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Designing Better Graphs by Including Distributional Information and Integrating Words, Numbers, and Images

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Running Head: Designing better graphs

Abstract

Statistical graphs are commonly used in scientific publications. Unfortunately, graphs in psychology journals rarely portray distributional information beyond central tendency and few graphs portray inferential statistics. Moreover, those that do portray inferential information generally do not portray it in a way that is useful for interpreting the data. We present several recommendations for improving graphs including: (1) bar charts of means with or without standard errors should be supplanted by graphs containing distributional information, (2) use good design to allow more information to be included in a graph without obscuring trends in the data, and (3) figures should include both (a) graphic images and (b) inferential statistics presented in words and numbers.

Designing Better Graphs by Including Distributional Information and Integrating Words, Numbers, and Images

Statistical graphs are useful both for the discovery of knowledge (Larkin & Simon, 1987; Tukey, 1974, 1977) and communication of knowledge (Few, 2004; Kosslyn, 1985; Tversky, 1995; Wilkinson and the Task Force on Statistical Inference, APA Board of Scientific Affairs, 1999). Although graphs have been used since prehistoric times, statistical graphs are a relatively new phenomenon. Before the late 18th century, tabular data representations were popular and graphs were regarded as useless for analysis. This view changed when Playfair invented what are still the most commonly used graphs: the bar graph and line graph in 1786 and the pie chart in 1801 (Wainer, 2005). Herschel's invention of the scatterplot in 1832 further demonstrated the value of graphs (Friendly & Denis, 2005).

Numerous studies have assessed the relative value of graphical versus tabular presentation of data. Based on an extensive review of research available at the time, Jarvenpaa and Dickson (1988) concluded that graphs are better at summarizing data, showing trends, and showing points and patterns whereas tables are better for point/value reading. More recent studies and literature reviews are consistent with these conclusions (Gelman & Stern, 2006; Gillan, Wickens, Hollands, & Carswell, 1988; Meyer, Shamo, & Gopher, 1999). A further benefit of graphs cited in the APA publication manual is that graphs can make it easy to perceive the "overall pattern of results" (APA, 2001).

Not surprisingly, graphs are used frequently in the reporting of results in psychology as well as in other fields, although the degree of use varies greatly across fields (Cleveland, 1984; Smith, Best, Stubbs, Archibald, & Robertson-Nay, 2002). For example, Cleveland (1984) found that the proportion of journal space devoted to graphs was higher in the natural sciences than in the social sciences. Best, Smith, and Stubbs (2001) found a positive relationship between perceived scientific hardness of psychology journals and the proportion of area devoted to graphs. Interestingly, Smith *et al.* (2002) found an inverse relationship between area devoted to tables and perceived scientific hardness.

Despite advances in graphics and the availability of graphics software, graphs are often poorly constructed in practice. In a study of the graphs in the journal Science, Cleveland (1984) evaluated the frequencies of four types of errors: construction errors, degraded image errors, errors in the explanation, and discrimination errors (elements of a graph were difficult to distinguish). He found that 30% of the graphs contained at least one error. This proportion is surprisingly high considering that only major errors were included and smaller flaws ignored.

Tufte (2001) sampled graphics from scientific and news publications between 1979 and 1980 and evaluated their data density. He found that the average published graphic had a low data density, although nearly every publication contained a few data-rich graphs.

We consider how to portray three types of information in graphs: shapes of distributions (Wilkinson *et al*., 1999), trends (Gelman, Pasarica, &

Dodhia, 2002), and inferential statistics (Cumming & Finch, 2005; Masson & Loftus, 2003; Wilkinson *et al.*, 1999). We focus on how to include and juxtapose these types of information so as to support correct interpretations of the data. We do not discuss perceptual considerations for creating graphs such as label placement, scaling of axes, aspect ratio, and contrast in detail since these topics have been covered very well in previous works (Cleveland, 1994; Gillan *et al*, 1998; Kosslyn, 1985, 1993). However, we do refer to these and related works in our section on general principles for graph construction.

Distributional Information

Graph use in psychology has been criticized for focusing on the depiction of central tendency to the neglect of other distributional information. For example, Wilkinson *et al.* (1999) argued strongly that a common deficiency of graphs in psychology journals is their lack of information regarding the shape or distribution of the data, and that this lack of information hinders scientific evaluation.

Sándor and Lane (2007) conducted a survey of graph use in two leading psychology journals and obtained results consistent with Wilkinson *et al.*'s conclusions: The majority of the graphs were bar charts and only about 10% of the graphs showed distributional information beyond central tendency. Although bar charts are useful for displaying counts or percentages, we believe that there are few if any situations in which it would not be better to replace bar charts of means with box plots since box plots take up no more space and provide summary information about distributions (See the Appendix for more information about box plots).

Consider how to graph the results from a hypothetical experiment with a Condition (A, B, and C) x Group (Control and Experimental) between-subjects design and 12 cases per cell in which Condition is a categorical variable. As would be typical in a design such as this, assume the (hypothetical) experimenter was interested in the difference between the experimental and control groups as well as whether this difference varies across conditions. Figure 1 shows a graph typical of those appearing in psychology journals. Although it is clear that the difference between the control and experimental means was large in Condition A, small in Condition B, and medium-sized in Condition C, there is no information about the variability or shape of the distributions. The "data density" of the figure is very low since it portrays very few values.

Please insert Figures 1 and 2 about here

Figure 2 shows the same data portrayed by box plots. In recognition of the fact that the mean is often of great importance, a variation of box plots that displays the mean (indicated by a plus sign) was chosen. Figure 2 is much richer in information than is Figure 1: In addition to the differences in means shown in Figure 1, Figure 2 shows (1) that the range and interquartile range in the experimental group are larger than in the control group in all three conditions; (2) the distributions overlap greatly in Condition B, somewhat in Condition C, and very little in Condition A; and (3) that there are no outliers.

If other aspects of the distribution were theoretically relevant, then three sets of back-to-back stem-and-leaf displays would be a good alternative. If the sample sizes were much larger, then back-to-back histograms could be shown (See the Appendix for more information about back-to-back stem-and-leaf displays and back-to-back histograms).

We stress that there is room for subjective opinions in the choice of graph type and the details of graph construction. Our point is not that graphs should be constructed precisely in the manner shown here. Rather, it is that the kinds of information contained in Figure 2 and subsequent example graphs should be routinely depicted in graphs appearing in psychology journals and other scientific publications.

One possible objection to including more distributional information is that this information may distract readers from what is typically the main objective of the graph: portraying the pattern of means. The tradeoff between including more information and obscuring important patterns is a general one, and can often be dealt with effectively by emphasizing graphic elements that show the important pattern. A good example is provided by Tufte (2006, p. 116-121) in his discussion of a graph showing the relationship between the body weight and brain weight of animals. In the original version of the graph presented by Sagan (1977), the animal names were printed next to the points in the scatterplot. Cleveland (1994) argued that these labels cluttered the graph and should have been omitted.

However, Tufte disagreed noting that with the proper design, clutter can be reduced without a content reduction. Accordingly, Tufte presented two excellent redesigns of the graph, each containing the information about the identities of the individual points while clearly showing the pattern of the relationship. In the first, the points in the scatterplot were dark whereas the labels for the points were light grey. In the second, drawings of the animals replaced the points themselves.

The idea that a design solution is preferable to a content reduction can be applied to the statistical graphs discussed here. For example, if one wished to emphasize differences among means in parallel box plots, one could, as shown in Figure 3, make the representation of the means more prominent and the other elements of the box plot less prominent. Alternatively, if one wished to emphasize differences in variability among conditions, one could make the range and the interquartile range of each distribution more prominent as in Figure 4.

Please insert Figures 3 and 4 about here

There is an understandable desire on the part of researchers to show their data in a positive light. As a result, some may resist showing distributional data that reveal the variability and possible irregularities not apparent in a plot of means. However, this is clearly not a justifiable basis on which to omit distributional information (Wilkinson *et al.*, 1999).

Trends and Distributional Information

Please insert Figure 5 about here

Portraying distributional information and trends in the same figure can be difficult. For example, consider a hypothetical experiment in which the researcher is interested in differences in the rate of learning as a function of condition. Figure 5 displays the means in order to show the trends and the standard deviations to show the most basic kind of distributional information, variability. There are several problems with this graph. First, the lines showing the standard deviations are distracting and make it more difficult to view the trends. Moreover, it is difficult to tell which bar goes with which condition. Second, the standard deviations are misleading since they are based on between-subjects error whereas the tests of the trends and Condition x Trend interactions are based on the within-subjects error term (See Loftus & Masson, 1994 for a discussion of this issue). Third, and probably most important, the distribution of scores at specific combinations of condition and trial are not likely to be the distributions most relevant to the research question. For example, if the researcher were interested in whether performance increased across trials and whether there were differences among conditions in the rate of increase, the distributions of the linear components of trend (computed by applying linear trend coefficients to the raw scores for each subject) for the various conditions would be more relevant.

Figure 6 shows the trend information in the lower portion and box plots of the linear components of trend in the upper portion[1]. Although one could show the distribution of the linear components of trend in a separate figure, we believe a single figure showing both graphs communicates the results more effectively.

Please insert Figure 6 about here

The box plots in Figure 6 provide important distributional information that is not available in either Figure 4 or 5. First, one can see that there are no outliers and that none of the distributions have substantial skew. Second, one can see that the within-group variances do not differ greatly. Finally, one can get a good sense of the size of the effect. For example, it can be seen that the 25th percentile of Condition A is approximately the same as the median of Condition B, which is slightly above the 75th percentile of Condition C. Also evident is the fact that the difference between the means of Conditions A and B as well as the difference between the means of Conditions B and C are approximately one standard deviation ($d \approx 1$).

We believe that it would be valuable to supplement most graphs showing trend with a graph showing the distributions of one or more components of the trend. The construction of the graph would, of course, depend on the details of the data being portrayed.

Inferential Statistics

The descriptive statistics shown in graphs, no matter how well presented, do not stand on their own. Wainer (1996) argued convincingly that an effective display of data must (1) reveal the uncertainty in the data, (2) characterize the uncertainty as it relates to inferences to be made from the data, and (3) help prevent the drawing of incorrect conclusions due to lack of appreciation of the precision of the information conveyed. Consistent with Wainer, many authors have recommended that graphs contain information relevant for inferential statistics (Belia, Fidler, Williams, & Cumming, 2005; Cumming & Finch, 2005; Loftus, 1993; Wilkinson *et al.*, 1999). Cumming *et al*. (2007) found that the percentage of graphs containing inferential statistics is increasing: The mean percentage of psychology articles with figures containing inferential information increased from 11% in 1998 to 25% in 2003-2004, and to 38% in 2005-2006. Even with this increase, it is clear that many graphs do not include inferential information. Similarly, Sándor and Lane (2007) found that inferential statistics are more often omitted than included in graphs.

A frequently-used method to portray inferential information is to display means with standard error bars. Figure 7, which is based on the same fictitious data as Figures 1 and 2, is typical of these graphs. The problem with Figure 7 is that the only inferences directly supported by the graph involve individual cell means. For example, the graph shows that the mean for the control group in Condition C is approximately seven with the error bar stretching somewhat more than one in each direction. Since a 95% confidence interval is approximately the mean plus and minus two standard errors, the graph provides meaningful information about the precision of the estimate of the population mean.

Please insert Figure 7 about here

The problem is that estimates of individual means are rarely the critical issue. More relevant here is, for example, the difference between the experimental and control groups in Condition C.

Figure 7 shows that the difference between group means in Condition C is approximately five. The standard error of the difference between means can be computed using the following formula:

$s_{\text{Mean Difference}} = \sqrt{\frac{2\text{MSE}}{n}}$ where MSE is the Mean Square Error or average variance within the groups and n is the sample size. Although considerable mental gymnastics would be required to obtain a rough approximation of this standard error of the difference between means, it could be done by using the average standard error as an estimate of $\sqrt{\frac{\text{MSE}}{n}}$ and 1.4 as an approximation of the square root of 2. Since the standard error for the control group is a little above 1.0 and the standard error of the experimental group is about 2.5, the average standard error is about 1.8. Multiplying by 1.4, one obtains a value approximately equal to 2.5. Therefore, the 95%

confidence interval on the difference between means ranges from about 0 to about 10 (i.e. 5 ± 2 x 2.5).

Since the main purpose of published graphs is to communicate, it is important to consider how well the target audience understands the graphs. Belia, Fidler, Williams, and Cumming (2005) investigated how well authors in psychology, behavioral neuroscience, and medical journals understand the relationship between standard error bars, confidence intervals, and significance tests. Participants were shown a graph of two means and either standard error bars or confidence intervals depending on the experiment. Their task was to adjust the difference between means so that the difference would be just statistically significant at the .05 level. The results were dramatic: Fewer than a quarter of the subjects set the means so that the *p* value was between half the target value (.025) and twice the target value (.10).

It is clear that the calculations to make a theoretically-meaningful inference from a graph containing means with standard error bars are not easy to do in one’s head and not well understood by most authors (not to mention readers) of journals. Therefore, including standard error bars is often not sufficient for communicating inferential information.

Figure 8 shows an alternative method of displaying inferential information. Although easy to interpret, we believe that graphs specifying the means that are significantly different from each other have several drawbacks. First, by marking differences as either significant or not, this type of graph encourages the “all or none” rejection of a null hypothesis, a practice that has been widely and severely criticized (Gelman & Stern, 2006; Loftus, 1993; Tukey, 1991; Wilkinson *et al.*, 1999).

Second, this type of graph emphasizes hypothesis testing and neglects confidence intervals. Finally, this type of graph may distract visually from the pattern of means if several means are being compared.

Please insert Figure 8 about here

Cumming and Finch (2005) proposed ways to design graphs so that they would show inferential statistics in a more meaningful way. Specifically, they gave examples of how graphs could display confidence intervals relevant to the inference in question. As noted previously, standard error bars and confidence intervals are typically drawn around condition means even though the relevant inference pertains to the difference between means. Cumming and Finch suggested that in a two-condition experiment, one should include a graph showing the difference between means and a confidence interval on this difference in addition to the group means and their respective confidence intervals. Figure 9 shows an example of this type of graph.

