1. **Answer: 21**

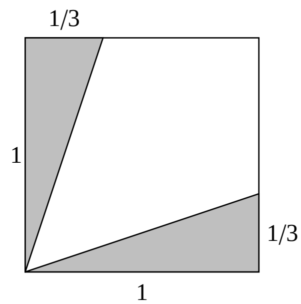
   The gladiators are able to attempt to attack $n-1$ times. The expected number of hits is therefore $\frac{n-1}{2}$, and since hits are just as likely as misses, the mean is also the median. There is thus a $\frac{1}{2}$ chance that the gladiators hit at least $\frac{n-1}{2}$ times, so $\frac{n-1}{2} \geq 10$, giving $n \geq 21$.

2. **Answer: $\cos^{-1}\left(\frac{24}{25}\right)$**

   The vectors from start to first drop-off and from there to the second are $\vec{v_1} = (3,4)$ and $\vec{v_2} = (4,3)$, both with magnitude 5. The angle θ between these two vectors is the angle turned; $\vec{v_1} \cdot \vec{v_2} = |v_1|\,|v_2| \cos\theta$, so $12 + 12 = 5 \cdot 5 \cdot \cos\theta$.

3. **Answer: $\frac{1}{3}$**

   Call the numbers x, y and suppose that $x > y$. Then the condition is that $x - y > \frac{x+y}{2}$, or solving, $y < \frac{x}{3}$. Therefore if $y > x$, we must have $x < \frac{y}{3}$. From the diagram below, we see that the probability is therefore $\frac{1}{6} + \frac{1}{6}$.



4. **Answer: $\frac{\pi^2}{6} - \frac{205}{144}$**

$$\begin{aligned}\sum_{n=1}^{\infty} \frac{1}{n^2 + 8n + 16} &= \sum_{n=1}^{\infty} \frac{1}{(n+4)^2} \\ &= \sum_{n=1}^{\infty} \frac{1}{n^2} - 1 - \frac{1}{4} - \frac{1}{9} - \frac{1}{16}\end{aligned}$$

5. **Answer: $\frac{1}{280}$**

   There are $\binom{9}{3,3,3} = \frac{9!}{3!3!3!} = 1680$ distinguishable ways of arranging the 9 letters on the grid. To satisfy the conditions of the problem, we must have one of each letter per column and per row. We have 3 choices for X in row 1, then 2 for row 2, and only 1 choice for row 3, which makes $3 \cdot 2 = 6$ options for the first row. Adding a single letter in the second row determines the entire rest of the square, and there will only be two choices for that letter since one is already above it in the first row. Therefore, the desired probability is $6 \cdot 2/1680 = 1/140$.

6. **Answer: 2008!**

   Since b_n is a fixed point, $b_n = (f(b))_n$, which is, as seen in the problem statement, a recursive definition for factorial.

7. **Answer: 24**

   Since each Muggle cursed has a 50 percent chance of dying, there is always a 50 percent chance that the number of Muggles killed will be even (as long as the Dark Lord curses at least one Muggle). To see this, note that the parity of the number of dead Muggles depends entirely on whether or not the last Muggle cursed dies. No matter how many of the previous Muggles die, this last one will determine whether or not an even number total die.

Now for a little trickery: Suppose Voldemort curses n Muggles. Let E_n be the expected value of the number of Muggles killed given that this number is even, and F the expected value given that the number killed is odd. Note that the overall expected value of the number of Muggles killed is $\frac{n}{2}$, which must be equal to $\frac{E_n+F_n}{2}$, so $E_n + F_n = n$. If $n = 2k$, then for each j, the probability of killing $k+j$ Muggles is the same as that of killing $k-j$ Muggles, so $E_n = F_n = k$. Now if $n = 2k+1$, $E_n = \frac{1}{2}E_{n-1} + \frac{1}{2}(F_{n-1}+1)$, since if the number of Muggles killed is even, this is equally likely to result from an even number of the first $n-1$ dying and the last surviving, and an odd number of the first $n-1$ dying and the last also dying. But this gives $E_n = \frac{n}{2}$, so $E_n = F_n$.

Since $E_n = F_n = \frac{n}{2}$ for all n, Voldemort's expected satisfaction from cursing n Muggles is $3F_n + 6E_n = \frac{9}{2}n$. This is greater than the expected satisfaction of $2n$ from kicking n puppies, so Voldy should ignore the puppies and curse Muggles all day long.

8. **Answer: $\boldsymbol{(4+\pi)(2+\sqrt{2})}$ or $\boldsymbol{8+4\sqrt{2}+2\pi+\pi\sqrt{2}}$**

   The second square has diagonal the same length as the diameter of the first circle and therefore the same as the edge of the first square. Therefore, each square is $\frac{1}{\sqrt{2}}$ the size of the previous, and the circles scale in the same way. The first square has perimeter 4, and the first circle has circumference π. The total length is therefore $(4+\pi)\sum\limits_{n=0}^{\infty}\left(\frac{1}{\sqrt{2}}\right)^n = (4+\pi)\left(2+\sqrt{2}\right)$.

9. **Answer: 8**

   Let $z_k = e^{i\theta_k} = \cos\theta_k + i\sin\theta_k$ for $k = 1, 2$. Recalling that multiplication corresponds to addition of angles in the complex plane, while division corresponds to subtraction, we see that $\theta_1 - \theta_2$ must be a multiple of π (real axis) while $\theta_1 + \theta_2$ must be an odd multiple of $\pi/2$ (imaginary axis). The first condition need only be used for $0 \cdot \pi$ and $1 \cdot \pi$ (giving vectors in the complex plane either in the same or in opposite directions). The second condition then reduces to either $2\theta_1$ or $2\theta_1 + \pi$ being an odd multiple of $\pi/2$, so θ_1 must be an odd multiple of $\pi/4$, giving four possibilities from 0 to 2π. For each of these, θ_2 can either be the same or offset by π, giving a total of eight possibilities.

10. **Answer: $\boldsymbol{\frac{20}{9}}$**

    Split the sum into three parts: One where $y = x$, one where $y = x + d$, and one where $y = x - d$ for some integer $d > 0$. The first is $S_1 = \displaystyle\sum_{x=0}^{\infty}\frac{1}{2^{2x}} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$. The second is $S_2 = \displaystyle\sum_{d=1}^{\infty}\sum_{x=0}^{\infty}\frac{1}{2^{2x+2d}} = \sum_{d=1}^{\infty}\frac{1}{2^{2d}}\sum_{x=0}^{\infty}\frac{1}{2^{2x}} = \frac{4}{3}\sum_{d=1}^{\infty}\frac{1}{2^{2d}} = \frac{4}{9}$. The third sum, S_3, is just obtained by switching x and y, so $S_3 = S_2$. Then $S = S_1 + 2S_2 = \frac{20}{9}$.