Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. List all triples of positive integers $\{p_1, p_2, p_3\}$ where p_1, p_2, and p_3 are all prime, $p_1 < p_2 < p_3$, and p_1, p_2 differ by 2, p_2, p_3 differ by 2. For example $\{6, 8, 10\}$ is a triple that satisfies the last two properties, but not the first one, so it is not included in the answer.

2. An ant is walking on the edges of an icosahedron of side length 1. Compute the length of the longest path that he can take if he never crosses the same edge twice, but is allowed to revisit vertices.

3. Compute the number of trailing zeros of 2016!.

4. A positive integer $n > 1$ is called *multiplicatively perfect* if the product of its proper divisors (divisors excluding the number itself) is n. For example, 6 is multiplicatively perfect since $6 = 1 \times 2 \times 3$. Compute the number of multiplicatively perfect integers less than 100.

5. Let $d(n)$ be the number of positive integer divisors of a positive integer n. For example, $d(6) = 4$, because the divisors of 6 are 1, 2, 3, and 6. Compute

$$\sum_{n=1}^{\infty} \frac{d(n)}{n^2},$$

given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

6. Suppose $n > 0$ is an integer which, when written in base 10, has all digits either 0 or 1. If 17 evenly divides n, find the smallest possible value of n.

7. Lennart and Eddy are playing a betting game. Lennart starts with 7 dollars and Eddy starts with 3 dollars. Each round, both Lennart and Eddy bet an amount equal to the amount of the player with the least money. For example, on the first round, both players will bet 3 dollars. A fair coin is then tossed. If it lands heads, Lennart wins all the money bet; if it lands tails, Eddy wins all the money bet. They continue playing this game until someone has no money. What is the probability that Eddy ends with 10 dollars?

8. Consider a 2011×2012 grid of points from $(1, 1)$ to $(2011, 2012)$ with the point $(1066, 1453)$ removed. Starting at $(1, 1)$ and only moving up or to the right at each step, compute the number of different ways you can get to $(2011, 2012)$. You may express your answer using multiple binomial coefficients.

9. Let $X_1, X_2, X_3, ..$ be a sequence of strings of 0s and 1s derived in the following manner: $X_1 =$ "1", and X_{n+1} is formed by replacing every "0" in X_n with a "1", and replacing every "1" in X_n with "11000". Thus $X_1 =$ "1", $X_2 =$ "11000", $X_3 =$ "1100011000111", and so on. How many times does "01" occur in X_{2016}?

10. A continuous real-valued function f on the positive real numbers has the property that for all positive x and y, $f(xy) = xf(y) + yf(x)$. Determine all such functions f.