Please insert Figure 9 about here

For between-group designs, this type of graph has the advantage of showing the relevant confidence interval directly. It has an even bigger advantage for within-subjects designs because the error term for these designs is often much smaller than the error terms for the groups individually and cannot be computed from the confidence intervals on individual means. Although the type of graph suggested by Cumming and Finch (2005) is valuable and represents a significant improvement over typical methods of portraying confidence intervals graphically, it does not represent a general solution to the problem of graphing confidence intervals. As acknowledged by Cumming and Finch, "However, if more than a few effects are of interest, the graphical challenge is very great, and no convincing and proven graphical designs have yet emerged" (p. 178). Moreover, it is often desirable to present inferential information more precisely than is practical to do with graphs. For example, the recommendation to report the exact *p* levels (APA, 2001 p. 25; Wilkinson *et al*., 1999) implies that at least some kinds of inferential information should be presented with text rather than graphically.

The failure of graphs to portray inferential information successfully is a serious problem since inferential statistics are important for interpreting a graph. A graph showing only means does not provide sufficient information for the reader to know how seriously to take sample differences or to judge the likely size of the difference in the population. One solution is to refer the reader to the text for the inferential statistics. Although this is not entirely unsatisfactory, it would be preferable for the reader to see the graph and the inferential statistics without having to jump back and forth between the text and the graph, which may even be on different pages. As described in the following section, we believe a much better solution is to create figures that integrate graphics and text.

The Archaic Separation of Graphics and Text

Wainer (1997) and more recently Tufte (2006) have bemoaned the separation of graphics and text that is now typical in scientific journals including those in psychology. Tufte (2006) argued that displays of evidence should bring together verbal, visual, and quantitative information and that the process of publishing causes these elements to be segregated unnecessarily.

Unlike modern scientific journals that separate graphics, text, and quantitative analyses, Leonardo da Vinci's notebooks (see Wainer, 1997, p. 145) integrated text and graphics. For example, "Studies of Embryos" published in the early 16th century contains pages that integrate sketches, geometric diagrams, and text. Similarly, Galileo's "The Starry Messenger" (Galileo, 1610) is 30% images and diagrams all fully integrated in the text (Tufte, 2006, p. 83).

The modern practice of separating these types of information is unfortunate since it often requires the reader to skip back and forth between figures, tables, and text in order to interpret the data. Gillan *et al*. (1998) recognized this problem and its importance for graph design. They argued that graph design should take into account the cognitive demands on the reader including the task of integrating the meaning of the graph with the text. Cognitive demands are lower if the various modes of information are displayed together in an integrated format.

Strong empirical support for integrating graphics and text was obtained by Sweller, Chandler, Tierney, and Cooper (1990). Based on a series of six experiments they concluded that the conventional method of separating graphics and text leads to poorer performance than presenting material in a way that does not require learners to split their attention.

As is evident from the casual inspection of any psychology journal and documented more formally by Sándor and Lane (2007), data presentations in psychology predominantly use a split-source format in which the numerically-presented details of the statistical analysis such as significance tests, confidence intervals, and measures of effect size are presented in text separated spatially from the figure. Design principles (Tufte, 2006; Wainer, 1997), theoretical considerations (Gillan *et* al., 1998) and empirical evidence (Sweller *et* al., 1990) all support the proposition that figures should integrate both graphical and numerical information[2].

In the following section we present examples of how graphics and text can be integrated in figures. The inferential statistics included in these examples are in line with the recommendations of Wilkinson *et al.* (1999). However, they should be considered only as examples since our purpose is to demonstrate how graphics and text can be integrated rather than to advocate any particular approach to inferential statistics. Our argument is applicable to non-parametric, resampling, Bayesian, and other approaches to inference.

Examples Integrating Graphics and Text

A reader examining the box plots in Figure 2 would likely wonder how seriously to take the finding of a larger difference between the control and experimental groups in Condition A than in the other two conditions. Figure

10 adds inferential statistics to this figure including the results of an analysis of variance that shows that the interaction is significant, $p = .012$.

Please insert Figure 10 about here

An advantage of integrating graphics and inferential information is that the graphics display distributional information relevant to the assumptions of the inferential statistics. Since differences in interquartile ranges between the control and experimental conditions shown in Figure 10 indicate a violation of the assumption of homogeneity of variance, it would be incumbent on the author to justify the use of ANOVA. In this case, a comment about the robustness of ANOVA to violations of the assumption of homogeneity of variance when the sample sizes are equal would probably suffice. In other cases a more extended discussion might be required.

Since an interaction means that the simple effects differ, tests of differences between simple effects are often informative. The upper right-hand portion of the figure shows the *p* values for the three differences between simple effects. Effect size estimates, confidence intervals, and significance tests for each simple effect are shown at the bottom of the figure. Note that a graphical scheme that indicated only whether or not an effect was significant would not reveal that the simple effect at C approached significance and would subtly hint that this possible simple effect should be ignored.

Please insert Figure 11 about here

A graph integrating distributional, trend, and inferential information is shown in Figure 11. This figure adds inferential information to Figure 6, which shows distributional and trend information. Figure 12 shows how one might integrate the display of bivariate data and relevant inferential statistics. The inferential statistics in this figure consist of tests showing that the slopes and correlations for the two conditions are significantly different as well as being significantly different from zero in each condition. Confidence intervals for all statistics reported are included.

Please insert Figure 12 about here

Our next example is a bit more complex. In a hypothetical experiment, a researcher is interested in the relationship between the independent variables X1, X2, X3, and the dependent variable (DV). Although the researcher did not necessarily anticipate an interaction between X1 and X2, the regression analysis found a significant $X1_{(linear)}$ x $X2_{(linear)}$ interaction with no test involving a quadratic term or other interaction term approaching significance. The regression equation after *centering* X1 and X2 (*i.e.,* transforming them to deviation scores) is:

$$DV' = 0.533\ X1 + 0.370\ X2 + 0.427\ X3 - 0.37\ X1\ X2 + 92.826$$

This equation indicates that the slope of the relationship between X1 and DV decreases by 0.37 for each increase of one in X2. The question is how to construct a graph to describe the interaction. A method suggested by Aiken and her colleagues (Aiken & West, 1991; Cohen, Cohen, West, & Aiken, 2003) is to graph the regression line for the prediction of DV by X1 separately for three levels of X2: the mean of X2, one standard deviation below the mean of X2 and one standard deviation above the mean of X2. For these (fictitious) data, these three values (after centering) are -10, 0, and 10. The three regression lines are shown in Figure 13.

Please insert Figure 13 about here

This method of graphing the interaction is valuable in that it shows the form of the interaction clearly. For these data it is easy to see that the slope is high for the low level of X2 and decreases linearly as X2 increases. However, we do not believe that this method of graphing should be the only one used to depict the interaction since it produces a graph of a model of the data rather than a graph of the actual data. As such, it contains no distributional data and highlights the regularities in the data without revealing any possible irregularities.

An additional way of graphing the interaction is to divide the data into groups based on X2 and to examine the relationship between X1 and DV for each of these groups. Since X2 and X3 are controlled in the statistical analysis, they should also be controlled in the graph. Figure 14 shows the relationship between X1 and DV separately for four levels of X2 with both X2 and X3 controlled. The first steps were to compute the residuals in X2 after being regressed on X3 (X2.3) and the residuals in X1 after being regressed on X2 and X3 (X1.23). Next, the data were divided into quartiles based on the values of X2.3. Finally, DV was regressed on X1.23 within each of these quartiles. The slopes in these four regressions show how the partial slope of

X1 (in predicting DV) decreases as a function of X2. Unlike Figure 13, Figure 14 shows that the decrease in the slope from Quartile 1 to Quartile 2 is larger than the decrease from Quartile 2 to Quartile 3 or from Quartile 3 to Quartile 4. Since the $X1_{(linear)}$ x $X2_{(quadratic)}$ interaction did not approach significance, this change in the difference in slopes should not serve as a basis for concluding that the interaction is not strictly linear x linear. However, it is important because it provides a hint that the model may not be correct and that some caution in interpretation is warranted. The distributional information in Figure 14 allows the reader to see that the data are generally well behaved and no points appear to exert undo influence.

Please insert Figure 14 about here

Figure 14 also contains significance tests and confidence intervals for each quartile. These tests are essentially simple effects tests following the significant linear x linear interaction and reveal that one can make a strong conclusion that there is a positive relationship between X1 and DV for low levels of X2 but leave the relationship for higher levels of X2 in doubt. The confidence intervals in Quartiles 3 and 4 make clear that a range of values of the slope other than zero are plausible and, as is always the case, a point null hypothesis should not be accepted.

In our final example, we consider how to display distributional information in an analysis of covariance (ANCOVA) design. As noted previously, a bar chart showing group means lacks distributional information. The alternative of using graphs such as box plots to portray group differences has two potential problems when used in an ANCOVA design. One is that since inferential tests are done on differences among adjusted means (controlling for the covariate), differences in means portrayed in the graph would not reflect the differences tested in ANCOVA. Second, the variability of the distribution shown in the graph would include the variability potentially controlled by the covariate and would therefore be greater than it should be. As a result, differences among means relative to this variability would appear smaller than they really are.

The solution proposed here is to remove the effect of the covariate from the data before creating the box plots. As an example, consider a hypothetical experiment designed to assess the difference between an experimental condition and a control condition. A total of 50 participants was randomly divided between the two conditions and a covariate thought to be related to the criterion was measured for each subject. Subsequently, the experimental procedures were administered.

Before conducting an ANCOVA, the assumptions of linearity and homogeneity of regression slopes should be assessed. Figure 15 shows separate regression lines for the two conditions. It is clear from the graph that the slopes of the lines are very similar and the inferential statistics shown on the graph indicate that the difference in slopes did not approach significance. Figure 15 also shows that the covariate is strongly related to the criterion and that the relationship is at least approximately linear.

Please insert Figure 15 about here

As noted previously, constructing box plots without regard to the covariate would portray the effects as being smaller (relative to variability) than they actually are. Therefore, the following procedure was used to eliminate variance due to the covariate from the graph:

1. A linear model was developed to predict the dependent variable (DV) from the covariate and from "condition."
2. The adjusted means for each condition (sometimes called "least squares means" or "estimated marginal means") were computed. The adjusted means are estimates of what the sample means on the dependent variable would have been if all group means on the covariate had been the grand mean on the covariate. Most if not all major statistics packages have an option to report adjusted means.
3. The residuals from the model were saved. The means for each condition's residuals are necessarily 0.
4. The adjusted mean for each condition was added to the residual score of every subject in the condition thus making the condition means equal to the adjusted means.
5. Box plots were constructed based on the scores computed in Step 4.

It is interesting to note that if one were to do an ANCOVA on these derived scores, the results would be the same as on the raw scores except that the slope and sums of squares for the covariate would be zero. This reflects the fact that variation related to the covariate was removed from the data.

The box plots created following these steps are shown in Figure 16 and reveal a clear treatment effect. Specifically, the box plots show that (1) the 25^{th} percentile of the experimental group is approximately equal to the median of the control group and (2) the median of the experimental group is approximately equal to the 75^{th} percentile of the control group. The variability shown in the box plots is considerably less than it would have been if the scores had not been adjusted for the covariate. For example, the difference between the top and bottom of the box plots (the range when there are no outliers as is the case here) is approximately 26 whereas it is about 36 for the raw data. Similarly, the height of the box (the H-Spread or interquartile range) is about 11 whereas it is about 14 for the raw data.

Figure 16 also shows that the effects of the covariate and of condition are both significant and that the mean difference is 0.81 standard deviations. Thus, one can draw a confident conclusion that the experimental treatment leads to higher scores than does the control treatment.

Please insert Figure 16 about here

We believe that the examples shown here that integrate graphs with inferential information do a better job communicating experimental findings than the procedure of artificially separating text, tables, and graphs typically used in journal articles. Since integrating text and graphics is no longer difficult or expensive for either publishers or authors, we suggest that authors follow the excellent examples of da Vinci and Galileo and create figures integrating words, numbers, and images.

Creating figures that integrate text and graphics is not technically difficult. The first step is to create the statistical graph using one of the many widely available statistics packages. One should be flexible in the choice of software rather than rely exclusively on one program. A spreadsheet program is often sufficient to create simple graphs such as line graphs. For more complex graphs, one could choose the graphing program or statistical package best suited for the specific graph to be created.

The integration of text and graphics can be best accomplished by graphics-editing programs such as Adobe Illustrator or CorelDRAW that provide considerable control and flexibility. However, even programs with basic page-layout capabilities such as Microsoft's PowerPoint, Microsoft Word, Apple's Keynote, and Apple's Pages can produce excellent integrated figures.

Principles for Constructing Good Graphs

Although the focus of this article has been on suggestions for including distributional and inferential information in graphs, we understand that other aspects of graphs are also of great importance. Since there is a sizeable literature relevant to the construction of effective graphs (*e.g.*, Kosslyn, 1989, 1993; Cleveland, 1993, 1994; Few, 2004; Gelman, Pasarica & Dodhia, 2002; Gillan *et al.*, 1988; Tufte, 2001, 2006; Wainer, 1997), space limitations preclude a comprehensive review of this literature. Therefore we present only some of the more important themes.

Well-constructed graphs help focus on important aspects of data, provide visual clarity, and make interpretation easy. Accordingly, one should normally avoid shaded backgrounds that reduce contrast, eliminate unnecessary or redundant elements, and use sufficiently dark lines and points. It is usually not necessary to include horizontal lines to extend tick values on the Y axis. When it is necessary, these lines should be made lighter than other graphical elements.

