1. **Answer:** $\frac{1}{2}$

   Using the product to sum trigonometric identity, we get:

$$\begin{aligned}\int_0^{\pi/2} \sin x \cos x \, dx &= \int_0^{\pi/2} \frac{1}{2}(\sin 2x - \sin 0) \, dx \\ &= \tfrac{1}{2} \int_0^{\pi/2} \sin 2x \, dx \\ &= \tfrac{1}{2} \left(-\tfrac{1}{2}\cos 2x\right)\Big|_0^{\pi/2} \\ &= \tfrac{1}{2}\left(1/2 - (-1/2)\right) \\ &= \tfrac{1}{2}(1) \\ &= 1/2\end{aligned}$$

2. **Answer: 30**

   Two applications of L'Hôpital's rule are necessary to obtain a fraction which does not divide by zero; the result is

$$\frac{20-3x}{\frac{2}{3}e^{x^2} + \frac{4}{9}x^2 e^{x^2}} = \frac{20-0}{\frac{2}{3}+0} = 30$$

3. **Answer:** $\frac{8}{3}$

   The graph is a figure-8. By symmetry, the area it encloses is four times the area under the curve that we get from just considering $0 \le t \le \frac{\pi}{2}$. For these values of t, we can write $y = 2\sin(t)\cos(t) = 2x\sqrt{1-x^2}$ for $0 \le x \le 1$. But this is integrable easily enough by letting $u = 1 - x^2$. The antiderivative that we get is $-\frac{2}{3}u^{\frac{3}{2}}$, which gives an area of $\frac{2}{3}$. Thus the total area enclosed by the parametric graph is $\frac{8}{3}$.

4. **Answer:** $n!$

   You can think of taking the derivatives this way, in an extended version of the product rule: For each $j \le n$, let $f_j(x) = \sin(x)$. Then $f(x) = f_1(x)f_2(x)...f_n(x)$. Each time you take the derivative, you pick one of the f_j's and replace it with its derivative. Some f_j's might be differentiated more than once in this process. To find the derivative of f, take the results from each way of choosing the f_j's to differentiate and add them all up.

   That said, since $\sin(0) = 0$, if *any* f_j doesn't get differentiated, it'll make the whole product zero when $x = 0$, so these terms don't matter. But we're only taking the nth derivative, so if every f_j gets differentiated at least once, then they all have to be differentiated exactly once. There are $n!$ remaining terms (the number of ways to choose what order we differentiate the f_j's in), each of which is $\cos^n(0) = 1$, so the value we want is just $n!$.

5. **Answer:** $\frac{625}{32}$

   Let $w(t)$ and $s(t)$ denote the amounts of water and salt, respectively, in the tank at time t. We can immediately see that $w(t) = t + 100$. Since the tank is constantly mixed, we know that

$$\begin{aligned}\frac{ds}{dt} &= -2\frac{s(t)}{w(t)} \\ \frac{ds}{s} &= -2\frac{dt}{t+100} \\ \ln(s) &= -2\ln(C(t+100)) \\ s &= \frac{C}{(t+100)^2}\end{aligned}$$

   Since $s(0) = 50$, $C = 500000$, so $s(60) = 625/32$.

6. **Answer:** $\frac{1}{10}e^{3x}\left(3\sin(x) - \cos(x)\right)$

Let I be the answer. Integrating by parts twice:

$$\begin{aligned} I &= \frac{1}{3}e^{3x}\sin(x) - \int \frac{1}{3}e^{3x}\cos(x)dx \\ &= \frac{1}{3}e^{3x}\sin(x) - \frac{1}{9}e^{3x}\cos(x) - \frac{1}{9}I \\ I &= \frac{9}{10}\left(\frac{1}{3}e^{3x}\sin(x) - \frac{1}{9}e^{3x}\cos(x)\right) \end{aligned}$$

7. **Answer:** $\frac{1}{2}e^2$

Multiply the sum by 2 and you get $\sum_{n=0}^{\infty} \frac{2^n}{n!}$.

Now notice that the nth derivative of e^{2x} is $2^n e^{2x}$, which gives 2^n evaluated at $x = 0$. The Taylor series for e^{2x} around $x = 0$ is thus $\sum_{n=0}^{\infty} \frac{2^n}{n!}x^n$. Evaluated at $x = 1$, this gives the sum above, so the sum we want is equal to $\frac{1}{2}e^2$.

8. **Answer:** $\tan^{-1} x$ **or** $\tan^{-1}(x) + ax + b$

The limit is, by definition, $\frac{1}{f''(x)}$. Therefore:

$$\begin{aligned} \frac{1}{f''(x)} &= -\frac{x^4+2x^2+1}{2x} = -\frac{(1+x^2)^2}{2x} \\ f''(x) &= -\frac{2x}{(1+x^2)^2} \\ f'(x) &= \frac{1}{1+x^2} \\ f(x) &= \tan^{-1} x \end{aligned}$$

9. **Answer:** $\frac{a_{n-7}-(n-1)a_{n-1}}{n(n-1)}$

Plugging the power series into the differential equation gives:

$$\begin{aligned} \sum n(n-1)a_n t^{n-2} + \sum n a_n t^{n-1} &= \sum a_n t^{n+5} \\ \sum n(n-1)a_n t^{n-2} + \sum (n-1)a_{n-1} t^{n-2} &= \sum a_{n-7} t^{n-2} \\ n(n-1)a_n t^{n-2} + (n-1)a_{n-1} t^{n-2} &= a_{n-7} t^{n-2} \\ n(n-1)a_n + (n-1)a_{n-1} &= a_{n-7} \end{aligned}$$

10. **Answer:** $\frac{x(2e)^x}{1+\ln 2} - \frac{(2e)^x)}{(1+\ln 2)^2}$

$$\begin{aligned} \int_{-\infty}^{x} t2^t e^t dt &= \left[t\frac{1}{\ln(2e)}(2e)^t\right]_{-\infty}^{x} - \int_{-\infty}^{0} \frac{1}{\ln 2e}(2e)^t dt \\ &= \frac{x(2e)^x}{1+\ln 2} - \left[\frac{(2e)^t}{(1+\ln 2)}\right]_{-\infty}^{x} \\ &= \frac{x(2e)^x}{1+\ln 2} - \frac{(2e)^x}{(1+\ln 2)^2} \end{aligned}$$