**Time limit:** 50 minutes.
**Instructions:** This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
**No calculators.**

1. In a Super Smash Brothers tournament, $\frac{1}{2}$ of the contestants play as Fox, $\frac{1}{3}$ of the contestants play as Falco, and $\frac{1}{6}$ of the contestants play as Peach. Given that there were 40 more people who played either Fox or Falco than who played Peach, how many contestants attended the tournament?

2. Find all pairs $(x, y)$ that satisfy
$$x^2 + y^2 = 1$$
$$x + 2y = 2$$

3. Find the unique $x > 0$ such that $\sqrt{x} + \sqrt{x + \sqrt{x}} = 1$.

4. Find the sum of all real roots of $x^5 + 4x^4 + x^3 - x^2 - 4x - 1$.

5. Let $f(a, b) = \frac{1}{a+b}$ when $a + b \neq 0$. Suppose that $x, y, z$ are distinct integers such that $x + y + z = 2015$ and $f(f(x, y), z) = f(x, f(y, z))$ (where both sides of the equation exist and are well-defined). Compute $y$.

6. Compute all pairs of real numbers $(a, b)$ such that the polynomial $f(x) = (x^2 + ax + b)^2 + a(x^2 + ax + b) - b$ has exactly one real root and no complex roots.

7. Let $a, b, c, d$ be real numbers that satisfy
$$ab + cd = 11$$
$$ac + bd = 13$$
$$ad + bc = 17$$
$$abcd = 30$$
Find the greatest possible value of $a$.

8. The polynomial $x^7 + x^6 + x^4 + x^3 + x + 1$ has roots $r_1, r_2, r_3, r_4, r_5, r_6, r_7$. Calculate
$$\sum_{n=1}^{7} r_n^3 + \frac{1}{r_n^3}$$

9. Given that real numbers $x, y$ satisfy the equation $x^4 + x^2y^2 + y^4 = 72$, what is the minimum possible value of $2x^2 + xy + 2y^2$?

10. Consider a sequence defined recursively by $a_n = 1 + (a_0 + 1)(a_1 + 1) \cdots (a_{n-1} + 1)$. Let $-2 < a_0 < -1$ such that
$$\sum_{n=0}^{2015} \frac{a_n}{a_n^2 - 1} = -\frac{a_0 + 4}{a_0^2 - 1}.$$
What is the value of $a_0$?