One should be careful to avoid apparent inconsistencies between the graph and the results of the statistical analysis. For example, in a within-subjects design, the graph should control for variation due to subjects just as the statistical analysis does (see Loftus & Masson, 1994 and Figure 11 of this article for examples of how to do this). Similarly if variance due to a covariate is controlled in the statistical analyses, it should also be controlled in the graph (see Figure 16). Special care should be taken in graphing means from designs containing both between- and within-subjects variables since the error terms for various comparisons between means are different.

Graphs should be designed to minimize mental load. For example, consider how to designate the conditions in a line graph such as the one shown in the lower portion of Figure 11. In graphs such as this, it is better to place the condition information next to each line as shown rather than to have a separate caption since having a separate caption would require the reader to remember which symbol represents which condition. For more complex designs, the choice of symbols should be consistent. For example, consider an Age (2) x Condition (2) design in which conditions are designated by two levels of shape (circle and square) and two levels of “fill color” (white and black). It is important that the assignment of symbols to conditions be consistent, as in the left portion of Figure 17 rather than inconsistent as in the right portion of the figure.

Please insert Figure 17 about here

The design of a graph should be informed by the well-established Gestalt laws of perceptual grouping including good continuation (objects following a line or smoothed curve tend to be grouped together), proximity (objects near each other tend to be grouped together), similarity (similar objects tend to be grouped together), common fate (lines going in the same direction tend to be grouped together), and good form (enclosed shapes tend to be seen as single units). See Kosslyn (1993) for examples of how the application of these laws can result in more easily apprehended graphs. Finally, place graphical elements that are likely to be compared near each other.

It is important to consider the amount of information contained in the graph: Too much information can make a graph difficult to interpret whereas two little information can waste space and fail to provide the benefits typically associated with graphs. As noted earlier, bar graphs provide relatively little information for the amount of space they take up. Box plots and stem-and-leaf displays are among the types of graphs that contain more information in roughly the same amount of space.

Graphs can be used to mislead rather than to inform. Although the blatant use of misleading graphs in scientific publications is rare due to the review process and the sophistication of the readership, one should always be on guard against creating a graph that is unintentionally misleading. It is well known that group differences can be obscured or exaggerated by the scaling of the Y-axis. Therefore, both the Y-axis origin and the scaling of the units should be chosen carefully and in accordance with theoretical notions of what constitutes small and large effect sizes for a particular domain. Graphs such as box plots that show the minimum and maximum values put constraints on the scale of the Y-axis and normally lead to good choices. For example, the box plots in Figure 10 lead naturally to a sensible Y-axis scale.

The use of double Y-axis graphs can be very misleading as is described in the following example. Wainer (1997, p. 93-94) shows three ways to portray the relationship between per-pupil expenditures for education and SAT using a double Y-axis graph. Year (from 1978-1990) is graphed on the X-axis. The graph contains two lines: one for per-pupil expenditures and one for SAT. Since these two variables are measured on vastly different scales two Y-axes are used: the per-pupil expenditures scale shown on the left Y-axis and the SAT scale shown on the right Y-axis. In the original graph published in Forbes magazine, the Y-axes were scaled so that it appeared that there was a large increase in expenditures over time with little change in SAT. In one of Wainer's alternate versions of the graph differing in the scaling of the Y-axes, it appears that both expenditures and SAT increased greatly. In a second alternative scaling, it appears that expenditures increased only slightly while SAT scores increased greatly over time.

The perceptual aspects of graphs can mislead in unexpected ways. As an example, Kosslyn (1993) presented a bar chart showing the results for three conditions. The heights of the bars increased from left to right, as did the darkness of the shading of the bars. This change in the darkness of the shading made the largest bar the most salient, a change that has been shown by previous research by Kosslyn to lead the reader to overestimate the size of the increase.

As noted previously, a comprehensive review of the work on designing effective graphs is beyond the scope of this article. Readers wishing to produce effective graphs are strongly encouraged to consult among others (a) Kosslyn (1993) for concrete recommendations for applying perceptual principles to the design of graphs, (b) Gillan *et al*. (1988) for numerous research-based guidelines and a set of rules for determining which type of graph to select and whether to use a figure or a table, (c) Wainer (1997) for an insightful analysis of many aspects of graphs and their design, and (d) Tufte (2001, 2006) for practical principles of designing good graphs and elegant examples of how to communicate quantitative information.

References

Aiken, L. S., & West, S. G. (1991) *Multiple Regression: Testing and Interpreting Interactions*. Newbury Park, CA: Sage.

American Psychological Association. (2001). Publication manual of the American Psychological Association (5th ed.). Washington, DC: Author.

Belia, S., Fidler, F., Williams, J., & Cumming, G. (2005). Researchers misunderstand confidence intervals and standard error bars. *Psychological Methods, 10*, 389-396.

Best, L. A., Smith, L. D., & Stubbs, D. A. (2001). Graphs use in psychology and other sciences. *Behavioural Processes, 54*, 155-165.

Cleveland, W. S. (1984). Graphs in scientific publications. *American Statistician, 38*, 261-269.

Cleveland W. (1993) A Model for Studying Display Methods of Statistical Graphics. *Journal of Computational and Graphical Statistics*, *2*, 323-343.

Cleveland, W. S. (1994). *The Elements of Graphing Data*. Murray Hill, NJ: AT&T Bell Laboratories.

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003) Applied *Multiple Regression/Correlation Analysis for the Behavioral Sciences* (3rd Edition). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Cumming, G., Fidler, F., Leonard, M., Kalinowski, P., Christiansen, A., Klenig, J. L., McMenamin, N., Wilson, S. (2007) Statistical reform in psychology: Is anything changing? *Psychological Science*, *18*, 230-232.

Cumming, G., & Finch, S. (2005). Inference by eye: Confidence intervals, and how to read pictures of data. *American Psychologist, 60*, 170-180.

Few, S. (2004). *Show Me the Numbers: Designing Tables and Graphs to Enlighten.* Oakland: Analytic Press.

Friendly, M., & Denis, D. (2005). The early origins and development of the scatterplot. *Journal of the History of the Behavioral Sciences*, *41*, 103-130.

Frigge, M, Hoaglin, D. C., & Iglewicz, B. (1989) *The American Statistician*, *43*, 50-54.

Galileo (1610). The Starry Messenger (Sidereus Nuncius) Venice.

Gelman, A., Pasarica, C., & Dodhia, R. (2002). Let's practice what we preach: Using graphs instead of tables. *American Statistician, 56*, 121-130.

Gelman, A., & Stern, H. (2006). The difference between "significant" and "not significant" is not itself statistically significant. *American Statistician, 60*, 328-331.

Gillan, D. J., Wickens, C. D., Hollands, J. G., Carswell, C. M. (1988) Guidelines for presenting quantitative data in HFES publications, *Human Factors*, *40,* 28-41.

Jarvenpaa, S., & Dickson, G. W. (1988) Graphics and managerial decision making: Research based guidelines. *Communications of the ACM*, *31*, 764-774.

Kosslyn, S. M. (1985). Graphics and human information processing. *Journal of the American Statistical Association, 80*, 449-512.

Kosslyn, S. M. (1993) *Elements of Graphic Design* New York: W.H. Freeman & Company

Larkin, J. H., & Simon, H. A. (1987). Why a diagram is (sometimes) worth 10,000 words. *Cognitive Science, 11*, 65-100.

Loftus, G. R. (1993). A picture is worth 1000 p-values - On the irrelevance of hypothesis-testing in the microcomputer age. *Behavior Research Methods, Instruments & Computers, 25*, 250-256.

Loftus, G. R., & Masson, M. E. J. (1994). Using Confidence Intervals in Within-Subject Designs. *Psychonomic Bulletin & Review*, 476-490.

Masson, M. E. J., & Loftus, G. R. (2003). Using confidence intervals for graphically based data interpretation. *Canadian Journal of Experimental Psychology, 57*, 203-220.

Meyer, J., Shamo, M. K., & Gopher, D. (1999) Information structure and the relative efficacy of tables and graphs. *Human Factors*, *41*, 570-587.

Sagan, C. (1977). *The Dragons of Eden: Speculations on the Evolution of Human Intelligence*. New York: Random House.

Sándor, A. & Lane, D. (2007) Graph use in psychology journals. Poster Presented at the Annual Conference of the Houston Chapter of the Human Factors and Ergonomics Society, Houston, Texas, May, 2007.

Smith, L. D., Best, L. A., Stubbs, D. A., Archibald, A. B., & Robertson-Nay, R. (2002). Constructing knowledge: The role of graphs and tables in hard and soft psychology. *American Psychologist, 57*(10), 749-761.

Sweller, J., Chandler, P., Tierney, P., & Cooper, M. (1990) Cognitive load as a factor in structuring technical material. *Journal of Experimental Psychology: General*, *119*, 176-192.

Tufte, E. R. (2001). *The Visual Display of Quantitative Information* (2nd ed.). Cheshire, CT: Graphics Press.

Tufte, E. R. (2006). *Beautiful Evidence*. Cheshire, CT: Graphics Press.

Tukey, J. W. (1974). Mathematics and the picturing of data. In*: Proceedings of The International Congress of Mathematics, 523-531*.

Tukey, J. W. (1977). *Exploratory Data Analysis*. Reading, MA: Addison-Wesley.

Tukey, J. W. (1991). The philosophy of multiple comparisons. *Statistical Science, 6*, 100-116.

Tversky, B. (1995). Cognitive origins of graphic conventions. In F. T. Marchese (Ed.), *Understanding Images* (pp. 29-53). New York: Springer-Verlag.

Wainer, H. (1996). Depicting error. *American Statistician, 50*, 101-111.

Wainer, H. (1997). *Visual Revelations*. New York: Springer-Verlag.

Wainer, H (2005). *Graphic Discovery: A Trout in the Milk and Other Visual Adventures,* Princeton University Press, Princeton, NJ.

Wilkinson, L. and the Task Force on Statistical Inference, APA Board of Scientific Affairs (1999). Statistical Methods in Psychology Journals. Guidelines and Explanations. *American Psychologist, 54*, 594-604.

Appendix

Box Plots

The first step in creating a box plot is to determine the 25th percentile, the median, and the 75th percentile. For the box plot shown in Figure 1 Appendix these values are 15, 20, and 23 respectively. A box is then drawn from the 25th to the 75th percentile with a line representing the median drawn inside the box. The mean is then drawn as a "+" sign. Next, the values of the inner and outer fences are computed. Defining a "step" as 1.5 times difference between the 75th and 25th percentiles (12 in this example), the upper inner fence is 1 step above the 75th percentile, the upper outer fence is 2 steps above the 75th percentile, the lower inner fence is 1 step below the 25th percentile and the lower outer fence (not shown) is 2 steps below the 25th percentile. The fences are drawn in Figure 1 Appendix for illustrative purposes only and do not appear in box plots.

Please insert Figure 1 Appendix about here

Lines are then drawn from the 25th percentile to the lowest value inside the fences and from the 75th percentile to the highest value inside the fences. Values between the inner and outer fences are each represented by the letter "o" whereas values outside the outer fences are represented by asterisks.

There are many variations of box plots (Frigge, Hoaglin, & Iglewicz, 1989). As stated in the text, we prefer versions that include the mean.

Although many statistics programs do not provide this option, the Statistical Analysis System (SAS) is one that does.

Stem-and-Leaf Displays

Stem-and-leaf displays can be used to display a relatively small dataset to two decimal places. Figure 2A Appendix contains a stem-and-leaf plot of the same data shown in Figure 1 Appendix. The "stems" are located on the left and range from 0 to 5 and represent the 10's place. The "leaves" are located on the right and represent the 1's place. The highest number in the data (55) is represented by a stem of 5 and a leaf of 5. The second highest number (37) is represented by a stem of 3 and a leaf of 7. The lowest number (6) is represented by a stem of 0 and a leaf of 6. Notice that there are two rows for each value of the stems: the higher is for leaves from 5-9 and the lower is for leaves from 0-4. The place represented by the stems (10's, 100's, etc.), the number of stem repetitions, and the sequence of stems can differ as a function of the details of the data.

Please insert Figure 2 Appendix about here

Figure 2B Appendix compares two distributions. The distribution on the right is the same as the distribution in Figure 2A Appendix. This type of graph is called a "back-to-back" stem-and-leaf display.

Back-to-Back Histograms

Creating a back-to-back histogram is a good way to portray differences between the distributions of two relatively large data sets. As in the example shown in Figure 3 Appendix, it is usually desirable to display the histograms vertically rather than horizontally.

Please insert Figure 3 Appendix about here

Footnotes

1. The scale of coefficients for a component of trend does not affect its significance test. To maximize interpretability, we scaled the coefficients so that the pooled standard deviation across cells is 1.0. The coefficients to accomplish this are: -0.15, -0.09, -0.03, 0.03, 0.09, 0.15

2. Ironically, an example of an error occurring because of the artificial separation of text and graphics was made by a reader of a previous version of this article. This reader apparently failed to relate the graph in Figure 8 to the text and thought we were presenting this graph as a positive example rather than as an example of what not to do. In the present version, we followed or own advice about integrating text and graphics in constructing Figure 8.

Figure Captions

1. A graph typical of those that appear in psychology journals. Note that the data are fictitious.

2. The data graphed in Figure 1 portrayed by box plots. Considerably more distributional information is revealed in approximately the same amount of space. Specifically, the medians are shown by the horizontal lines inside the boxes, the 25th and 75th percentiles are shown as the bottoms and tops of the boxes, and the minimum and maximum values are shown as the small horizontal lines below and above the boxes (if there were outliers they would be shown individually). The ranges are therefore the differences between the lower and upper horizontal lines and the interquartile ranges are the differences between the lower and upper portions of the boxes.

3. Parallel box plots emphasizing differences among means by making the representation of the means more prominent and the other elements of the box plot less prominent. As a result, the pattern of differences among means is easier to perceive.

4. Parallel box plots that emphasize differences in variability by making the ranges and interquartile ranges more prominent than the means and medians.

