

Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

No calculators.

1. a_1, a_2, a_3, a_4, a_5 is an arithmetic sequence of real numbers whose terms sum to 40. Exactly one of these terms is uniquely determined by this information. What is its value?
2. Given complex numbers $z_1 = 3 + 4i, z_2 = 5i, z_3 = 3 - 4i$, compute the positive real number z such that the expression $\left(\frac{z_1 - z_2}{z_1 - z}\right) \cdot \left(\frac{z_3 - z}{z_3 - z_2}\right)$ is real.
3. Let $f(x) = ax^2 + bx + c$ where $a \neq 0$. Find d , where $0 < d < 1$, such that $f(0) = 2014$, $f(d^2) = 2015$, $f(d) = 2016$, and the sum of the roots of f is 0.
4. Suppose a sequence $\{a_n\}$ of real numbers follows the rule $a_n = p(n)$, where p is a polynomial with real coefficients of degree less than or equal to 6. If $\{a_1, a_2, \dots, a_8\} = \{-2, -93, -458, -899, 366, 8623, 35302, 101337\}$, what is a_9 ?
5. Lynnelle and Moor are playing a game of Set. In Set, there are 27 red cards, 27 purple cards, and 27 green cards and at the end of the game, all the cards are divided between the two players. At the end of the game, the number of red cards Lynnelle has is the same as the number of green cards Moor has. We also know that Lynnelle has 17 more cards than Moor at the end of the game. How many purple cards does Lynnelle have?

6. Compute

$$\sum_{m=1}^{2016} \sum_{k=m-2016}^{m-2} \frac{1}{m^2 + k^2 - 2mk - m + k}$$

7. Find the real roots of $x^4 + 4x^3 + 6x^2 + 4x - 15$.
8. The polynomial $f(x) = x^3 - 4\sqrt{3}x^2 + 13x - 2\sqrt{3}$ has three real roots, a, b , and c . Find

$$\max\{a + b - c, a - b + c, -a + b + c\}.$$

9. Consider the path formed by an infinite number of line segments L_1, L_2, \dots of length l_1, l_2, \dots , where $l_1 = 1$; L_1 starts at the origin and goes in the positive y direction; and for $i \geq 2$, L_i 's start point is L_{i-1} 's end point, L_i is rotated 60° counterclockwise from L_{i-1} , and $l_i = k \cdot l_{i-1}$ for some constant $0 < k < 1$. The path looks like a hexagonal spiral that converges to a point. What are the coordinates for that point?
10. Let X_1, X_2, X_3, \dots be a sequence of strings of 0s and 1s derived in the following manner: $X_1 = \text{"1"}$, and X_{n+1} is formed by replacing every "0" in X_n with a "1", and replacing every "1" in X_n with "11000". Thus $X_1 = \text{"1"}$, $X_2 = \text{"11000"}$, $X_3 = \text{"1100011000111"}$, and so on. How many times does "01" occur in X_{2016} ?