Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. Clyde is making a Pacman sticker to put on his laptop. A Pacman sticker is a circular sticker of radius 3 inches with a sector of 120° cut out. What is the perimeter of the Pacman sticker in inches?

2. In a certain right triangle, dropping an altitude to the hypotenuse divides the hypotenuse into two segments of length 2 and 3 respectively. What is the area of the triangle?

3. Consider a triangular pyramid $ABCD$ with equilateral base ABC of side length 1. $AD = BD = CD$ and $\angle ADB = \angle BDC = \angle ADC = 90^\circ$. Find the volume of $ABCD$.

4. Two circles with centers A and B respectively intersect at two points C and D. Given that A, B, C, D lie on a circle of radius 3 and circle A has radius 2, what is the radius of circle B?

5. Consider two concentric circles of radius 1 and 2. Up to rotation, there are two distinct equilateral triangles with two vertices on the circle of radius 2 and the remaining vertex on the circle of radius 1. The larger of these triangles has sides of length a, and the smaller has sides of length b. Compute $a + b$.

6. In a triangle ABC, let D and E trisect BC, so $BD = DE = EC$. Let F be the point on AB such that $\dfrac{AF}{FB} = 2$, and G on AC such that $\dfrac{AG}{GC} = \dfrac{1}{2}$. Let P be the intersection of DG and EF, and extend AP to intersect BC at a point X. Find $\dfrac{BX}{XC}$.

7. A unit sphere is centered at $(0, 0, 1)$. There is a point light source located at $(1, 0, 4)$ that sends out light uniformly in every direction but is blocked by the sphere. What is the area of the sphere's shadow on the x-y plane? (A point (a, b, c) denotes the point in three dimensions with x-coordinate a, y-coordinate b, and z-coordinate c).

8. Consider the parallelogram $ABCD$ such that $CD = 8$ and $BC = 14$. The diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E and $AC = 16$. Consider a point F on the segment $\overline{ED}$ with $FD = \frac{\sqrt{66}}{3}$. Compute CF.

9. Triangle ABC is isoceles with $AB = AC = 2$ and $BC = 1$. Point D lies on AB such that the inradius of ADC equals the inradius of BDC. What is the inradius of ADC?

10. For a positive real number k and an even integer $n \geq 4$, the *k-Perfect* n-gon is defined to be the equiangular n-gon $P_1P_2 \ldots P_n$ with $P_iP_{i+1} = P_{n/2+i}P_{n/2+i+1} = k^{i-1}$ for all $i \in \{1, 2, \ldots, n/2\}$, assuming the convention $P_{n+1} = P_1$ (i.e. the numbering wraps around). If $a(k, n)$ denotes the area of the *k-Perfect* n-gon, compute $\frac{a(2,24)}{a(4,12)}$.