1. Calculate the least integer greater than $5^{(-6)(-5)(-4)\cdots(2)(3)(4)}$.

2. How many primes exist which are less than 50?

3. Give the positive root(s) of $x^3 + 2x^2 - 2x - 4$.

4. A right triangle has sides of integer length. One side has length 11. What is the area of the triangle?

5. One day, the temperature increases steadily from a low of 45°F in the early morning to a high of 70°F in the late afternoon. At how many times from early morning to late afternoon was the temperature an integer in both Fahrenheit and Celsius? Recall that $C = \frac{5}{9}(F - 32)$.

6. A round pencil has length 8 when unsharpened, and diameter $\frac{1}{4}$. It is sharpened perfectly so that it remains 8 inches long, with a 7 inch section still cylindrical and the remaining 1 inch giving a conical tip. What is its volume?

7. At the Rice Mathematics Tournament, 80% of contestants wear blue jeans, 70% wear tennis shoes, and 80% of those who wear blue jeans also wear tennis shoes. What fraction of people wearing tennis shoes are wearing blue jeans?

8. Terence Tao is playing rock-paper-scissors. Because his mental energy is focused on solving the twin primes conjecture, he uses the following very simple strategy:

   - He plays rock first.
   - On each subsequent turn, he plays a different move than the previous one, each with probability $\frac{1}{2}$.

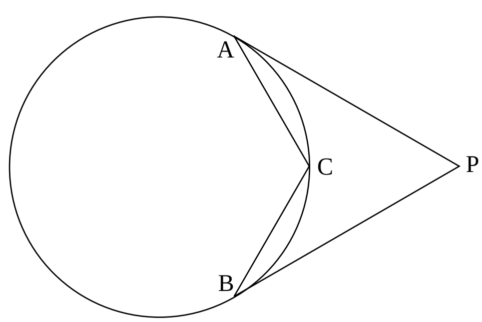
   What is the probability that his 5^{th} move will be rock?

9. What is the sum of the prime factors of 20!?

10. Six people play the following game: They have a cube, initially white. One by one, the players mark an X on a white face of the cube, and roll it like a die. The winner is the first person to roll an X (for example, player 1 wins with probability $\frac{1}{6}$, while if none of players 1–5 win, player 6 will place an X on the last white square and win for sure). What is the probability that the sixth player wins?

11. Simplify: $\sqrt[3]{\frac{17\sqrt{7}+45}{4}}$

12. If in the following diagram, $m\angle APB = 16°$ and $\overline{AP}$ and $\overline{BP}$ are tangent to the circle, what is $m\angle ACB$?



13. Let N be the number of distinct rearrangements of the 34 letters in SUPERCALIFRAGILISTICEX-PIALIDOCIOUS. How many positive factors does N have?

14. Suppose families always have one, two, or three children, with probability $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{1}{4}$ respectively. Assuming everyone eventually gets married and has children, what is the probability of a couple having exactly four grandchildren?

15. While out for a stroll, you encounter a vicious velociraptor. You start running away to the northeast at 10 m/s, and you manage a three-second head start over the raptor. If the raptor runs at $15\sqrt{2}$ m/s, but only runs either north or east at any given time, how many seconds do you have until it devours you?

16. Suppose convex hexagon HEXAGN has 120°-rotational symmetry about a point P—that is, if you rotate it 120° about P, it doesn't change. If $PX = 1$, find the area of triangle $\triangle GHX$.

17. An "expression" is created by writing down five random characters, taken from the digits 1 to 9, the four basic operations, and parentheses. What is number of possible mathematically valid expressions that can be created this way, without using implicit multiplication?

18. Cody enjoys his breakfast tacos with his favorite hot sauce, which comes in 88 mL bottles. On any given day he eats 3, 4, or 5 tacos with probabilities $\frac{1}{6}$, $\frac{1}{3}$, and $\frac{1}{2}$, respectively. For the first taco, he always uses 2 mL of hot sauce, but for each additional taco he uses 1 mL more than the previous. If he starts with a new bottle, what is the probability that it is empty after five days?

19. Four pirates are dividing up 2008 gold pieces. They take turns, in order of rank, proposing ways to distribute the gold. If at least half the pirates agree to a proposal, it is enacted; otherwise, the proposer walks the plank. If no pirate ever agrees to a proposal that gives him nothing, how many gold pieces does the highest-ranking pirate end up with? (Assume all pirates are perfectly rational and act in self-interest, i.e. a pirate will never agree to a proposal if he knows he can gain more coins by rejecting it.)

20. What is the smallest number which can be written as the sum of three distinct primes, the product of two distinct primes and the sum of three distinct squares?

21. Find the value of $\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\cdots}}}}$, assuming all square roots refer to the positive values.

22. What is the smallest possible surface area of an object constructed by joining the faces of five cubes of edge length one?

23. You are standing at point A, which is 1000 feet from point B. You choose a random direction and walk 1000 feet in that direction. What is the probability that you end up within 1000 feet of point B?

24. An isosceles right triangle with legs of length 1 has a semicircle inscribed within it and a semicircle inscribed around it. Both have their diameter lying along the hypotenuse of the triangle. Find the ratio of their radii (larger to smaller).

25. Three unit circles are mutually externally tangent. All three are internally tangent to a larger circle. What is the radius of the larger circle?