

1. **Answer: 17**

Let the answer be v . The raptors gain $20(v - 10)$ while the train is moving, then $13v$ after it stops, for a total of $33v - 200$. In this time they have gained 595, so we have $395 = 33v - 200$, so $v = 17$.

2. **Answer: 350π**

We see that since this is at the corner of the barn, the snake is free to travel in a three quarter arc around this point such that it does not intersect the barn. On the corners, we notice that the leash will bend, and basically act as a shorter leash fixed at the other two corners. Thus, there is a leash of length 10 for each of two quarter circles, which sum to half of an arc:

$$\frac{3}{4}(20)^2\pi + \frac{1}{2}(10)^2\pi = 350\pi.$$

3. **Answer: $\frac{5}{4}$**

This can be expanded, and we see that it is quadratic:

$$(2x + 15)(10 - x) = -2x^2 + 5x + 150 \implies \frac{-b}{2a} = \frac{5}{4} = x$$

4. **Answer: 144°**

The two different angles in a parallelogram sum to 180° . If x is the smaller angle, we have $x + 4x = 5x = 180^\circ$, so $x = 36^\circ$ and the larger angle is $4x = 144^\circ$.

5. **Answer: 15501**

Each upgrade will reduce the total costs by \$1000.1. Therefore the first ten upgrades, up to the one that costs \$1000, will save money. The cost of the fizzbangs will then be one dollar each, costing \$10001, and the upgrades will cost a total of \$5500.

6. **Answer: 1**

Suppose the point (x, y) is the intersection point. The distance from (y, x) to the origin is the same, and it will also be on the line $y = 10 - x$. The slope of the line from the origin to (y, x) is $1/m$, providing a second solution. The product of these two solutions is 1.

7. **Answer: 69**

The number of tarts made is 140, the least common multiple of 4, 7, and 10. This uses $\frac{140}{4} + \frac{140}{7} + \frac{140}{10} = 35 + 20 + 14$

8. **Answer: $\frac{225\pi}{64}$**

The cone of melted ice cream is similar to the original cone, so it has a radius of $\frac{3}{8} \cdot 5 = \frac{15}{8}$. The volume of the cone is therefore $\frac{1}{3} \cdot 3 \cdot \left(\frac{15}{8}\right)^2$.

9. **Answer: 13**

Let x be the length of wall without windows. The length of wall with windows is $20 - x$, so the total number of bricks used is $2120 = 120x + 80(20 - x)$. Solving for x gives $x = \frac{2120 - 1600}{40} = 13$.

10. **Answer: 49**

Let x be the normal number of 3-button mice, and y the normal number of 5-button mice. We have $3x + 5y = 207$ and $5x + 3y = 281$. Subtracting 3 times the first equation from 5 times the second gives $25x - 9x = 5 \cdot 281 - 3 \cdot 207$, so $16x = 784$ and $x = 49$.

11. **Answer: 61**

Notice that $3599 = 60^2 - 1^2$. Factoring the difference of two squares gives $60^2 - 1^2 = (60 - 1)(60 + 1)$. 59 and 61 are both prime, so the answer is 61.

12. Answer: 8

The number of factors of $p_1^{n_1} p_2^{n_2} \dots p_m^{n_m}$, where each of p_i is a prime, $1 \leq i \leq m$, is

$$(n_1 + 1)(n_2 + 1) \dots (n_m + 1).$$

Since $12345 = 3 \times 5 \times 823$, we have $(1 + 1)(1 + 1)(1 + 1) = 8$ factors.

13. Answer: 14

We have three different colors, but each glove must have a matching pair of gloves. Thus, there are $\frac{8}{2} = 4$ white lefthanded gloves, $\frac{12}{2} = 6$ black lefthanded gloves, and $\frac{6}{2} = 3$ gray lefthanded gloves. We have $4 + 6 + 3 = 13$. But pulling out one more glove ensures a pair since all the remaining gloves are righthanded. Thus, we need 14 gloves to make sure we have a matching pair.

14. Answer: $\frac{1}{2}$

The first 19 flips are irrelevant. The probability that the last flip will be heads is still $\frac{1}{2}$.

15. Answer: 273

Nathan can prove theorems at a rate of $\frac{1}{6}$ theorems per hour, while the Lemm-o-Matic 1729 can prove theorems at a rate of $\frac{1}{5}$ theorems per hour. Working together, Nathan and the Lemm-o-Matic can prove $\frac{1}{5} + \frac{1}{6} = \frac{11}{30}$ theorems per hour. To prove 100 theorems, it will take Nathan and the Lemm-o-Matic a total of $\frac{100}{11/30} \approx 273$ hours.

16. Answer: 0

There are 0 such values of N . By kicking out one of their own members, the Silly party can at most reduce the number of people needed to pass a bill by one, but at the same time, they reduce the number of voters in the Silly party by one. Thus, the number of votes needed from the Sensible party does not decrease.

17. Answer: 1

You can see immediately that 1 is a root, so factor the equation as $(x - 1)(x^3 - 2x^2 + 2x - 1)$. The second factor still has 1 as a root, so factor it further as $(x - 1)^2(x^2 - x + 1)$. The second factor now has no real roots. The answer is 1, since 1 is the only real root.

18. Answer: 500000500000

Pair the terms up: 1 pair with 1000000, 2 pairs with 999999, etc. There are 500000 pairs, each of which sums to 1000001, so the sum is 500000500000.

19. Answer: 5

Suppose we have just three triangles. The only way to join them (up to rotations and reflections) is to make an isosceles trapezoid. If we add a fourth triangle to this, we can get at most six sides. However, with a fifth triangle, we can get 7 sides. The arrangement that works comes from making a regular hexagon out of six triangles, then removing one of them.

20. Answer: $2\sqrt{3}$

The foci, at points $(c, 0)$ and $(-c, 0)$, satisfy the equation $a^2 - b^2 = c^2$ where a and b are the lengths of the semimajor and the semiminor axes. Thus, we have that $c^2 = 2^2 - 1^2 = 3$. The distance between the two foci is then $2\sqrt{3}$.

21. Answer: 1

The expression factors as $(n^2 - n + 1)(n^2 + n + 1)$. If it is to be prime, one of these must equal 1. But the only way that can happen, for positive integers n , is if $n = 1$.

22. Answer: $\frac{189}{256}$

Probability is independent, so the probability that any given coin is eaten is $1 - (1/2 * 1/2) = 3/4$. The probability of getting 3 or 4 coins is $\binom{4}{3}(3/4)^3(1/4) + \binom{4}{4}(3/4)^4 = 108/256 + 81/256 = 189/256$.

23. **Answer:** $\frac{54}{11}$ minutes

The area of pizza Cody, Frank, and Jeffrey can eat per minute are π , $\frac{\pi}{2}$, $\frac{\pi}{3}$, respectively. So together, they can eat $\pi + \frac{\pi}{2} + \frac{\pi}{3} = \frac{11\pi}{6}$ square meters of pizza per minute. Thus, it takes $\frac{6}{11\pi} \times 9\pi = \frac{54}{11}$ minutes to eat a pizza of area 9π .

24. **Answer:** 10076

The answer is the number of times $8! = 40320$ is divisible by 5. This is equal to $\lfloor \frac{40320}{5} \rfloor + \lfloor \frac{40320}{25} \rfloor + \lfloor \frac{40320}{125} \rfloor + \dots$, which is $8064 + 1612 + 322 + 64 + 12 + 2 = 10076$.