1. **Answer:** $\boldsymbol{\frac{-1}{x^2+1}}$

   Notice that as $t \to 0$, both the numerator and the denominator approach 0. Thus, applying L'Hopital's rule on t (keeping x constant):

$$\frac{d}{dt}\mathrm{Tan}^{-1}\left(\frac{1}{x+t}\right)|_{t=0} = -\frac{1}{1+x^2}$$

2. **Answer: 1**

   Let $f(x) = e^x - x - \frac{x^3}{3}$. Then $f'(x) = e^x - 1 - x^2$. When $x < 0$, $e^x < 1$ and $1 + x^2 > 1$, so $f'(x) = e^x - (1+x^2) < 0$. Thus, f is decreasing on $(-\infty, 0)$. When $x = 0$, $f'(x) = f'(0) = e^0 - 1 - 0^2 = 1 - 1 = 0$. Finally, for $x > 0$, $f'(x) = e^x - 1 - x^2 > 0$ by a Maclaurin series expansion, so f is increasing on $(0, \infty)$. Thus, f must attain its minimum when $x = 0$, at which point f has the value $e^0 - 0 - \frac{0^3}{3} = 1$.

3. **Answer:** $\boldsymbol{\sqrt{2}}$

   Consider:

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin^{-1}(t-\sqrt{1/2})\bigg|_{t=0} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{-\infty}^{\infty} e^{tx}f(x)\mathrm{d}x\bigg|_{t=0} = \int_{-\infty}^{\infty} xe^{tx}f(x)\mathrm{d}x\bigg|_{t=0} = \int_{-\infty}^{\infty} xf(x)\mathrm{d}x$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\sin^{-1}(t-\sqrt{1/2})\bigg|_{t=0} = \frac{1}{\sqrt{1-\left(\sqrt{1/2}-t\right)^2}}\Bigg|_{t=0} = \frac{1}{\sqrt{1-(1/2)}} = \sqrt{2}.$$

4. **Answer:** $\boldsymbol{x = -\frac{2}{3}}$ **and** $\boldsymbol{x = 0}$

   Notice that $f(x) \to 0$ as $x \to \pm\infty$. Since $9x^2 + 6x + 2$ has no real roots, the maximum value of $f(x)$ is attained at the maximum of the absolute values of the critical points of $\frac{3x+1}{9x^2+6x+2}$.

   The extrema of $\frac{3x+1}{9x^2+6x+2}$ occur at $x = -\frac{2}{3}$ and $x = 0$. It is easily checked that maxima of $f(x)$ occur at both of these points.

5. **Answer:** $\boldsymbol{\frac{128\sqrt{3}}{27}}$

   Let the circular island be a circle of radius 2 centered at the origin. Without loss of generality, let the length of the rectangular base be from $-x$ to x and the width from $-y$ to y. Notice that by the equation of a circle, $x^2 = 4 - y^2$. Then

$$V = \frac{1}{3}(2x)^2(2y) = \frac{8}{3}x^2y = \frac{8}{3}(4-y^2)y = \frac{8}{3}(4y-y^3)$$

$$\frac{dV}{dy} = \frac{8}{3}(4-3y^2) = 0 \to y = \sqrt{\frac{4}{3}}$$

$$V = \frac{8}{3}\left(\frac{8}{3}\right)\sqrt{\frac{4}{3}} = \frac{128}{9\sqrt{3}} = \frac{128\sqrt{3}}{27}.$$

6. **Answer: 13**

   This is the evaluation of the mean of a Poisson distribution: for any λ,

$$\sum_{k=0}^{\infty} ke^{-\lambda}\frac{\lambda^k}{k!} = \sum_{k=1}^{\infty} ke^{-\lambda}\frac{\lambda^k}{k!} = \lambda\sum_{k=1}^{\infty} e^{-\lambda}\frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda}\sum_{m=0}^{\infty}\frac{\lambda^m}{m!} = e^{-\lambda}e^{\lambda} = \lambda.$$

7. **Answer:** $\frac{-2\cos(t^2)}{t}$

By the Leibniz integral rule, the above integral becomes

$$\int_{-\ln 1/t}^{\ln 1/t} -e^x \sin(te^x)dx + \cos(te^{\ln(1/t)})(-1/t) - \cos(te^{-\ln(1/t)})(1/t) = \frac{\cos(te^x)}{t}\Big|_{-\ln 1/t}^{\ln 1/t} - \frac{\cos(1)+\cos(t^2)}{t}$$
$$= \frac{-2\cos(t^2)}{t}.$$

8. **Answer:** $\ln 3$

The partial sums of this sum are equal to

$$\left(\frac{1}{1}+\frac{1}{2}+\cdots+\frac{1}{3n}\right) - 3\left(\frac{1}{3\cdot 1}+\frac{1}{3\cdot 2}+\cdots+\frac{1}{3\cdot n}\right)$$
$$= \frac{1}{n+1}+\frac{1}{n+2}+\cdots+\frac{1}{3n} = \frac{1}{n}\left(\frac{1}{1+\frac{1}{n}}+\frac{1}{1+\frac{2}{n}}+\cdots+\frac{1}{1+\frac{2n}{n}}\right)$$

This is a Riemann sum, so as $n \to \infty$ the partial sums converge to

$$\int_0^2 \frac{1}{1+x}\,dx = \ln 3.$$

9. **Answer:** 3

Since the parabola $f(x) = x(4-x) - k$ is symmetric about $x = 2$, the problem is equivalent to minimizing $\int_0^2 |f(x)|dx$. The vertex of the parabola equals $(2, f(2)) = (2, 4-k)$. When $k = 4$, $f(x)$ lies completely below the x-axis in the interval [0,2] and hence $k > 4$ would only translate f(x) down and increase the integral. Similarly, at $k = 0$, $f(x)$ lies completely above the x-axis so $k < 0$ would only increase the integral. Thus, we can split the integral into two regions

$$a = \int_0^{2-\sqrt{4-k}} \left(x^2 - 4*x + k\right) dx = -\frac{16}{3} + \frac{8\sqrt{4-k}}{3} + 2k - \frac{2}{3}\sqrt{4-k}k$$
$$b = \int_{2-\sqrt{4-k}}^2 \left(-x^2 + 4*x - k\right) dx = \frac{2}{3}(4-k)^{3/2}$$

We want to solve for the critical point of

$$a+b$$

.

$$\frac{d(a+b)}{dk} = 2 - \frac{4}{3\sqrt{4-k}} - \frac{5\sqrt{4-k}}{3} + \frac{k}{3\sqrt{4-k}} = \frac{2\left(-4+\sqrt{4-k}+k\right)}{\sqrt{4-k}}$$

The numerator equals 0 when $k = 3$. It is clear that a global minimum results since this is a global minimum on $(-\infty, 4]$ and $F(k)$ is clearly increasing for $k > 4$.

10. **Answer:** $y = -4x^2 + 5x - 7$

Such a parabola intersects $f(x)$ precisely where $f'(x) = 0$. Hence, the value of the intersection points do not change when we replace $f(x)$ by $f(x)+g(x)f'(x)$ for any $g(x)$. Therefore, since $f'(x) = 6x^5 - 12x + 6$, we must have that $f(x) - 1/6xf'(x) = -4x^2 + 5x - 7$ passes through the three critical points. Since three points determines a parabola uniquely, this must be the unique parabola passing through the three critical points.