Time limit: 50 minutes.
Instructions: For this test, you work in teams of eight to solve 15 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Submit a single answer sheet for grading. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. How many squares are there in the xy-plane such that both coordinates of each vertex are integers between 0 and 100 inclusive, and the sides are parallel to the axes?

2. According to the Constitution of the Kingdom of Nepal, the shape of the flag is constructed as follows:

   Draw a line AB of the required length from left to right. From A draw a line AC perpendicular to AB making AC equal to AB plus one third AB. From AC mark off D making line AD equal to line AB. Join BD. From BD mark off E making BE equal to AB. Touching E draw a line FG, starting from the point F on line AC, parallel to AB to the right hand-side. Mark off FG equal to AB. Join CG.

   If the length of AB is 1 unit, what is the area of the flag?

3. You have 17 apples and 7 friends, and you want to distribute apples to your friends. The only requirement is that Steven, one of your friends, does not receive more than half of the apples. Given that apples are indistinguishable and friends are distinguishable, compute the number of ways the apples can be distributed.

4. At $t = 0$ Tal starts walking in a line from the origin. He continues to walk forever at a rate of $1\frac{m}{s}$ and lives happily ever after. But Michael (who is Tal's biggest fan) can't bear to say goodbye. At $t = 10s$ he starts running on Tal's path at a rate of n such that $n > 1\frac{m}{s}$. Michael runs to Tal, gives him a high-five, runs back to the origin, and repeats the process forever. Assuming that the high-fives occur at time $t_0, t_1, t_2...$, compute the limiting value of $\frac{t_z}{t_{z-1}}$ as $z \to \infty$.

5. In a classroom, there are 47 students in 6 rows and 8 columns. Every student's position is expressed by (i, j). After moving, the position changes to (m, n). Define the change of every student as $(i - m) + (j - n)$. Find the maximum of the sum of changes of all students.

6. Consider the following family of line segments on the coordinate plane. We take $(0, \frac{\pi}{2} - a)$ and $(a, 0)$ to be the endpoints of any line segment in the set, for any $0 \le a \le \frac{\pi}{2}$. Let A be the union of all of these line segments. Compute the area of A.

7. Compute the smallest $n > 2015$ such that $6^n + 8^n$ is divisible by 7.

8. Find the radius of the largest circle that lies above the x-axis and below the parabola $y = 2-x^2$.

9. Let C_1 be the circle in the complex plane with radius 1 centered at 0. Let C_2 be the circle in the complex plane with radius 2 centered at $4 - 2i$. Let C_3 be the circle in the complex plane with radius 4 centered at $3 + 8i$.

   Let S be the set of points which are of the form $\frac{k_1+k_2+k_3}{3}$ where $k_1 \in C_1, k_2 \in C_2, k_3 \in C_3$. What is the area of S? (Note: a circle or radius r only contains the points at distance r from the center and does not include the points inside the circle)

10. 3 points are independently chosen at random on a circle. What is the probability that they form an acute triangle?

11. We say that a number is *ascending* if its digits are, from left-to-right, in nondecreasing order. We say that a number is *descending* if the digits are, from left-to-right, in nonincreasing order. Let a_n be the number of n-digit positive integers which are ascending, and b_n be the number of n-digit positive integers which are descending. Compute the ordered pair (x, y) such that $\lim_{n\to\infty} \frac{b_n}{a_n} - xn - y = 0$.

12. Let $f(x)$ be a function so that $f(f(x)) = \frac{2x}{1-x^2}$, and $f(x)$ is continuous at all but two points. Compute $f(\sqrt{3})$.

13. Compute:
$$\sum_{k=1,k\neq m}^{\infty} \frac{1}{(k+m)(k-m)}.$$

14. Let $\{x\}$ denote the fractional part of x, the unique real $0 \leq \{x\} < 1$ such that $x - \{x\}$ becomes integer. For the function $f_{a,b}(x) = \{x+a\} + 2\{x+b\}$, let its range be $[m_{a,b}, M_{a,b})$. Find the minimum of $M_{a,b}$ as a and b ranges along all reals.

15. An ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is tangent to each of the circles $(x-1)^2 + y^2 = 1$ and $(x+1)^2 + y^2 = 1$ at two points. Find the ordered pair (a, b) that minimizes the area of the ellipse.