GENERAL TEST
2003 RICE MATH TOURNAMENT
FEBRUARY 22, 2003

1. Bobo needs to rent a truck. He could rent from Company Alpha for $250 per week plus 10 cents per mile or he could rent from Company Beta for $150 a week plus 25 cents per mile driven. At what weekly mileage are the rental fees of the two companies equal?

2. You have 12 red socks and 5 blue socks in your drawer. You take two socks at random. What is the probability of having matching socks?

3. Six athletes - F, G, H, M, N, O - swim in two separate 100-meter events, numbered 1 and 2. Each athlete finishes both events, and there are no ties. The same athlete who finishes second in event 1 finished fifth in event 2. M finished sixth in event 1. F finishes in one of the last three places in both events. If M finishes fourth in event 2, in which place must F finish in event 2?

4. Suppose that $a * b = a^2 + ab + 3b + 1$. List all numbers a such that there is no b for which $a * b = 2$.

5. Suppose you are writing numbers in the following font:

How many integers from 0 to 2003 inclusive have the property that if you rotate them 180°, you get the same number back (e.g. 1, 956)?

6. Ani is taking a true/false test with 6 questions. She gets 1 point for each question she gets right, loses one point for each question she gets wrong, and 0 points for each question she skips. In how many ways can she get a score of 5 on the exam?

7. An ant is allowed to walk along the edges of a tetrahedron. She starts at vertex A and wants to go to vertex B. She can use any *edge* at most once, and her journey ends once she gets to B. How many different paths can she take?

8. What is the perimeter of an isosceles right triangle that has an area of 1 square inch?

9. A "Multiplication Day" is defined as a date for which the product of the number of the month and the number of the day equals the last two digits of the year. How many Multiplication Days occur between January 1, 1995 and December 31, 2003, inclusive?

10. What are the dimensions of a rectangular tract of land if its perimeter is 40 and its area 96?

11. Evaluate $2^{2^{2^2}}/2^{2^2}$.

12. A mixture of 12 liters of Chemical A, 16 liters of Chemical B and 26 liters of Chemical C is required to kill the evil Wiffle bug. A bottle of commercial spray X contains 1, 2 and 2 liters respectively of these chemicals. A bottle of commercial spray Y contains only 1 liter of Chemical C and a bottle of commercial spray Z contains only 1 liter of Chemical A and 1 liter of Chemical B. How much of each spray (X, Y, Z) is needed to get the desired mixture?

13. A card is chosen randomly from a regular 52-card deck. Given that aces are the highest in their suit, what is the probability that the card is red and is a six or less?

14. On a twelve-hour digital clock displaying hours and minutes only, for how many minutes during each day is the sum of the digits 12?

15. Homer has three kids, Bart, Lisa and Maggie. In 2003, Lisa's age is four times Maggie's age plus two. Bart goes into a time machine and goes back 11 years; then his age (today) is twice Lisa's age (in 1992) minus three. Bart then goes back to the present and Maggie takes a 15 year trip at light-speed such that she doesn't age but everyone else does. Upon Maggie's return, Bart finds he's seven times as old as Maggie. What is Maggie's age?

16. A tropical island has eight towns - N, O, P, R, S, T, U, and W. Tourists must fly into P and then take roads to the other towns. There are roads joining N to O, N to P, N to R, O to U, P to S, P to T, S to T and T to W. A tourist travelling from O to W and visiting as few towns as possible must visit how many different towns between O and W?

17. Each square in the grid below has side length 1. What is the area of the gray region?
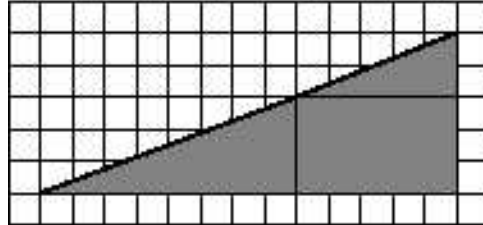
18. A rancher has 100 meters of fencing to enclose a rectangular corral whose area is $350\,\mathrm{m}^2$. He also wants to use some of the fencing to divide the corral into two equal sized areas, and he wants this inner fence to be as short as possible. Find the length of the inner fence.