5. A line graph showing the standard deviations for each combination of condition and trial. This method of combining trend and distributional informational information does not work well. Note that the data are fictitious.

6. A graph showing the trend information in the lower portion and box plots of the linear components of trend for the three conditions in the upper portion. The trend coefficients applied to the raw scores were scaled so that the mean within-group standard deviation is 1.0. Note that the data are fictitious.

7. A typical bar graph showing standard error bars. Note that the data are fictitious.

8. A graph showing significant differences among pairs of means with asterisks.

9. The type of graph recommended by Cumming and Finch (2005) to show confidence intervals in a two-group between-subjects design. Confidence intervals for the mean of Condition A, the mean of Condition B, and the difference between means are shown. The axis on the left represents raw scores and the axis on the right represents differences from the mean of Condition A. Note that the data are fictitious.

10. This graph shows box plots of the six combinations of condition and group. The upper-left hand portion of the figure shows the analysis of variance results. The box in the upper right shows the *p* values for the differences among the three simple effects of Group. Cohen's d for the difference between Control and Experimental groups and the 95% confidence interval on the difference between means are presented below each condition. Note that the data are fictitious.

11. A graph showing distributional, trend, and inferential information. The box plots show the distribution of the linear components of trends for the three conditions; the table shows inferential statistics for the linear component computed by applying the linear trend coefficients to the scores for each subject. In the pairwise comparisons, t_s stands for the studentized t. Note that the data are fictitious.

12. A graph showing a scatterplot of variables X and Y separately for Conditions A and B. Precise information such as the values of correlations and slopes as well as inferential information is presented in text. The data themselves and the least-squares regression lines are shown graphically.

13. The regression of DV on X1 for three levels of X2. This graph shows the shape of the linear x linear interaction clearly but does not contain any distributional information. Note that the data are fictitious.

14. The regression of DV on the part of X1 independent of both X2 and X3 (X1.23) as a function of the portion of X2 independent of X3 (X2.3). The upper-left portion of the figure contains a significance test of the linear x linear interaction. The full regression equation is shown to the right. The scatterplot for each quartile of X2.3 includes the regression line, the slope of the line, a 95% confidence interval on the slope of the line and the probability value. Note that the data are fictitious.

15. The regression of the dependent variable (DV) on the covariate separately for the experimental and the control conditions. The inferential statistics in the lower right show the R^2 for the model in which the slopes of the two lines are allowed to differ and the R^2 assuming a common slope. The common slope and a significance test for the difference between the slopes are also shown. Note that the data are fictitious.

16. Box plots and inferential statistics relevant to the comparison of the experimental and the control conditions. The box plots were created based on residual scores computed by removing variability linearly related to the covariate.

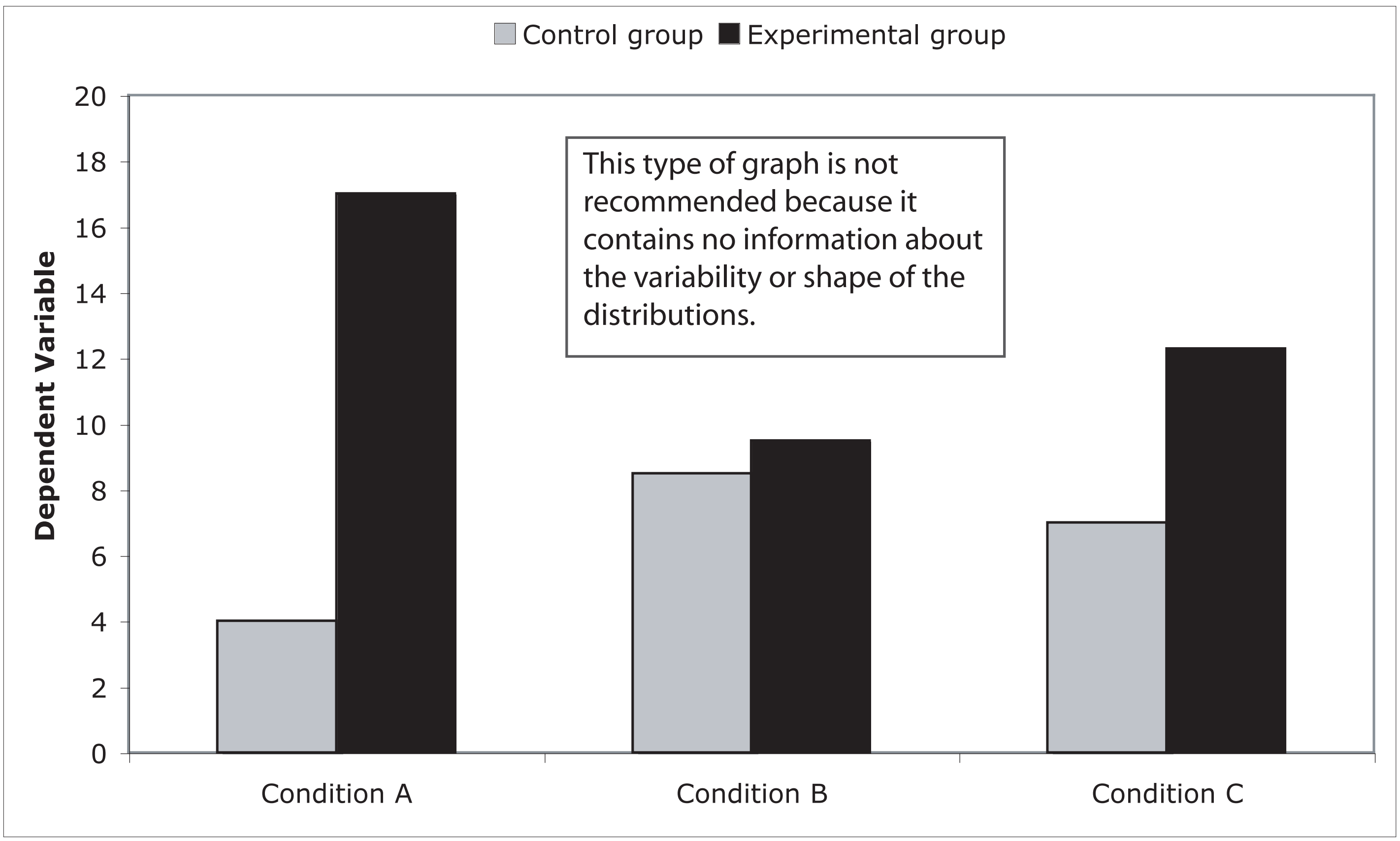
17. Consistent and inconsistent assignment of symbols to conditions. For the consistent assignment, the level “young” is represented by circles whereas the level “old” is represented by squares. The level “control” is represented

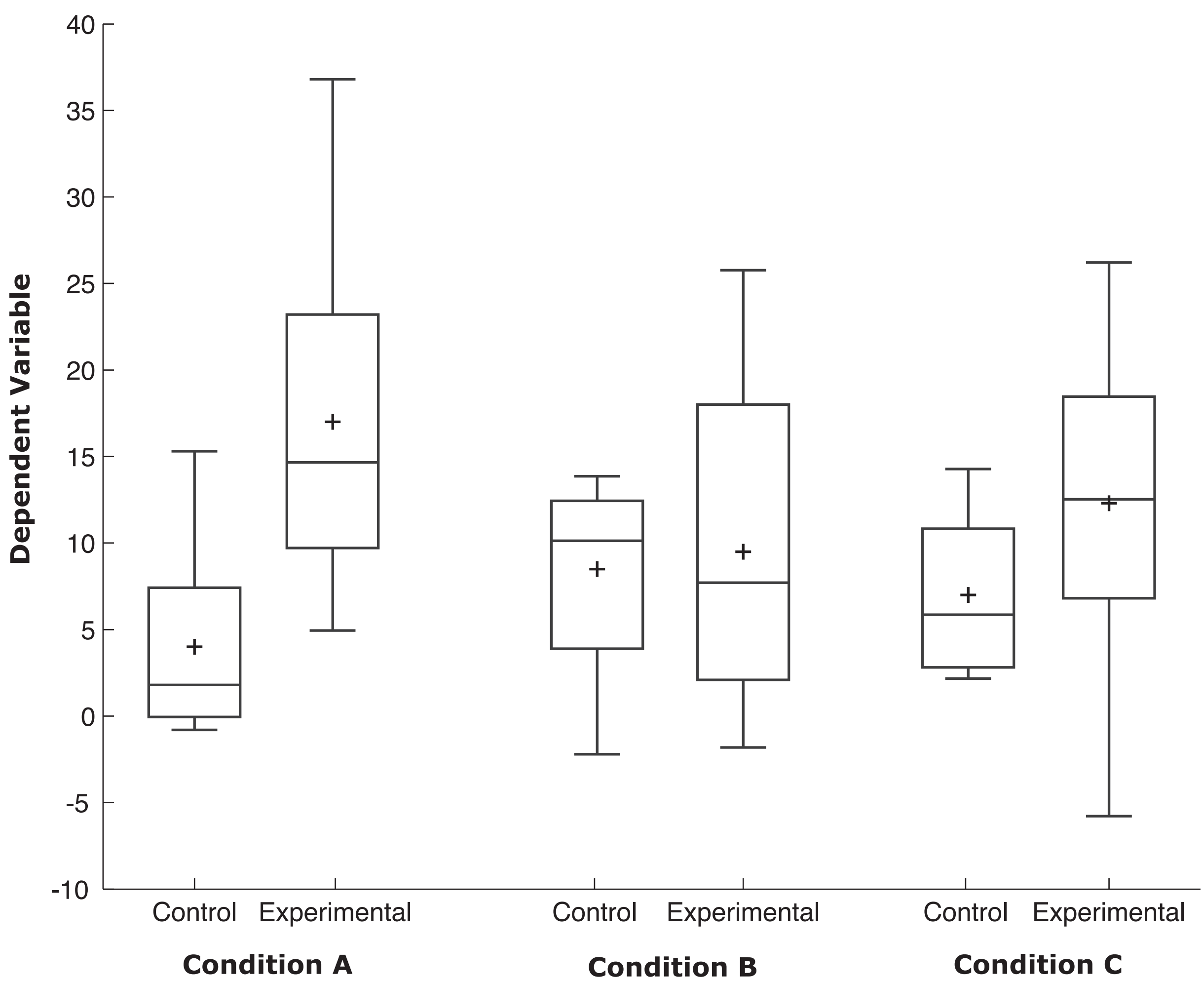
by a filled symbol whereas the level “experimental” is represented by an unfilled symbol.

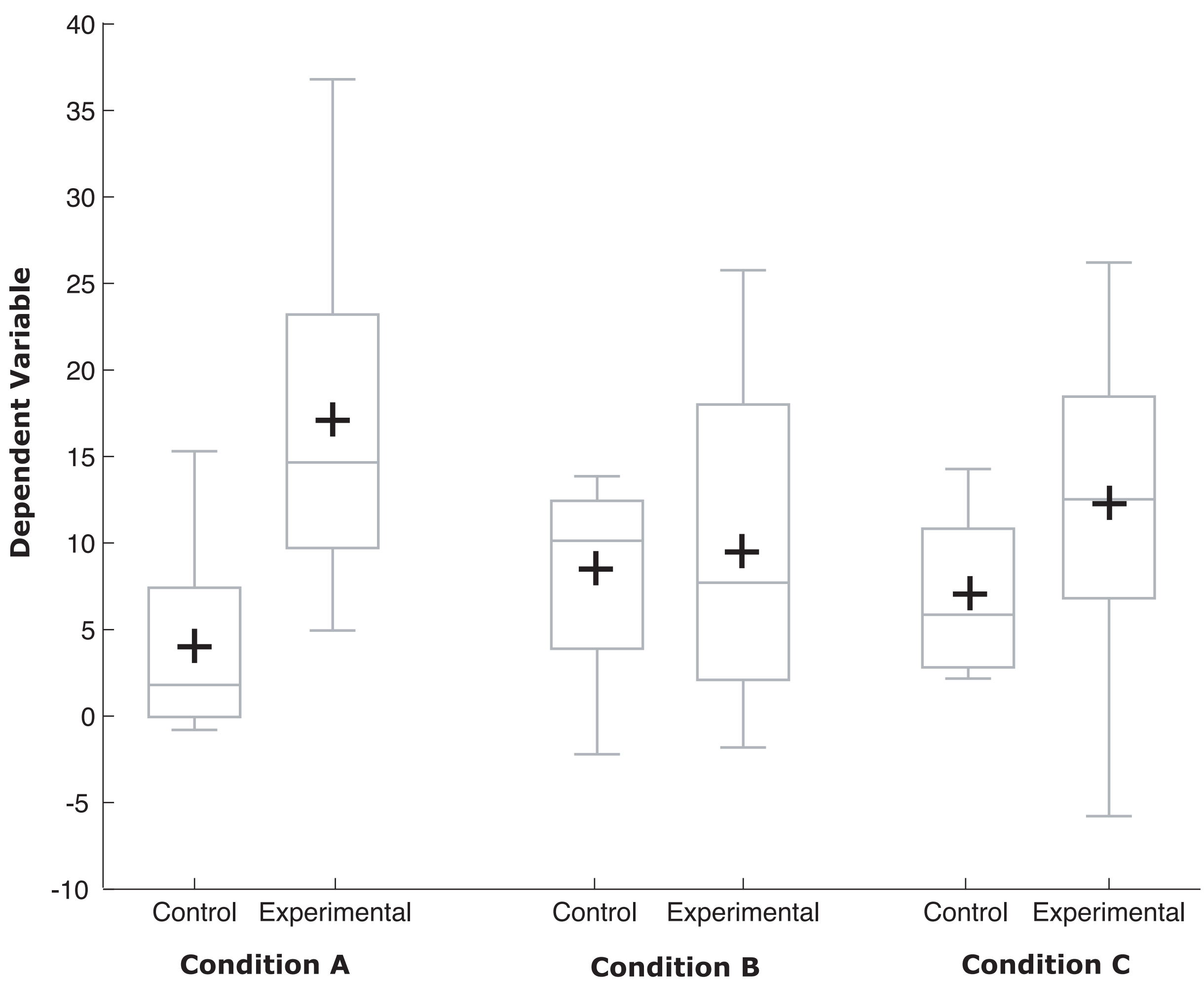
1 Appendix. An example of a box plot. The fences are included for illustrative purposes only and should not be shown in the final version of a box plot.

