Time limit: 50 minutes.

Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.

No calculators.

1. Let $f(x) = x^4$ and let $g(x) = x^{-4}$. Compute $f''(2)g''(2)$.

2. There is a unique positive real number a such that the tangent line to $y = x^2 + 1$ at $x = a$ goes through the origin. Compute a.

3. Moor has \$1000, and he is playing a gambling game. He gets to pick a number k between 0 and 1 (inclusive). A fair coin is then flipped. If the coin comes up heads, Moor is given $5000k$ additional dollars. Otherwise, Moor loses $1000k$ dollars. Moor's happiness is equal to the log of the amount of money that he has after this gambling game. Find the value of k that Moor should select to maximize his expected happiness.

4. The set of points (x, y) in the plane satisfying $x^{2/5} + |y| = 1$ form a curve enclosing a region. Compute the area of this region.

5. Compute the improper integral
$$\int_0^2 \left(\sqrt{\frac{4-x}{x}} - \sqrt{\frac{x}{4-x}} \right) dx.$$

6. Compute
$$\lim_{x \to \infty} \left[x - x^2 \ln\left(\frac{1+x}{x}\right) \right].$$

7. For a given $x > 0$, let a_n be the sequence defined by $a_1 = x$ for $n = 1$ and $a_n = x^{a_{n-1}}$ for $n \geq 2$. Find the largest x for which the limit $\lim_{n \to \infty} a_n$ converges.

8. Evaluate
$$\int_{-2}^2 \frac{1+x^2}{1+2^x} dx.$$

9. Let f satisfy $x = f(x)e^{f(x)}$. Calculate $\int_0^e f(x)\, dx$.

10. Given that $\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$, compute the sum
$$\sum_{n=1}^{\infty} \frac{1}{2^n n^2}.$$