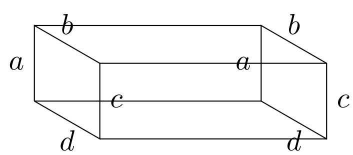
1. Let $f_1(n)$ be the number of divisors that n has, and define $f_k(n) = f_1(f_{k-1}(n))$. Compute the smallest integer k such that $f_k(2013^{2013}) = 2$.

2. In unit square $ABCD$, diagonals $\overline{AC}$ and $\overline{BD}$ intersect at E. Let M be the midpoint of $\overline{CD}$, with $\overline{AM}$ intersecting $\overline{BD}$ at F and $\overline{BM}$ intersecting $\overline{AC}$ at G. Find the area of quadrilateral $MFEG$.

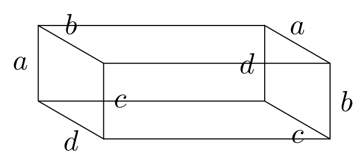
3. Nine people are practicing the triangle dance, which is a dance that requires a group of three people. During each round of practice, the nine people split off into three groups of three people each, and each group practices independently. Two rounds of practice are different if there exists some person who does not dance with the same pair in both rounds. How many different rounds of practice can take place?

4. For some positive integers a and b, $(x^a + abx^{a-1} + 13)^b(x^3 + 3bx^2 + 37)^a = x^{42} + 126x^{41} + \cdots$. Find the ordered pair (a, b).

5. A polygonal prism is made from a flexible material such that the two bases are regular 2^n-gons ($n > 1$) of the same size. The prism is bent to join the two bases together without twisting, giving a figure with 2^n faces. The prism is then repeatedly twisted so that each edge of one base becomes aligned with each edge of the other base exactly once. For example, when $n = 2$, the untwisted and one-twist cases are shown below; in both diagrams, each edge of one base is to be aligned with the edge of the other base with the same label.



$n = 2$, untwisted



$n = 2$, one twist

For an arbitrary n, what is the sum of the number of faces over all of these configurations (including the non-twisted case)?

6. How many distinct sets of 5 distinct positive integers A satisfy the property that for any positive integer $x \leq 29$, a subset of A sums to x?

7. Find all real values of u such that the curves $y = x^2 + u$ and $y = \sqrt{x - u}$ intersect in exactly one point.

8. Rational Man and Irrational Man both buy new cars, and they decide to drive around two racetracks from time $t = 0$ to time $t = \infty$. Rational Man drives along the path parametrized by

$$x = \cos(t)$$
$$y = \sin(t)$$

and Irrational Man drives along the path parametrized by

$$x = 1 + 4\cos\frac{t}{\sqrt{2}}$$
$$y = 2\sin\frac{t}{\sqrt{2}}.$$

Find the largest real number d such that at any time t, the distance between Rational Man and Irrational Man is not less than d.

9. Charles is playing a variant of Sudoku. To each lattice point (x, y) where $1 \leq x, y < 100$, he assigns an integer between 1 and 100, inclusive. These integers satisfy the property that in any row where $y = k$, the 99 values are distinct and are never equal to k; similarly for any column where $x = k$. Now, Charles randomly selects one of his lattice points with probability proportional to the integer value he assigned to it. Compute the expected value of $x + y$ for the chosen point (x, y).

10. A unit circle is centered at the origin and a tangent line to the circle is constructed in the first quadrant such that it makes an angle $5\pi/6$ with the $y-$axis. A series of circles centered on the $x-$axis are constructed such that each circle is both tangent to the previous circle and the original tangent line. Find the total area of the series of circles.

11. What is the smallest positive integer with exactly 768 divisors? Your answer may be written in its prime factorization.

12. Suppose Robin and Eddy walk along a circular path with radius r in the same direction. Robin makes a revolution around the circular path every 3 minutes and Eddy makes a revolution every minute. Jack stands still at a distance $R > r$ from the center of the circular path. At time $t = 0$, Robin and Eddy are at the same point on the path, and Jack, Robin, Eddy, and the center of the path are collinear. When is the next time the three people (but not necessarily the center of the path) are collinear?

13. A board has 2, 4, and 6 written on it. A person repeatedly selects (not necessarily distinct) values for x, y, and z from the board, and writes down $xyz + xy + yz + zx + x + y + z$ if and only if that number is not yet on the board and is also less than or equal to 2013. This person repeats this process until no more numbers can be written. How many numbers will be written at the end of this process?

14. You have a 2 meter long string. You choose a point along the string uniformly at random and make a cut. You discard the shorter section. If you still have 0.5 meters or more of string, you repeat. You stop once you have less than 0.5 meters of string. On average, how many cuts will you make before stopping?

15. Suppose we climb a mountain that is a cone with radius 100 and height 4. We start at the bottom of the mountain (on the perimeter of the base of the cone), and our destination is the opposite side of the mountain, halfway up (height $z = 2$). Our climbing speed starts at $v_0 = 2$ but gets slower at a rate inversely proportional to the distance to the mountain top (so at height z the speed v is $(h - z)v_0/h$). Find the minimum time needed to get to the destination.