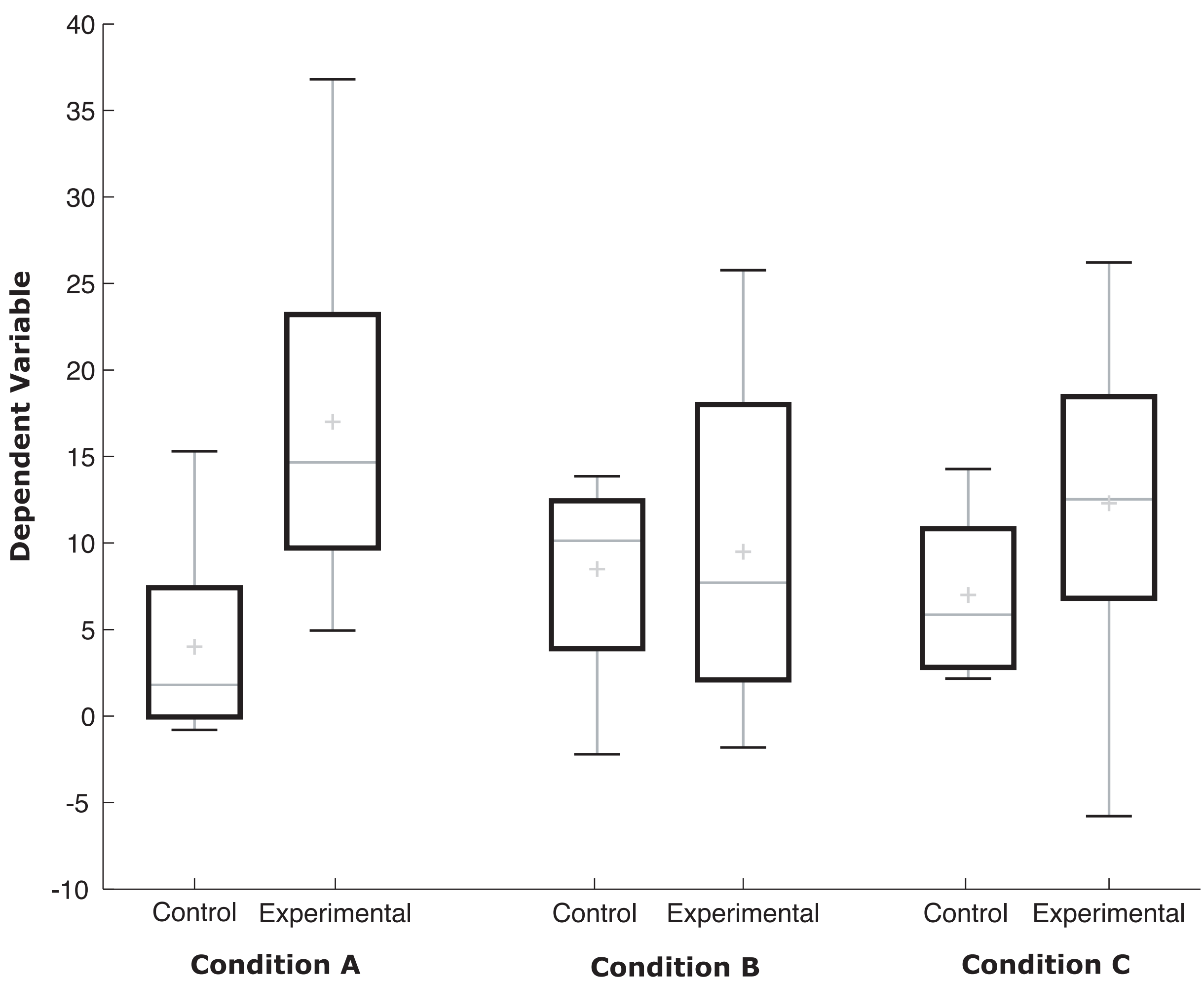
2 Appendix. Examples of stem-and-leaf displays. Graph A shows the distribution of one variable. The data values are equal to 10 times the stem plus the leaf. Graph B shows back-to-back stem-and-leaf displays.

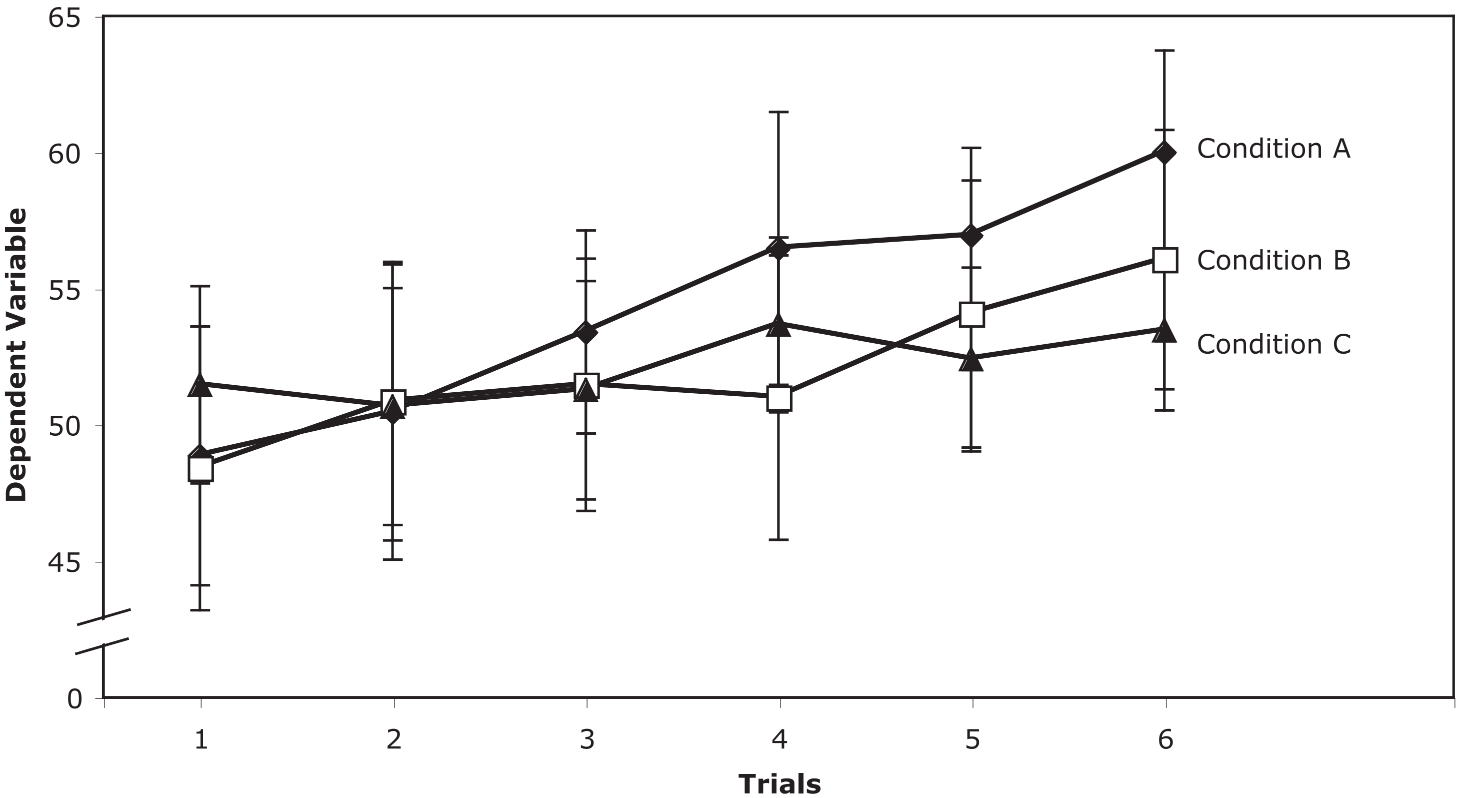
3 Appendix. An example of back-to-back histograms.

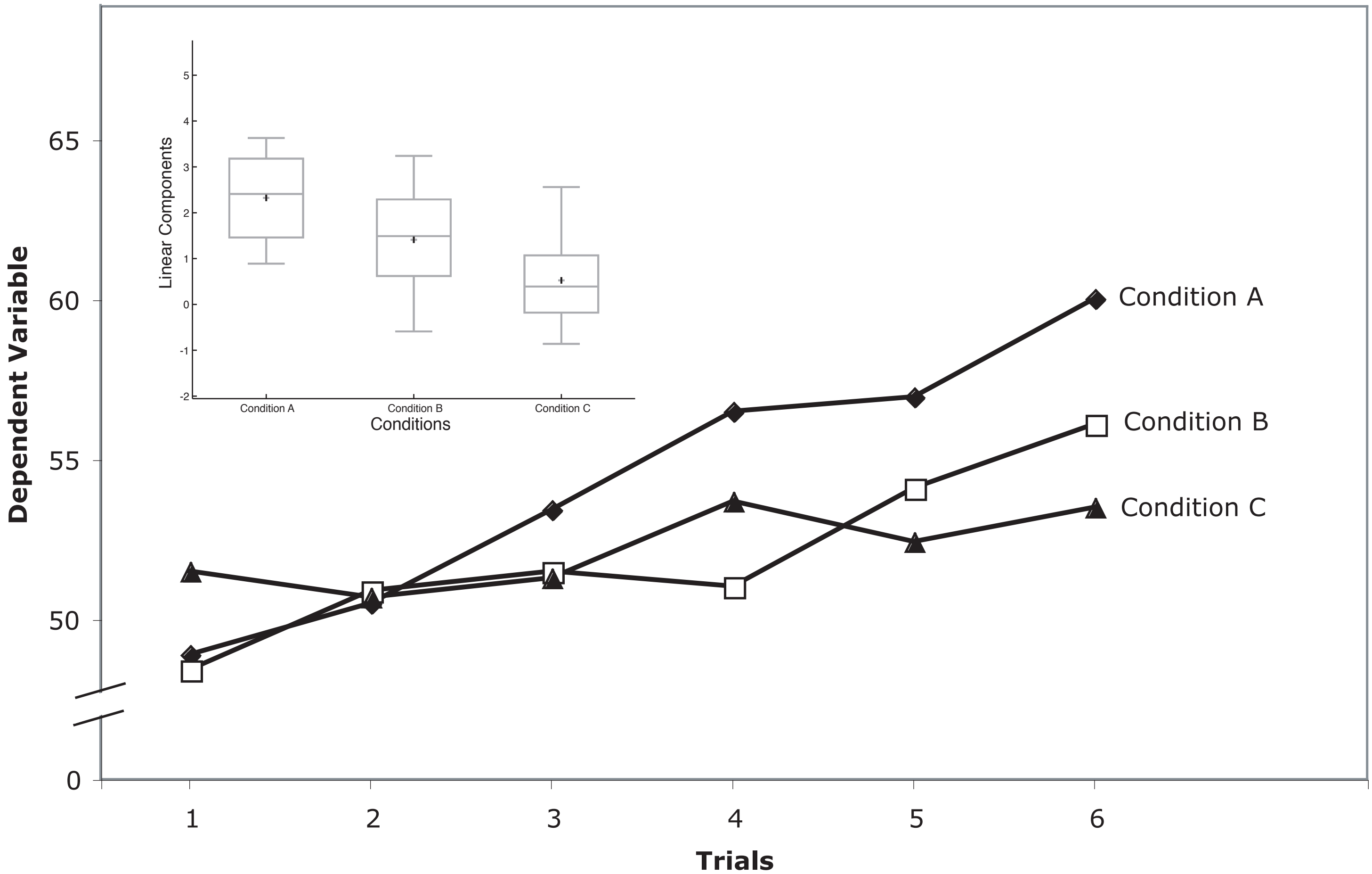

Control group
Experimental group
Dependent Variable
0
2
4
6
8
10
12
14
16
18
20
This type of graph is not recommended because it contains no information about the variability or shape of the distributions.
Condition A
Condition B
Condition C


Dependent Variable
40
35
30
25
20
15
10
5
0
-5
-10
Control
Experimental
Control
Experimental
Control
Experimental
Condition A
Condition B
Condition C