19. Two fair 5-sided dice are rolled. The faces are labelled 1, 2, 3, 4, and 5. What is the probability their sum is odd?

20. What is the smallest positive number k such that there are real numbers a and b satisfying $a + b = k$ and $ab = k$?

21. How many three-digit numbers satisfy the conditions that there is no repetition in the digits, the number must contain a 5, and is less than 800? (Note that a 0 in the leading digit is not allowed.)

22. A merchant plans to sell two models of CD players at costs of $250 and $400. The $250 model yields a profit of $45 and the $400 model yields one of $50 per unit. The merchant estimates a monthly demand of 250 total units. For security purposes, he doesn't want to have more than $70,000 in inventory (based on selling price) at once. Find the number of $250 CD players he should stock to maximize profit.

23. How many distinct permutations are there of the group of letters A, A, G, E, E, E, M?

24. A 1000 liter tank contains 50 liters of a 25% brine solution. If you have plenty of liters of a 75% brine solution, how many liters of the 75% solution do you need to add to the tank to get a 30% brine solution?

25. Peter is going bowling. On each roll he always hits one of pins 1 through 7, as numbered below, with equal likelihood.
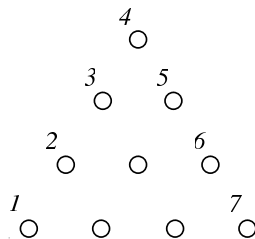
When he hits pin x, then he knocks over all the pins in the largest equilateral triangle with x as its topmost point.

Now, suppose Peter bowls a full frame (two consecutive rolls with no replacement of the pins in between). If he would hit pin x on his second roll, but has already knocked it down on his first roll, then he doesn't hit any pins at all. What is the probability that Peter knocks down exactly 6 pins in a frame?

26. Write $\frac{1}{x^2+3x+2}$ as a sum of fractions, each of which has a denominator of the form $ax + b$.

27. Thirteen pirates are trying to divide up their booty. When everyone is given the same number, one is left over. In the ensuing fight, two pirates fall overboard. The remaining pirates again try to divide the coins evenly amongst themselves and find there is one coin left over. After another fight, four pirates are shot out of cannons. The remaining pirates again find that when they divide up the coins, one is leftover. They give up and donate all the coins to the Rice Math Club.

    Afterwards, Greg counts the loot and finds out that the number of coins is the smallest odd number greater than 1 that would give the pirates all these problems. How many coins are there?

28. Find the number of diagonals in a regular 17-sided polygon.

29. Certificates of deposit (CDs) pay 10% annually and municipal bonds pay 8% interest annually. Over the year, an investor wants to invest one-fourth of his money into stocks, and stocks grow by 12% each year. He wants a combined return of 10% on his total investment of $500,000. How much is put into each type of investment? Give (CDs:bonds:stocks).

30. A rectangular box has a face with area 8, a face with area 15, and a face with area 10. Find the volume of the box.

31. Patty is picking peppermints off a tree. They come in two colors, red and white. She picks fewer than 30 total peppermints but at least one of each color. In addition, she always picks fewer white peppermints than five times the number of reds. How many different combinations of peppermints can she go home with?

32. We can express any four-digit number as $ABCD$, where A is the first digit, B is the second digit, etc. (note that $A \neq 0$). If any of the conditions below hold, we say that the number is "interesting":

    - A, B, C, and D are all even,
    - A, B, C, and D are all odd,
    - $A > B > C > D$,
    - $A < B < C < D$, or
    - $A = B = C = D$.

    How many "interesting" four-digit numbers are there?

33. Let O be an octagon with with vertices labelled $V_1, V_2, \ldots, V_8$ consecutively. Draw in all the diagonals of the octagon except for diagonals between V_1 and V_5, V_2 and V_6, V_3 and V_7, and V_4 and V_8. Now consider all triangles whose vertices are vertices of the octagon, and whose edges are the diagonals we have just drawn in or edges of the octagon. How many such triangles are there?