2}{3}$ of that, which is $\frac{\sqrt{3}}{3}$. This length and the height of the tetrahedron form a right triangle, with an edge as the hypotenuse. That is, $\frac{1}{3}s^2 + 16 = s^2$. Thus $s^2 = 24$, and so the area of a face is $6\sqrt{3}$. The volume is $\frac{1}{3} * 6\sqrt{3} * 4 = 8\sqrt{3}$.

10. **Answer: 6029.**

    If we consider a single 1×1 square, and find two regions within it on which the center of the coin of radius $\frac{1}{4}$ can land — the center $(\frac{1}{2} \times \frac{1}{2})$, of area $\frac{1}{4}$, and the outside edge, where an overlap will occur, of area $\frac{3}{4}$.

    The total area that the center of the coin can land on is thus

    $$\left(2010 - \frac{1}{2}\right)\left(2010 - \frac{1}{2}\right) = \frac{4019^2}{4}.$$

    Thus, the probability is $\frac{2010^2}{4019^2}$, so $a + b = 6029$.