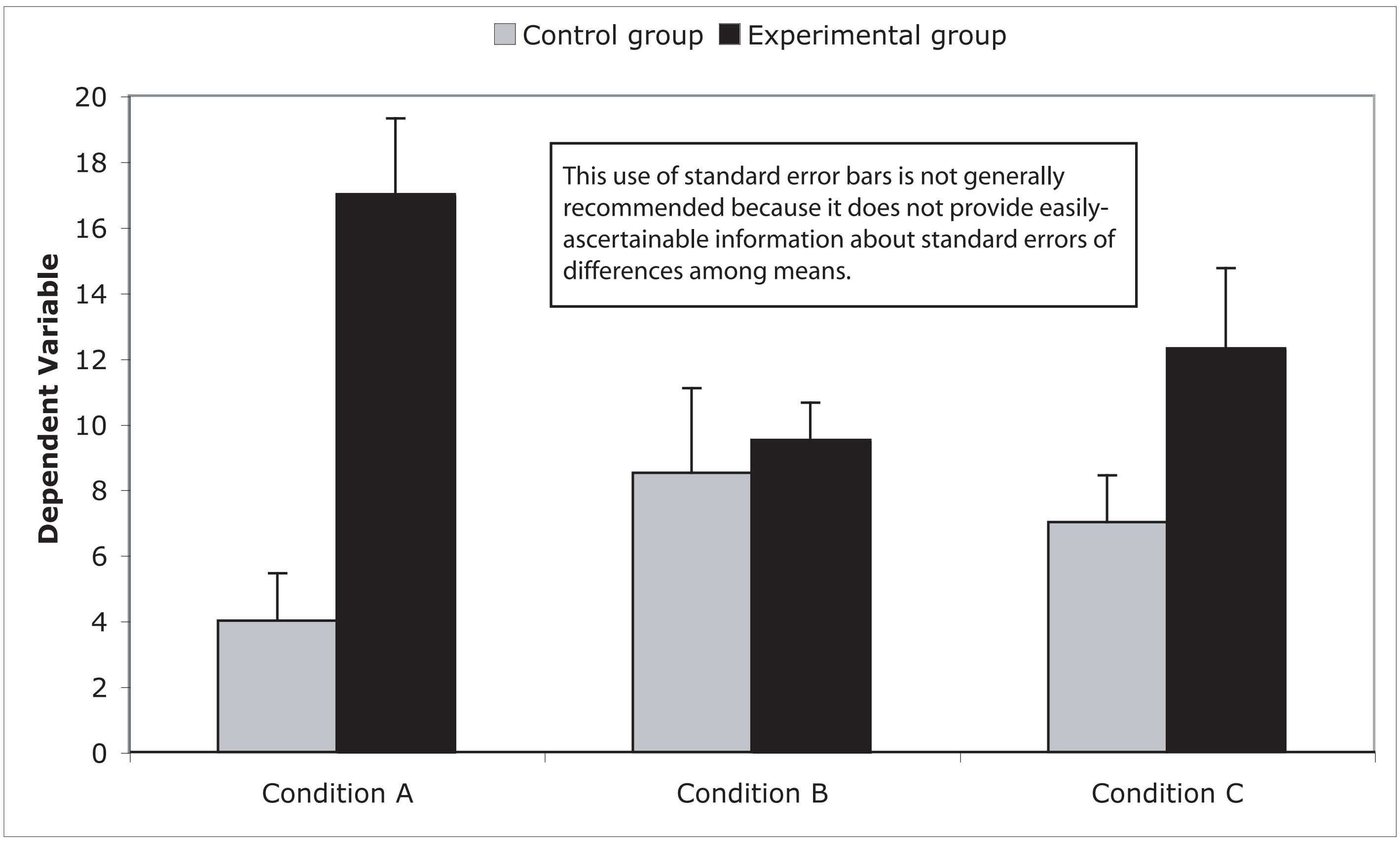

Dependent Variable
40
35
30
25
20
15
10
5
0
-5
-10
Control
Experimental
Control
Experimental
Control
Experimental
Condition A
Condition B
Condition C

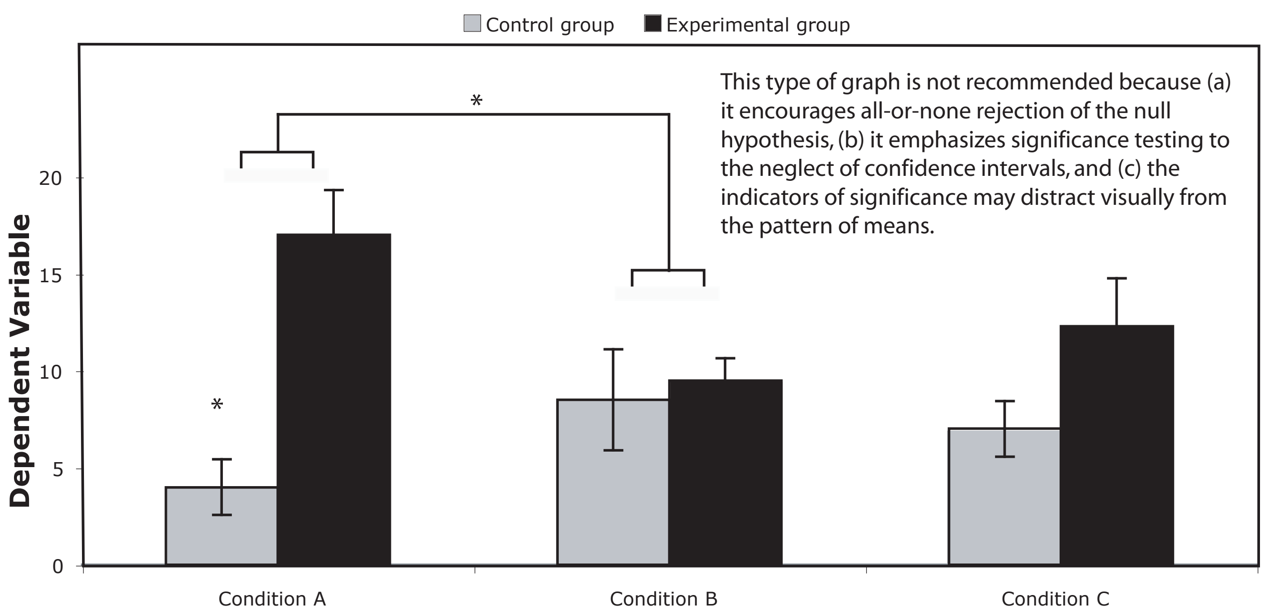

Dependent Variable
40
35
30
25
20
15
10
5
0
-5
-10
Control
Experimental
Control
Experimental
Control
Experimental
Condition A
Condition B
Condition C

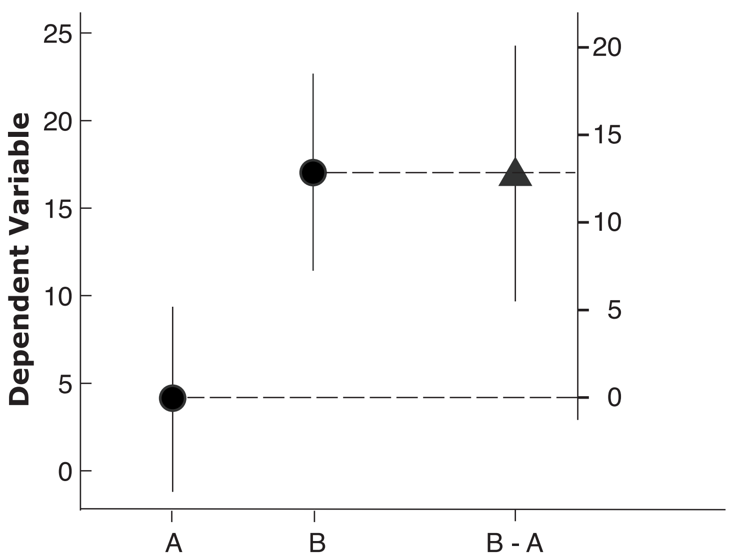

Dependent Variable
65
60
55
50
45
0
1
2
3
4
5
6
Trials
Condition A
Condition B
Condition C



Dependent Variable
65
60
55
50
0
1
2
3
4
5
6
Trials
Condition A
Condition B
Condition C
Linear Components
5
4
3
2
1
0
-1
-2
Condition A
Condition B
Condition C
Conditions


Control group
Experimental group
Dependent Variable
20
18
16
14
12
10
8
6
4
2
0
This use of standard error bars is not generally recommended because it does not provide easily-ascertainable information about standard errors of differences among means.
Condition A
Condition B
Condition C


Control group
Experimental group
This type of graph is not recommended because (a) it encourages all-or-none rejection of the null hypothesis, (b) it emphasizes significance testing to the neglect of confidence intervals, and (c) the indicators of significance may distract visually from the pattern of means.
*
*
Dependent Variable
20
15
10
5
0
Condition A
Condition B
Condition C



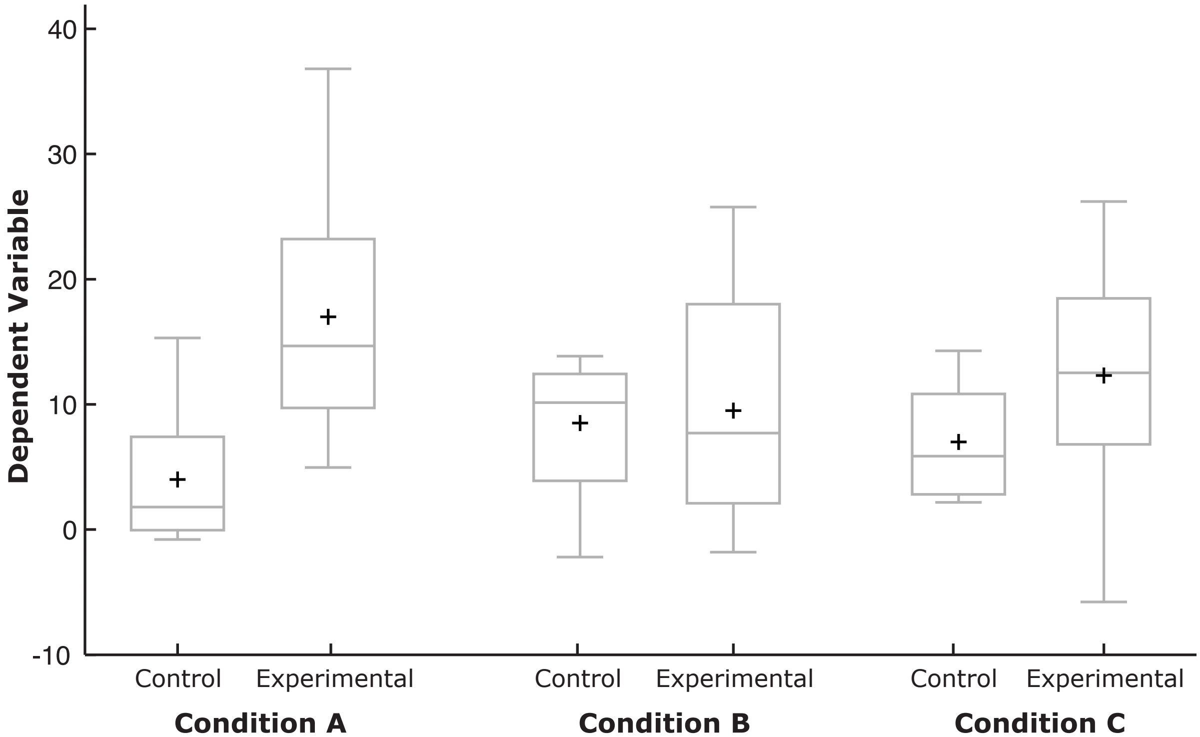
Dependent Variable
25
20
15
10
5
0
20
15
10
5
0
A
B
B - A

Analysis of Variance

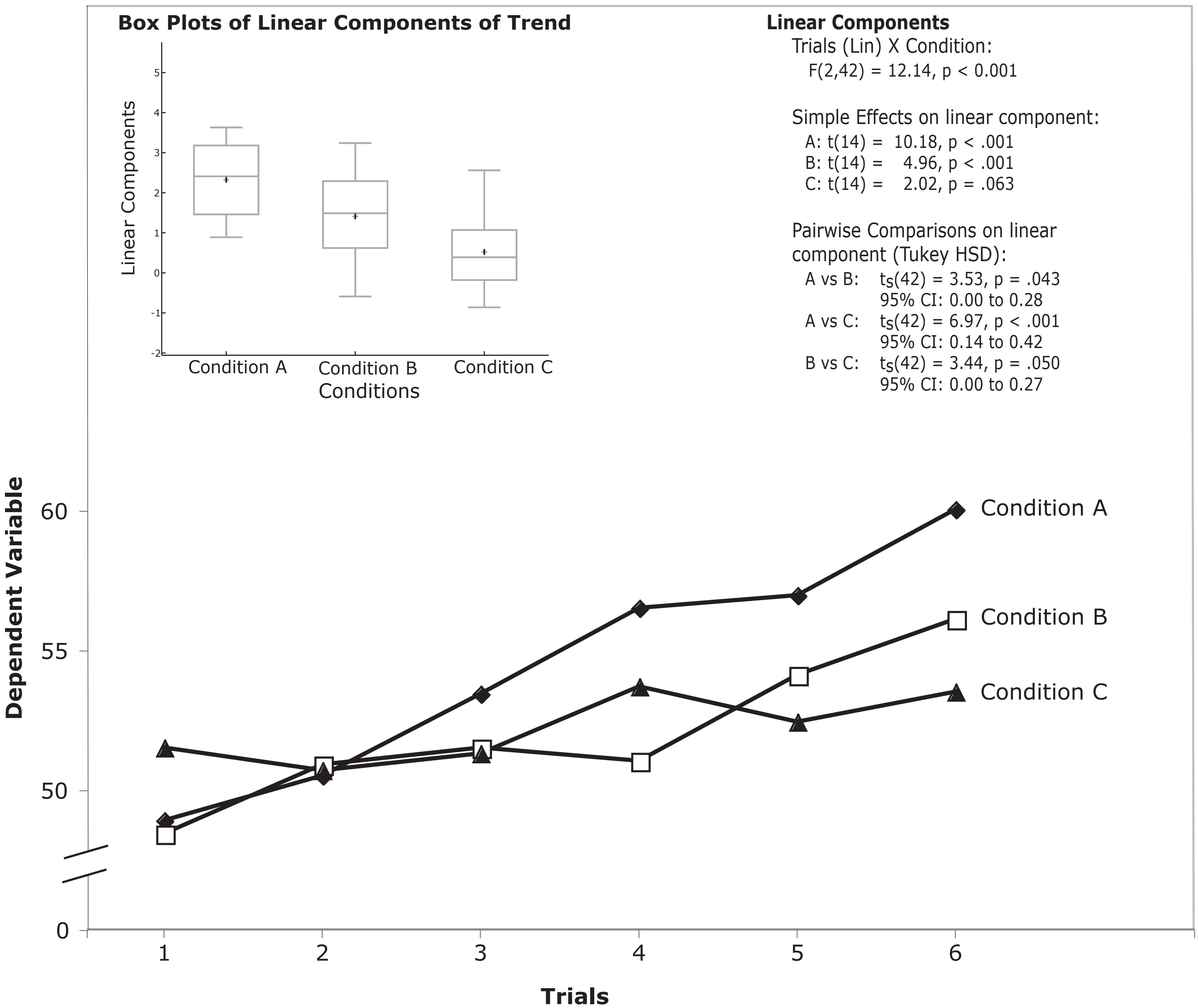
Group:	$F(2,66) = 15.78$, $p < 0.001$
Condition:	$F(2,66) = 0.29$, $p = 0.751$
Group x Condition:	$F(2,66) = 4.70$, $p = 0.012$
Pooled SD:	6.87

