

**Time limit:** 50 minutes.

**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.

**No calculators.**

1. Let  $f(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ . Compute  $f'(2)$ .
2. There are 5 contestants in the Rice Marathon Tournament, numbered 1 to 5. After  $t$  hours, contestant  $n$  has run  $\frac{t^n}{2}$  miles. Compute the average speed of the 5 contestants at time  $t = 2$ , in miles per hour.
3. Moor is trying to paint the interval  $[0, 5]$  using red and green paints. Painting at the point  $x$  using red paint costs  $2x$  dollars per unit length and using green paint costs  $x^2$  dollars per unit length. What is the minimum amount of money Moor needs to spend to paint the entire interval if he's allowed to change colors as he paints?
4. Let  $y(u)$  be the largest of the roots of  $x^2 + ux - 7 = 0$ . If  $u$  is increasing by 2 per second, what is the rate of change of  $y(u)$  when  $y(u) = 4$ ?
5. Given that  $\alpha$  and  $\beta$  are positive real numbers, compute the following limit (where it exists and is nonzero) in terms of  $\alpha$  and  $\beta$ :

$$\lim_{x \rightarrow 0^+} \frac{\sin x^\alpha}{\cos x^\beta - 1}.$$

6. Compute

$$\frac{d}{dx} \prod_{n=1}^{16} \left( x + \frac{1}{n} \right) \Big|_{x=0}.$$

7. Compute

$$\int_0^2 \sqrt{(2-x)(\sqrt{x} + \sqrt{x+2})^2} dx.$$

8. Compute

$$2014 \int_0^\infty \frac{(1+x)^{2013}}{(2+x)^{2015}} dx.$$

9. Given that it converges, compute the following infinite product:

$$\prod_{n=1}^{\infty} \frac{5^{2^{-n}} + 3^{2^{-n}}}{2}.$$

10. Consider the real-valued differential equation  $u''(x) = u^2(x) - u^5(x)$ . Suppose that  $u'(0) = 7$  and  $u(0) = 2$ . Compute the maximum possible value of  $|u'(x)|$ .