GEOMETRY TEST
2003 RICE MATH TOURNAMENT
FEBRUARY 22, 2003

1. $ABCD$ is a square with sides of length 1. Suppose that a point E is placed somewhere on the edge CD. Let M be the maximum possible area of $\triangle ABE$, and let m be the minimum possible area of $\triangle ABE$. What is m/M?

2. Points A, B, and C do not determine a unique triangle. If $A = (1,2)$ and $B = (5,6)$, and $AC = 3BC$, then what are the possible locations for C?

3. Patty and Selma are racing through a park. The park has two concentric circular paths joined by two radial paths, one of which is at the point where they enter the park. The exit is at the intersection of the other radial path and the outer circle. Patty follows the radial path to the inner circle, walks around the short way to the other radial path and down it to the exit. Selma just walks the short way around the outer circle to the exit. They move at the same rate and meet up at the exit at the same time. What is the smaller angle made by the two radial paths?

4. Circle O is inscribed in $\triangle ABC$ and has radius 1. Suppose that $AC = BC$ and $AB = 2\sqrt{3}$. Find the area of the shaded region.
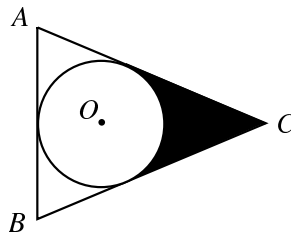

5. In trapezoid $ABCD$ with $AB \parallel CD$, $AB = 20$, $CD = 3$, $\angle ABC = 32°$ and $\angle BAD = 58°$. Compute the distance from the midpoint of AB to the midpoint of CD.

6. In circle O, $OA \perp OB$ and $OB \perp CD$. CD has length $\sqrt{3}$ and segment AC has length 6. $\angle AOC = 120°$. Find DB.
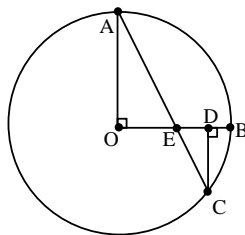

7. Two balls lie on the ground touching. If one ball has a radius of 8 inches and the point of contact is 10 inches above the ground, what is the radius of the other ball?

8. A circular pizza of diameter 16 is cut so that two perpendicular diameters are each divided into 4 equal lengths. Find the area of the shaded corner piece.
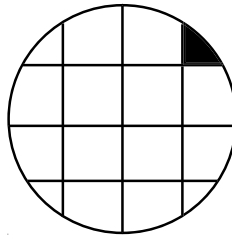
9. Let the area of the first figure below, the solid triangle, be 1. Then suppose that smaller triangles are successively removed step by step in the manner depicted below. All triangles are equilateral. What is the perimeter of the shaded region of the nth figure?
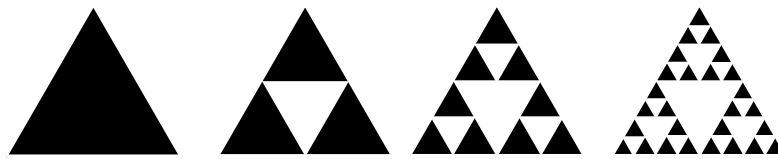
10. Consider 3 pairwise tangent circles with centers A, B, C. Given that $BC = 1$, $m\angle A = 45°$, $m\angle B = 60°$, $m\angle C = 75°$, what is the area of $\triangle ABC$ that is **not** inside of any of the three circles?