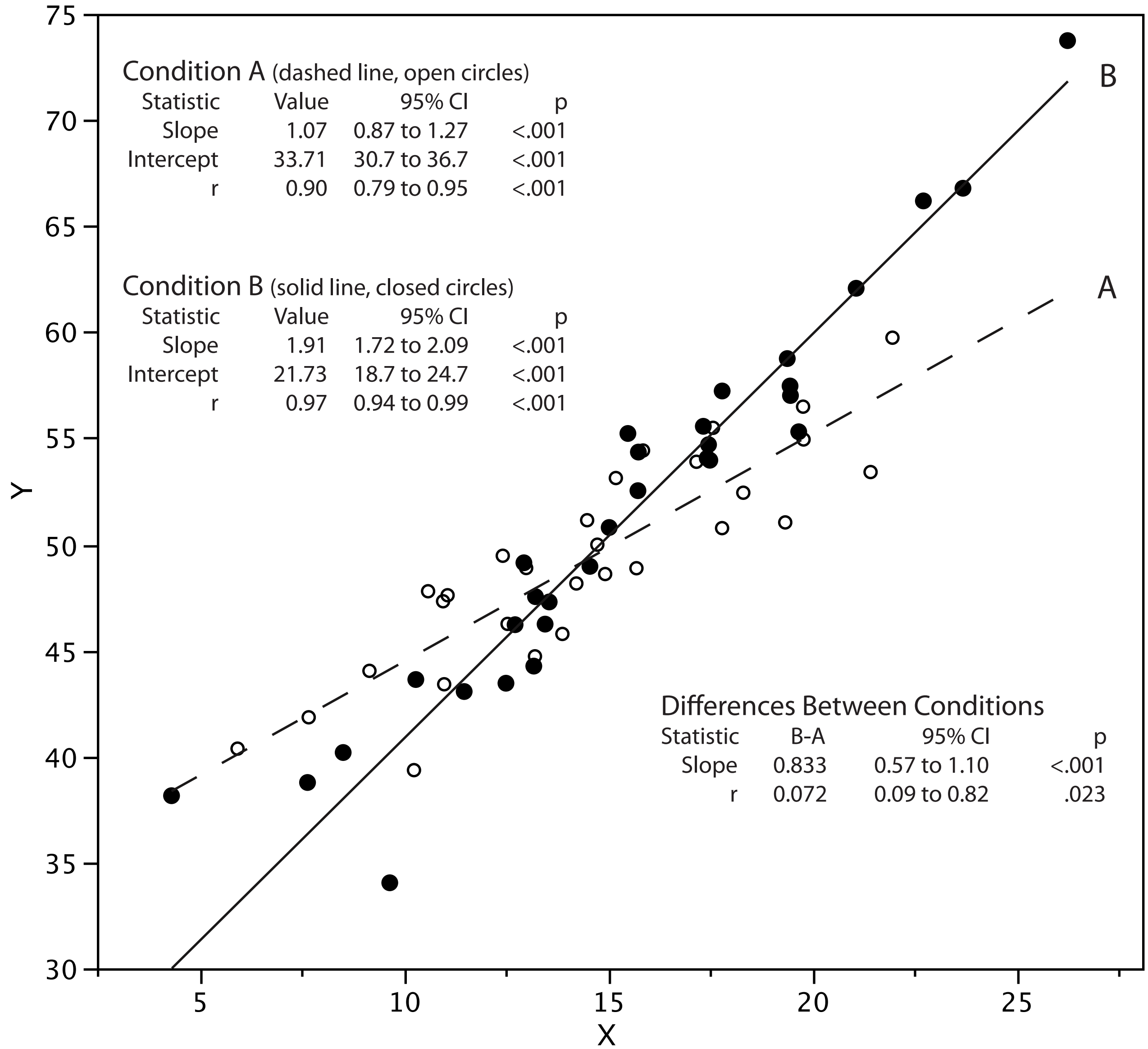
Differences Among Simple Effects (p's)

	A	B	C
A	x		
B	0.004	x	
C	0.057	0.282	x

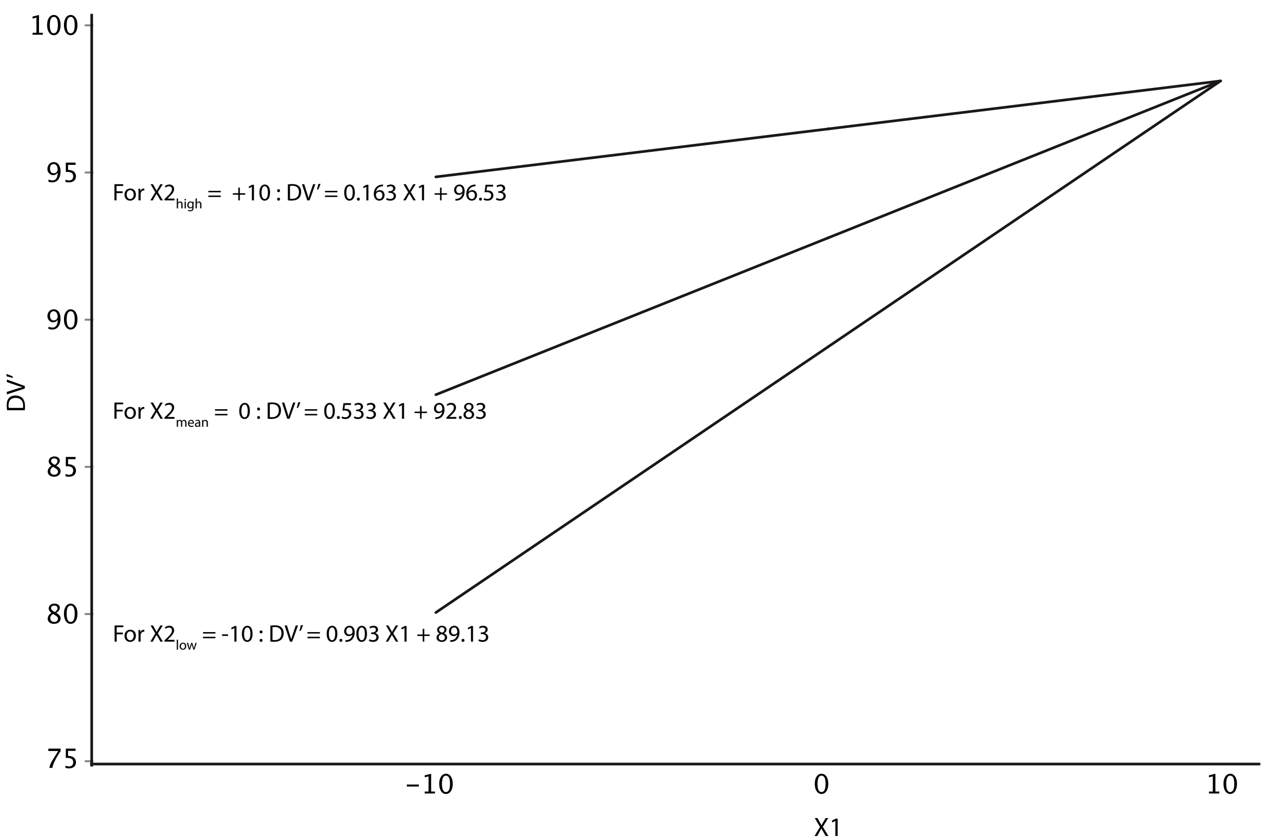


Condition A	Condition B	Condition C
d′ = 1.89, p < 0.001	d′ = 0.15, p = 0.723	d′ = 0.77, p = 0.063
95% CI: 6.56 to 19.43	95% CI: -4.60 to 6.60	95% CI: -0.30 to 10.90


Box Plots of Linear Components of Trend
Linear Components
5
4
3
2
1
0
-1
-2
Condition A
Condition B
Condition C
Conditions
Linear Components
Trials (Lin) X Condition:
$F(2,42) = 12.14$, $p < 0.001$
Simple Effects on linear component:
A: $t(14) = 10.18$, $p < .001$
B: $t(14) = 4.96$, $p < .001$
C: $t(14) = 2.02$, $p = .063$
Pairwise Comparisons on linear component (Tukey HSD):
A vs B: $t_S(42) = 3.53$, $p = .043$
95% CI: 0.00 to 0.28
A vs C: $t_S(42) = 6.97$, $p < .001$
95% CI: 0.14 to 0.42
B vs C: $t_S(42) = 3.44$, $p = .050$
95% CI: 0.00 to 0.27
Dependent Variable
60
55
50
0
Condition A
Condition B
Condition C
1
2
3
4
5
6
Trials



Condition A (dashed line, open circles)
Statistic Value 95% CI p
Slope 1.07 0.87 to 1.27 <.001
Intercept 33.71 30.7 to 36.7 <.001
r 0.90 0.79 to 0.95 <.001
Condition B (solid line, closed circles)
Statistic Value 95% CI p
Slope 1.91 1.72 to 2.09 <.001
Intercept 21.73 18.7 to 24.7 <.001
r 0.97 0.94 to 0.99 <.001
Differences Between Conditions
Statistic B-A 95% CI p
Slope 0.833 0.57 to 1.10 <.001
r 0.072 0.09 to 0.82 .023
A
B
Y
X
75
70
65
60
55
50
45
40
35
30
5
10
15
20
25



100
95
90
85
80
75
DV′
For X2high = +10 : DV′ = 0.163 X1 + 96.53
For X2mean = 0 : DV′ = 0.533 X1 + 92.83
For X2low = -10 : DV′ = 0.903 X1 + 89.13
−10
0
10
X1

X1 (linear) x X2 (linear) Interaction

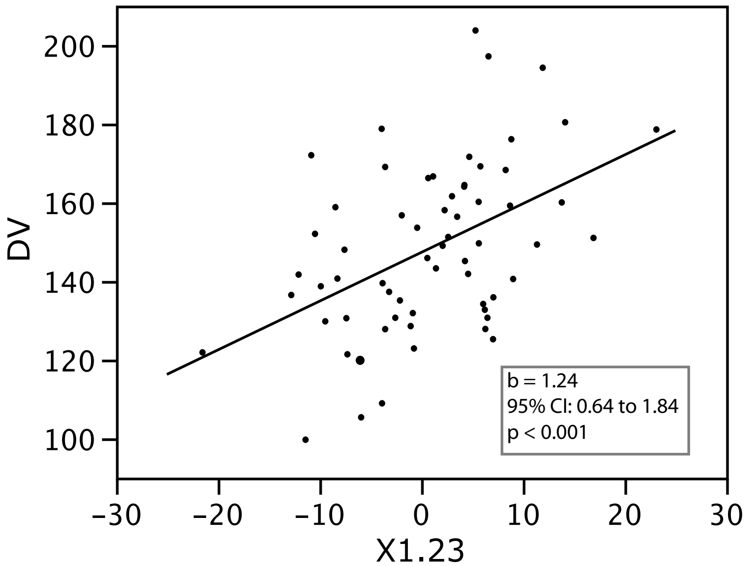
b = -0.037
95% CI: -0.562 to -0.018
Incremental R^2 = .039
$p < .001$

Regression Equation

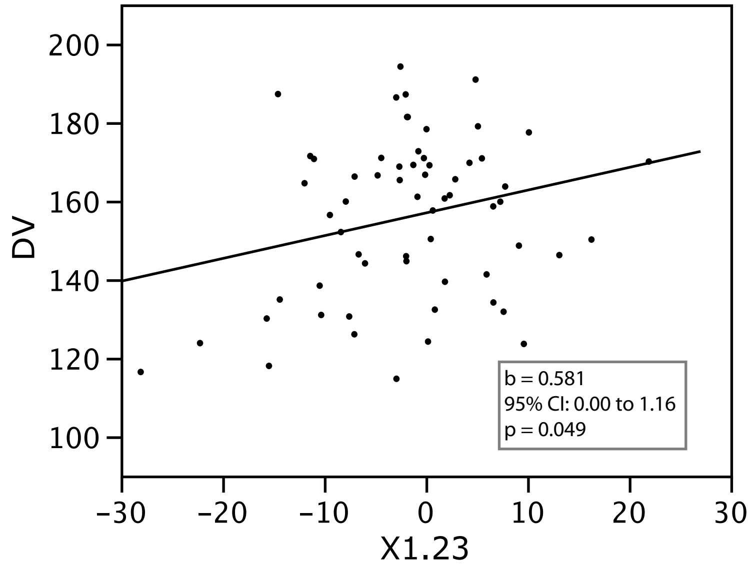
Term	Value
b1	0.533
b2	0.370
b12	-0.037
b3	0.427
A	92.826

Note: X1 and X2 were centered before interaction predictor was created.

