Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Form a triangle ABC with side lengths $AB = 12$, $AC = 8$, and $BC = 15$. Let the altitude from A to BC intersect BC at D and let AE be the angle bisector of $\angle BAC$, where E is on BC. Compute the length of DE.

2. Consider a unit cube and a plane that slices through it. The plane passes through the mid points of two adjacent edges on the top face, two on the bottom face, and the center of the cube. Compute the area of the cross section.

3. Compute the area of the largest square that can be inscribed in a unit cube. You may assume that the square's vertces lie on the edges of the cube.

4. Inside a circle of radius 1 are three circles of equal radius such that each of them is tangent to the other two and to the large circle. Determine the radius of one of the smaller circles.

5. When circles of the same radius are packed into the plane with maximum density they form a regular lattice. Compute the packing density of this arrangement, that is, the fraction of area covered by circles.

6. Consider a unit square $ABCD$. Let E be the midpoint of BC and F the intersection of AC and DE. Compute the area of triangle ADF.

7. Consider a circular sector of unit radius and angle $\arcsin\left(\frac{1}{3}\right)$. Let S be a square inscribed in the sector such that the axis of symmetry of the sector passes through the center of S, is parallel to two of the sides of S, and all four vertices of S are on the boundary of the sector. What is the area of S?

8. Natasha walks along a closed convex polygonal curve of length 2016. She carries a paintbrush of length 1 and walking all the way around paints all the area as far as she can reach on the outside of the curve. What is that area?

9. Four spheres of radius 1 are mutually tangent. What is the radius of the smallest sphere containing them?

10. Consider a regular pentagon and connect each vertex to the pair of vertices farthest from it by line segments. The line segments intersect at 5 points to form another smaller pentagon. If the large pentagon has side length 1, compute the area of the smaller pentagon. Express your answer without trigonometric functions.