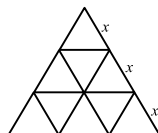


TEAM TEST
 2005 RICE MATH TOURNAMENT
 FEBRUARY 26, 2005

1. Find the largest prime whose cube divides $1! \cdot \dots \cdot 2005!$.
2. What is the number of sides of the regular polygon with the largest number of sides whose interior angles measure an integer multiple of 7° .
3. A solid is constructed out of an infinite number of cones with height h . The bottom cone has base diameter h . Each successive cone has as its base the circular cross-section halfway up the previous cone. Find the volume of the solid.
4. Suppose $\triangle ABC$ is a triangle with area 25. Points P , Q and R are on sides AB , BC and CA respectively so that $\frac{BP}{AP} = \frac{CQ}{BQ} = \frac{AR}{CR} = r$. If the area of $\triangle PQR$ is 7, what is the sum of all possible values of r ?
5. You are walking up a staircase with stairs that are 1ft. tall and 1ft. deep. The number of stairs from the ground floor to the first floor is 1. From the first floor to the second is 2 stairs. From the 99th to the hundredth stair is 100 steps. At each floor, the staircase makes a 90° turn to the right. At the top of the 100th floor, how far away from the bottom of the staircase are you?
6. A point in 3-dimensional space is called a lattice point if all three of its coordinates (x, y, z) are integers. When making a list of lattice points a_1, a_2, \dots, a_n , what is the minimum n that guarantees the midpoint between some 2 of the lattice points in the list is a lattice point?
7. Sparc is played with an octahedral and a dodecahedral die, numbered 1-8 and 1-12. If a player rolls a sum of 2, 6, 11, or 20 he wins. Of the other possible sums, a casino picks some which cause the player to lose. If the player rolls any of the other sums, they roll repeatedly until they get an 11 or their first roll. If they roll an 11 first they lose; if they roll their first roll, they win. Given that the probability of winning is $\frac{23242}{110880}$ and that given a choice between two equal probability rolls, the one with greater sum loses, which sums allow the player to keep rolling?
8. How many right triangles with integer side lengths have one leg (not the hypotenuse) of length 60?
9. Let S be the set of the first nine positive integers, and let A be a nonempty subset of S . The *mirror* of A is the set formed by replacing each element m of A by $10 - m$. For example, the mirror of $1, 3, 5, 6$ is $4, 5, 7, 9$. A nonempty subset of S is *reflective* if it is equivalent to its mirror. What is the probability that a randomly chosen nonempty subset of S is reflective?
10. Approximate to the nearest tenth $\sqrt{2000 \cdot 2010}$.
11. Each of the small equilateral triangles (9 total) have side length x and is randomly colored red or blue. What is the probability that there will be an equilateral triangle of side length $2x$ or $3x$ that is entirely red or entirely blue?



12. Craig Fratricks walks from home to a nearby Dunkin Donuts. He walks East a distance of 30 meters. Then he turns 15° to the left and walks 30 meters. He repeats this process until he has traveled 210 meters and arrives at Dunkin Donuts. If he had walked directly from home to Dunkin Donuts, how much distance could he have saved by walking directly from home to Dunkin Donuts (in one straight line).

13. Let

$$P = \cos \frac{\pi}{4} \cos \frac{\pi}{8} \cdots \cos \frac{\pi}{2^{1000}}.$$

What is $2\pi \cdot P$ to the nearest integer?

14. For all real numbers x , let the mapping $f(x) = \frac{1}{x-i}$. There are real numbers a, b, c and d for which $f(a)$, $f(b)$, $f(c)$, and $f(d)$ form a square in the complex plane. What is the area of the square?

15. The Fibonacci numbers are defined recursively so that

$$F_0 = 0, F_1 = 1,$$

and for $n > 1$,

$$F_n = F_{n-1} + F_{n-2}.$$

Calculate

$$\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n \cdot F_n.$$