1. **Answer:** 10

Label three consecutive vertices of the polygon A, B, and C. Let BP be the common side to the pentagons placed on sides AB and BC. Then \( \angle ABP = \angle PBC = 108^\circ \). Since \( \angle ABP + \angle PBC + \angle ABC = 360^\circ \), this gives \( \angle ABC = 144^\circ \). So the exterior angle of this polygon is 36, and thus it has 10 sides.

2. **Answer:** \( 8\sqrt{2} \)

Notice that the ball travels the length of the room twice and the width of the room twice, so it’s traveled a total of 8 meters in the horizontal direction and 8 meters in the vertical direction. Because the ball is bouncing (and thus its path after a bounce is the same as its path before the bounce, but reflected), we can rearrange the four segments of its path into a straight line by only reflection and translation. This line travels 8 meters horizontally and 8 meters vertically, so its length, which is the total length of the ball’s path, is \( 8\sqrt{2} \).

3. **Answer:** \( \sqrt{3}\pi \)

Since the space diagonal of the cube is a diameter of the sphere, we have \( s\sqrt{3} = 2r \). The ratio is then

\[
\frac{\frac{4}{3}\pi r^3}{\left( \frac{2r}{\sqrt{3}} \right)^3} = \frac{\sqrt{3}\pi}{2}
\]

4. **Answer:** \( 288\sqrt{3} - 432 \)

Let \( r \) be the radius of a small circle. The centers of the small circles form an equilateral triangle of side length \( 2r \). The length of the median of such a triangle is \( \sqrt{3}r \), so the distance from the center of the triangle (which is also the center of the large circle) to a vertex is \( \frac{2\sqrt{3}}{3}r \). Since each vertex of the triangle is distance \( r \) from the edge of the large circle, the radius of the large circle is \( \frac{2\sqrt{3}}{3}r + r = 144 \).

This gives \( (2\sqrt{3} + 3)r = 432 \), so \( r = \frac{432}{2\sqrt{3}+3} \cdot \frac{2\sqrt{3}-3}{2\sqrt{3}-3} = 144(2\sqrt{3} - 3) = 288\sqrt{3} - 432 \).

5. **Answer:** \( 15^\circ + \tan^{-1} x \) or \( \frac{\pi}{12} + \tan^{-1} x \)

From basic trigonometry, we have \( \tan(\angle B) = \frac{2 - \sqrt{3} + x}{1 - (2 - \sqrt{3})x} \). This is the tangent angle addition identity, for angles with tangents \( x \) and \( 2 - \sqrt{3} \). Since \( \tan(15^\circ) = 2 - \sqrt{3} \), \( \angle B \), the inverse tangent, is therefore \( 15^\circ + \tan^{-1} x \).

6. **Answer:** \( 44 \over 13 \)

Let \( \alpha = \angle E \) and \( \beta = \angle F \). Note that \( D \) is a right angle. Therefore, \( \sin \alpha = \frac{\sqrt{3}}{2} \). \( [CBF] = \frac{1}{2} \cdot 112 \cdot \sin \alpha = \frac{1}{2} \cdot 121 \cdot \frac{\sqrt{3}}{2} \). Similarly, \( [ABE] = \frac{1}{2} \cdot 2^2 \sin \beta = \frac{1}{2} \cdot 4 \cdot \cos \alpha = \frac{1}{2} \cdot 4 \cdot \frac{12}{13} \). Finally, \( [ACD] = \frac{31}{2} \). Subtracting these three areas from that of \( \triangle DEF \) gives the result.

7. **Answer:** \( 2008^2\pi \) or \( 4032064\pi \)

Note that we can scale the triangle down by a factor of 2008 to a 3,4,5 right triangle. Let \( AB \), \( AC \) be the legs of the triangle. The incircle splits \( AB \) into two segments of lengths \( x \) and \( y \). It similarly splits \( AC \) into segments of lengths \( x \) and \( z \) and \( BC \) into segments of lengths \( y \) and \( z \). Thus, we get:

\[
\begin{align*}
  x + y &= 3 \\
  x + z &= 4 \\
  y + z &= 5
\end{align*}
\]

Thus, \( x = 1 \), \( y = 2 \), \( z = 3 \). Thus, the incircle has a radius of 1, and so an area of \( \pi \). Scaling back up will increase the incircle’s radius by a factor of 2008, giving us an area of \( 2008^2\pi \).
8. Answer: $\sqrt{\frac{30}{3}}$

Let $O$ be the center of the circle, and $X$ be the center of the rhombus (the intersection of $AC$ and $BD$). Let $m \angle ABC = \theta = \cos^{-1}(-\frac{2}{3})$. Considering $\triangle OBX$ and $\triangle ABX$, using triangle angle sums and the fact that an inscribed angle has half the measure of the intercepted arc, we have $OX = \cos(\pi - \theta)$, so $AX = 1 + \cos(\pi - \theta)$. Also, $BX = \sin(\pi - \theta)$. The Pythagorean theorem then gives

$l = \sqrt{2(1 + \cos(\pi - \theta))} = \sqrt{2(1 + \frac{2}{3})}$.

9. Answer: 12

Let the trapezoid be $ABCD$ with $AB = 10$, $CD = 15$. Let $P$ be the intersection of the diagonals, and let $XY$ be the segment through $P$ parallel to the bases with $X$ on $AD$ and $Y$ on $BC$. Note that $\triangle PYC \sim \triangle ABC$, so $\frac{PY}{AB} = \frac{YC}{BC}$. Also, $\triangle PYB \sim \triangle DCB$, so $\frac{PY}{CD} = \frac{BY}{BC}$. Adding these equations gives $\frac{PY}{AB} + \frac{PY}{CD} = \frac{BY + YC}{BC} = 1$, so $PY(\frac{1}{10} + \frac{1}{15}) = PY \cdot \frac{1}{5} = 1$, hence $PY = 6$.

The same argument shows that $PX = 6$, so $XY = 12$.

10. Answer: 3

The polygon has angles of 171°, and the smallest triangle has two adjacent sides of the original polygon as two of its sides. The area of this triangle is $\frac{1}{2} \cdot 1 \cdot 1 \cdot \sin(171) = \frac{1}{2} \sin(9)$. So the question is, how many square roots do we need to express $\sin(9)$? Conveniently enough, $\sin(18) = \sqrt{\frac{5-1}{4}}$, so $\cos(18) = \sqrt{1 - \sin^2(18)}$, which requires two square roots to express. Then by the half-angle formula, $\sin(9) = \sqrt{\frac{1 - \cos(18)}{2}}$, which requires three square roots.