1. **Answer:** $\frac{1+\sqrt{5}}{2}$

   Let $x = \sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}$. Then $x^2 = 1+\sqrt{1+\sqrt{1+\ldots}}$. Thus $x^2 = x+1$. The roots of $x^2-x-1=0$ are $\frac{1\pm\sqrt{5}}{2}$.

2. **Answer:** $\frac{112563}{999999000}$**, or** $\frac{379}{3367000}$

   $2010.220\overline{112563}$ modulo $2010.220 = 0.000\overline{112563}$

$$
\begin{aligned}
0.000\overline{112563} = 0.001\cdot 112563\sum_{i=1}^{\infty}(10^{-6})^i &= 0.001\cdot 112563\left(\frac{10^{-6}}{1-10^{-6}}\right)\\
&= 0.001\cdot 112563\left(\frac{1}{999999}\right)\\
&= \frac{112563}{999999000}
\end{aligned}
$$

3. **Answer:** $\frac{\sqrt{5}-1}{4}$**.**

   Consider:

$$
\begin{aligned}
\sin 18^\circ = \cos 72^\circ = 2\cos^2 36^\circ - 1 &= 2(1-2\sin^2 18^\circ)^2 - 1\\
&= 1 - 8\sin^2 18^\circ + 8\sin^4 18^\circ\\
0 &= 8x^4 - 8x^2 - x + 1 = (x-1)(2x-1)(4x^2+2x+1).
\end{aligned}
$$

   Clearly, $x \neq 1, \frac{-1}{2}$, because $0 < \sin 18^\circ < \sin 90^\circ = 1$. We solve the remaining term:

$$
0 = 4x^2 + 2x + 1 \implies x = \frac{-2\pm\sqrt{4+4(4)}}{2(4)} = \frac{-1\pm\sqrt{5}}{4}.
$$

   The only root that is within our bounds is $\frac{\sqrt{5}-1}{4}$.

4. **Answer:** $\pm$**123**

   Note that $\left(x+\frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2 = 9$. Thus, $x + \frac{1}{x} = \pm 3$. Therefore,

$$
\begin{aligned}
x^5 + \frac{1}{x^5} &= \left(x+\frac{1}{x}\right)^5 - 5\left(x^3 - \frac{1}{x^3}\right) - 10\left(x+\frac{1}{x}\right)\\
&= \left(x+\frac{1}{x}\right)^5 - 5\left(x+\frac{1}{x}\right)^3 + 5\left(x+\frac{1}{x}\right)\\
&= \pm 123
\end{aligned}
$$

5. **Answer:** $\frac{8\pi}{3} - 2\sqrt{3}$

   The area of the intersection can be found by summing the areas of the four 60 deg disc sections in the region and subtracting the combined area of the two equilateral triangles that the intersecting pairs of these disc sections have in common. Thus, the area of the intersection is $\frac{2\pi r^2}{3} - \frac{\sqrt{3}}{2}r^2 = \frac{8\pi}{3} - 2\sqrt{3}$

6. **Answer: 1027**

   If $S(n)$ is the nth partial sum, note that if m is the kth triangular number, $S(m) = k^2$. Since $44^2 = 1936$ and $45^2 = 2025$, we want to begin our search at $44(44+1)/2 = 990$. Because $(2010-1936)/2 = 37$, 37 more $2s$ are needed, so the needed term is $n = 990 + 37 = 1027$.

7. **Answer: 21,26,31,36,41,46**

$$\begin{aligned} 6x+5 &\equiv -19 \mod 10 \\ \Leftrightarrow 6x &\equiv -24 \mod 10 \\ \Rightarrow x &\equiv -4 \mod \frac{10}{gcd(10,6)} \\ \Leftrightarrow x &\equiv -4 \mod 5 \\ \Leftrightarrow x &\equiv 1 \mod 5 \end{aligned}$$

That is, x is in the form $5k+1$ where k is an integer.

8. **Answer: (1, 1) and (3, 2)**

Solution: This requires $2^x = 3^y - 1$, or $2^x = 2(3^{y-1} + 3^{y-2} + ... + 3 + 1)$, by geometric summation. Inside the parentheses on the right side, every other term is congruent to 3 mod 8, and every other term is congruent to 1 mod 8. Therefore the number of terms must be divisible by 4 in order to have the sum in the parentheses divisible by 8. But the number of terms is y. Thus the right side cannot be divisible by 16, so x is at most 3. The solutions are $(1, 1)$ and $(3, 2)$.

9. **Answer: 31**

Factor the equation as $(x+2)(y-5)+10 = 30$, or $(x+2)(y-5) = 20$. x must be 2 less than a factor of 20. The solutions for x are thus 2, 3, 8, and 18, which sum to 31.

10. **Answer: $\frac{2}{1005}$**

We can rewrite this equation as

$$\frac{x^2}{x^2-1} + \frac{x^2}{x^2-2} + \frac{x^2}{x^2-3} + \frac{x^2}{x^2-4} =$$
$$\frac{1+(x^2-1)}{x^2-1} + \frac{2+(x^2-2)}{x^2-2} + \frac{3+(x^2-3)}{x^2-3} + \frac{4+(x^2-4)}{x^2-4}$$
$$= (2010x-4)+4 = 2010x.$$

Therefore, we have

$$\frac{x}{x^2-1} + \frac{x}{x^2-2} + \frac{x}{x^2-3} + \frac{x}{x^2-4} = 2010$$

Clearing denominators yields the polynomial equation

$$\begin{aligned} &x((x^2-2)(x^2-3)(x^2-4) + (x^2-1)(x^2-3)(x^2-4) + \\ &\qquad (x^2-1)(x^2-2)(x^2-4) + (x^2-1)(x^2-2)(x^2-3)) \\ &= 2010(x^2-1)(x^2-2)(x^2-3)(x^2-4) \end{aligned}$$

The solutions that we want are therefore the roots of the polynomial

$$2010x^8 - 4x^7 + (\text{lower order terms}) = 0$$

By Vieta's formulas, the sum of the roots of this polynomial equation is therefore $\frac{4}{2010}$.