Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written inside the boxes on the answer sheet will be considered for grading.
No calculators.

1. How many nonnegative integers less than 1000 have the property that the sum of their digits is a multiple of 3?

2. Compute the number of ways 6 girls and 5 boys can line up if all 11 people are distinguishable and no two girls stand next to each other.

3. A certain high school has exactly 1000 lockers, numbered from 1 to 1000, all initially closed. Mark first opens every locker whose number has exactly 3 factors, starting with locker 4. Matt then opens every locker whose number is a power of 2, starting with locker 1. If Matt encounters a locker that Mark has already opened, he leaves it open. Compute the number of lockers that will be open when both Mark and Matt finish.

4. Andy has two identical cups, the first one is full of water and the second one is empty. He pours half the water from the first cup into the second, then a third of the water in the second into the first, then a fourth of the water from the first into the second and so on. Compute the fraction of the water in the first cup right before the 2015th transfer.

5. Alexander and Eliza are betting on a roll of a standard pair of 6-sided dice. Alexander bets that a sum of 12 will occur first and Eliza bets that two consecutive sums of 7 will appear first. They continue to roll the dice until one player wins. What is the probability that Alexander wins?

6. There are four seats arranged in a circle and a person is sitting on one of the seats. He rolls a standard six-sided die 6 times. For each roll of the die, if it lands on 4, he moves one seat clockwise. Otherwise, he moves k seats counterclockwise where k is the number he rolled. Compute the probability that he ends up on the same seat he originally started on.

7. For a positive integer n, let $f(n)$ denote the number of ones in the base 2 representation of n. For example, $f(13) = 3$ because $13 = 1101_2$. Compute the number of positive integers n that satisfy $n \leq 2015$ and $f(n) \equiv f(n+1) \pmod{4}$.

8. Find all ordered triples of positive integers (x, y, z) such that $x^2 + xz = y^2$ and $x + y + z = 40$.

9. A sequence is formed of n 1s and m 0s in random order. A run is defined to be a consecutive string of 1s or 0s. What is the average number of runs?

10. One 1×1 square tile and 115 1×5 tiles cover an entire 24×24 grid. How many positions can the square tile occupy?