Time limit: 50 minutes.
Instructions: This test contains 10 short answer questions. All answers must be expressed in simplest form unless specified otherwise. Only answers written on the answer sheet will be considered for grading.
No calculators.

1. Alice and Bob are painting a house. If Alice and Bob do not take any breaks, they will finish painting the house in 20 hours. If, however, Bob stops painting once the house is half-finished, then the house takes 30 hours to finish. Given that Alice and Bob paint at a constant rate, compute how many hours it will take for Bob to paint the entire house if he does it by himself.

2. Compute $9^6 + 6 \cdot 9^5 + 15 \cdot 9^4 + 20 \cdot 9^3 + 15 \cdot 9^2 + 6 \cdot 9$.

3. Let x_1 and x_2 be the roots of $x^2 - x - 2014$, with $x_1 < x_2$. Let x_3 and x_4 be the roots of $x^2 - 2x - 2014$, with $x_3 < x_4$. Compute $(x_4 - x_2) + (x_3 - x_1)$.

4. For any 4-tuple (a_1, a_2, a_3, a_4) where each entry is either 0 or 1, call it *quadratically satisfiable* if there exist real numbers $x_1, \ldots, x_4$ such that $x_1 x_4^2 + x_2 x_4 + x_3 = 0$ and for each $i = 1, \ldots, 4$, x_i is positive if $a_i = 1$ and negative if $a_i = 0$. Find the number of *quadratically satisfiable* 4-tuples.

5. a and b are nonnegative real numbers such that $\sin(ax + b) = \sin(29x)$ for all integers x. Find the smallest possible value of a.

6. Find the minimum value of
$$\frac{1}{x-y} + \frac{1}{y-z} + \frac{1}{x-z}$$
for reals $x > y > z$ given $(x-y)(y-z)(x-z) = 17$.

7. Compute the smallest value p such that, for all $q > p$, the polynomial $x^3 + x^2 + qx + 9$ has exactly one real root.

8. $P(x)$ and $Q(x)$ are two polynomials such that
$$P(P(x)) = P(x)^{16} + x^{48} + Q(x).$$
Find the smallest possible degree of Q.

9. Let b_n be defined by the formula
$$b_n = \sqrt[3]{-1 + a_1 \sqrt[3]{-1 + a_2 \sqrt[3]{-1 + \ldots a_{n-1} \sqrt[3]{-1 + a_n}}}}$$
where $a_n = n^2 + 3n + 3$. Find the smallest real number L such that $b_n < L$ for all n.

10. Let $x_0 = 1, x_1 = 0$, and $x_i = -3x_{i-1} + x_{i-2}$ for $i \geq 2$. Let $y_0 = 0, y_1 = 1$, and $y_i = -3y_{i-1} + y_{i-2}$ for $i \geq 2$. Compute
$$\sum_{i=0}^{2013} \frac{(x_i y_{2014} - y_i x_{2014})^2}{y_{2014}^2}.$$
You may give your answer in terms of at most ten values of the x_i and/or y_i (but must otherwise simplify completely).