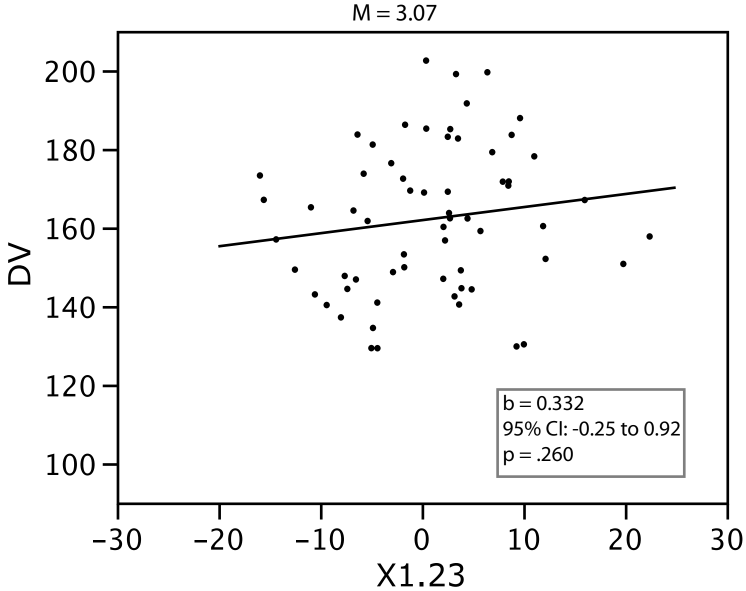
X2.3 Quartile1
M = -12.00



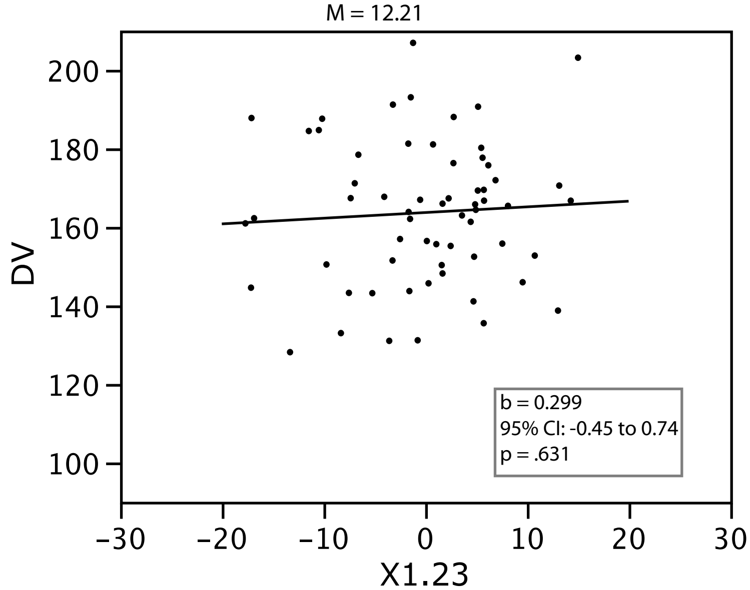
X2.3 Quartile 2
M = -3.14

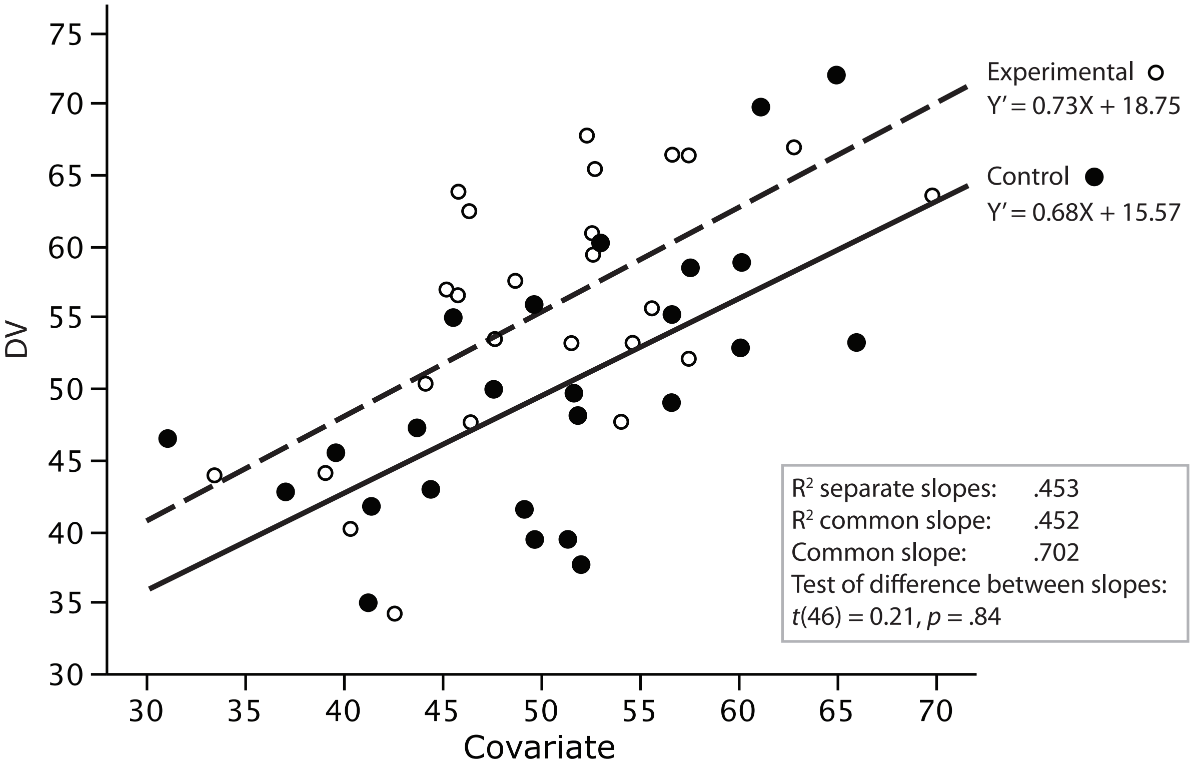


X2.3 Quartile 3
M = 3.07



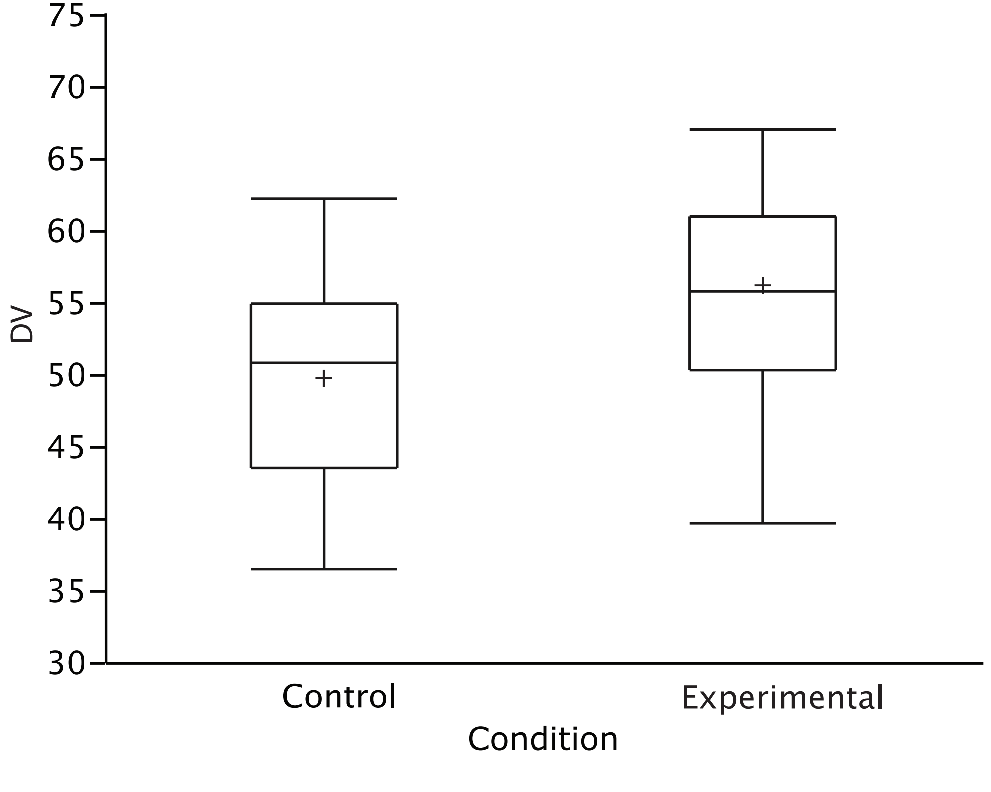
X2.3 Quartile 4
M = 12.21





Experimental
Y' = 0.73X + 18.75
Control
Y' = 0.68X + 15.57
R² separate slopes: .453
R² common slope: .452
Common slope: .702
Test of difference between slopes:
t(46) = 0.21, p = .84
DV
Covariate
75
70
65
60
55
50
45
40
35
30
30
35
40
45
50
55
60
65
70

Covariate:	$F(1,47) = 31.17$, $p < .001$
Condition:	$F(1,47) = 8.23$, $p = .006$
Pooled SD:	7.24
Mean Difference:	5.88, 95% CI: 1.75 to 10.00
d:	0.81

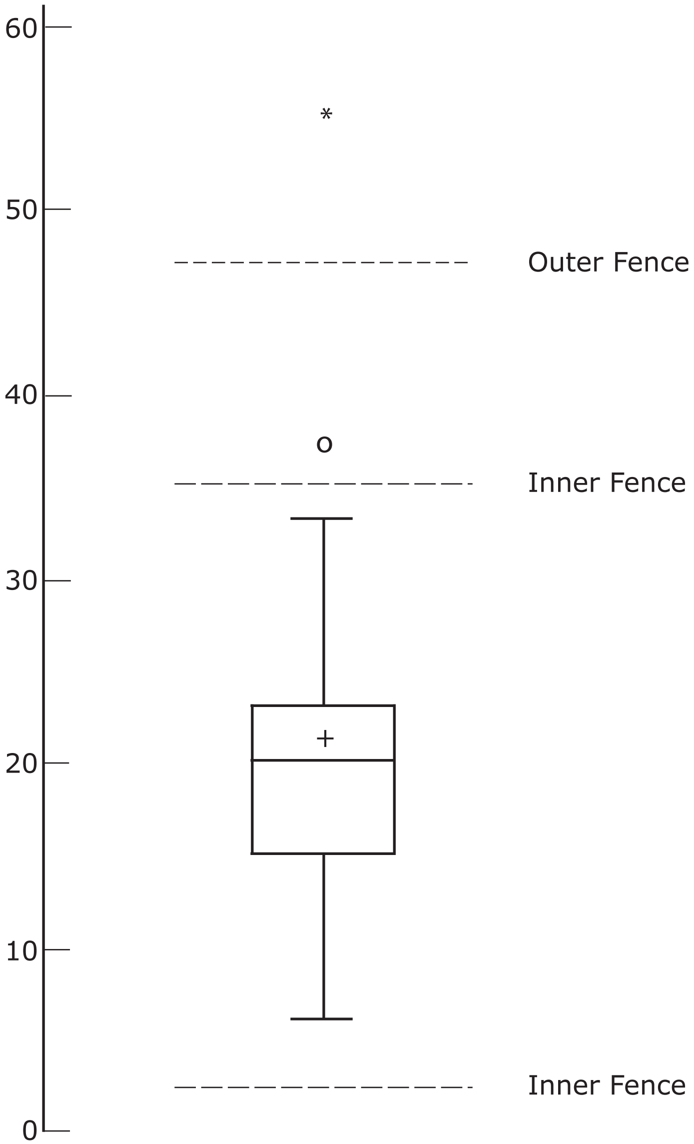


Consistent Assignment

● Young, Control
○ Young, Experimental
■ Old, Control
□ Old, Experimental

Inconsistent Assignment

● Young, Control
■ Young, Experimental
□ Old, Control
○ Old, Experimental

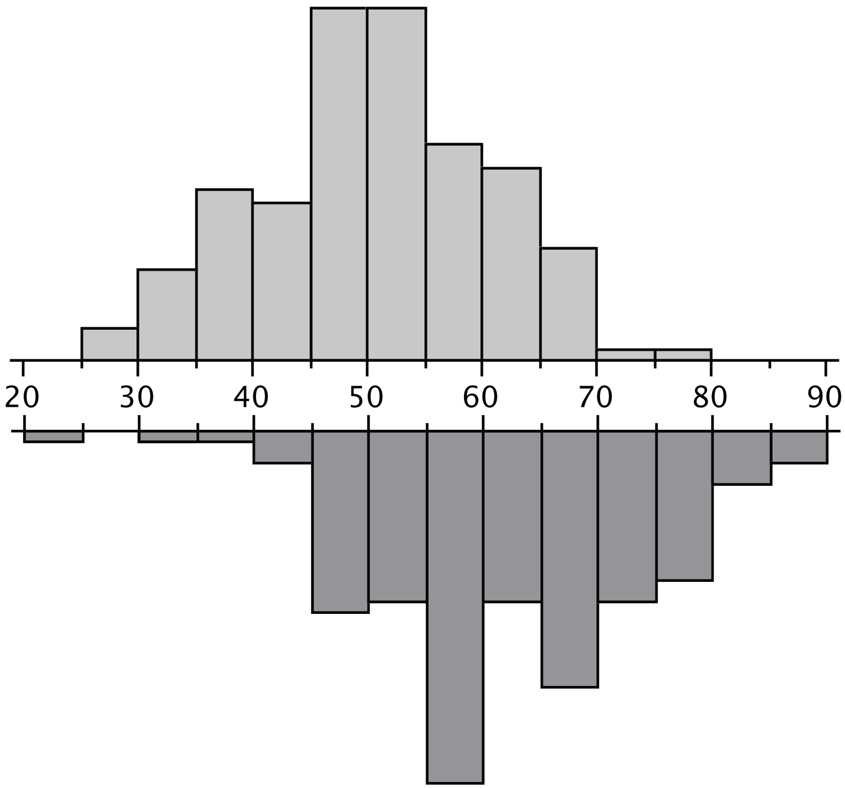

60
50
40
30
20
10
0
*
o
+
Outer Fence
Inner Fence
Inner Fence

A

```
5|5
5|
4|
4|
3|7
3|23
2|889
2|001112223
1|56888899
1|22444
0|69
```

B

```
        |5|5
        |5|
       8|4|
       2|4|
        |3|7
     012|3|23
  556678|2|889
01233344|2|001112223
   55789|1|56888899
     344|1|22444
       5|0|69
```


20
30
40
50
60
70
80
90