1. **Answer: 16**

   Solution: $1 - \frac{2}{3} - \frac{1}{8} = \frac{5}{24}$ of the birds that Robin sees are pigeons. Therefore, Robin sees exactly $\frac{5}{\frac{5}{24}} = 24$ birds, so $\frac{2}{3} \cdot 24 = \boxed{16}$ robins.

2. **Answer: 80**

   Solution: The limiting factor is the cheese. With only 20 lbs of cheese, the most pizzas that can be made is $20 \cdot 4 = \boxed{80}$.

3. **Answer: 56**

   Solution: There are a total of 8 queens and jacks, each of which is distinguishable from the others. Thus, the number of hands that Queen Jack likes is $\binom{8}{5} = \boxed{56}$.

4. **Answer: 9030**

   Solution: The first four primes are 2, 3, 5, and 7, so the number must be a multiple of $2 \cdot 3 \cdot 5 \cdot 7 = 210$. The least multiple of 210 that is greater than 9000 is $210 \cdot 43 = \boxed{9030}$.

5. **Answer: $\mathbf{12\sqrt{5}}$**

   Solution: Let d be the length of the shorter diagonal and $2d$ the length of the longer diagonal. If d_1, d_2 are the lengths of the diagonals of a rhombus, its area will be $\frac{d_1 \cdot d_2}{2}$. Thus, we have $d^2 = 36$ and $d = 6$. Therefore, the side length of the rhombus is $\sqrt{3^2 + 6^2} = 3\sqrt{5}$, so the perimeter is $\boxed{12\sqrt{5}}$.

6. **Answer: 15**

   Solution: Let d be the length of one lap in miles. Then he needs to complete the four laps in $\frac{4d}{10} = \frac{2d}{5}$ hours. He has already spent $\frac{3d}{9} = \frac{d}{3}$ hours on the first three laps, so he has $\frac{2d}{5} - \frac{d}{3} = \frac{d}{15}$ hours left. Therefore, he must maintain a speed of $\boxed{15}$ mph on the final lap.

7. **Answer: $\mathbf{\sqrt{5} - \sqrt{3}}$**

   Solution: The fly's path is the space diagonal of the cube, or the hypotenuse of the right triangle with one leg as a face diagonal of the cube (length $\sqrt{2}$) and the other leg as an edge of the cube (length 1). Thus, it has a length of $\sqrt{2+1} = \sqrt{3}$. The ant's path crosses two faces of the cube to reach the opposite corner and is minimized as the diagonal of the rectangle formed by these two faces when flattened out. Thus, it is the hypotenuse of a right triangle with legs of length 2 and 1 and has length $\sqrt{4+1} = \sqrt{5}$. The difference in length between the ant's path and the fly's path is $\boxed{\sqrt{5} - \sqrt{3}}$.

8. **Answer: 2018**

   Solution: If there are n pairs of shoes, then the number of mismatched pairs is $n(n-1)$. In 2017, there are 16 pairs of shoes, so there are $16 \cdot 15 = 240 < 500$ possible mismatchings. In 2018, there are 32 pairs of shoes, so there are $32 \cdot 31 = 992 > 500$ possible mismatchings.

9. **Answer: 10/11**

   Solution: After removing x from 10, and then increasing that amount by 10%, we must end up with at least the amount we started with, 10 pounds. That is, the maximum value of x must satisfy $\frac{11}{10}(10 - x) = 10$. Solving for x, we get that $x = \boxed{10/11}$.

10. **Answer: 5**

Solution: In order to find the remainder mod 7, evaluate the sequence mod 7: $1+3+1 \equiv 5 \bmod 7$, $5+3\cdot 1+1 \equiv 2 \bmod 7$, and so on. The sequence repeats itself after 6 iterations, producing

$$1, 1, 1, 5, 2, 4, 1, 1, 1, \ldots$$

Since $2013 \equiv 3 \bmod 6$, then $a_{2013} \equiv a_3 \equiv \boxed{5} \bmod 7$.

11. **Answer: 5π**

Solution: The radius of the cone is $\frac{2}{2} = 1$, so the lateral height of the cone is $\sqrt{1^2 + (2\sqrt{2})^2} = 3$ and the lateral surface area of the cone is $\pi \cdot 1 \cdot 3 = 3\pi$. Next, the surface area of the hemisphere is $2\pi r^2 = 2\pi$. Thus, the total surface area is $2\pi + 3\pi = \boxed{5\pi}$.

12. **Answer: $\frac{25}{4}$**

Solution: For an equation to have real solutions, the discriminant must be nonnegative. Thus, we have $b^2 - 4ac = 5^2 - 4c \geq 0$ or $\boxed{\frac{25}{4}} \geq c$.

13. **Answer: 4**

Solution: If Red only places 3 points, then Blue can get in between Red's first 2 points and block the third point from winning. Therefore, the answer is no smaller than 4.

Now, we will describe a strategy that enables Red to win in 4 moves. First, Red places a point r_1 and then Blue places a point b_1. Then, Red places a point r_2 such that r_1, r_2, and b_1 are not collinear. Blue must now place a point b_2 between r_1 and r_2 in order to avoid losing immediately. Red must now place a point r_3 between b_1 and b_2 to avoid losing immediately. But now, $\overline{r_3r_1}$ and $\overline{r_3r_2}$ are both lines without any blue points between them. So, no matter which line Blue chooses to block, Red can immediately place a point r_3 on the other line such that there are 3 red points in in a row.

Therefore, Red needs $\boxed{4}$ moves to guarantee a win.

14. **Answer: $\frac{17\sqrt{5}}{6}$**

Solution: Rob runs a distance of $\sqrt{17^2 + 34^2} = 17\sqrt{5}$ units. Therefore, Rob runs for a total of $\frac{17\sqrt{5}}{2}$ seconds. Peter must therefore run a total of $\frac{17\sqrt{5}}{2} - t$ seconds, and we know that $3\left(\frac{17\sqrt{5}}{2} - t\right) = 17\sqrt{5}$. Solving for t, we get $t = \boxed{\frac{17\sqrt{5}}{6}}$.

15. **Answer: $\frac{3}{8}$**

Solution: If we partition the hexagon into six equilateral triangles by drawing AD, BE, and CF, we get 6 congruent equilateral triangles. If we then take each equilateral triangle and partition each one into four smaller equilateral triangles by means of connecting the midpoints of the sides, we note that PQR contains 9 of the small equilateral triangles while $ABCDEF$ contains 24 of the small equilateral triangles. The probability therefore follows as $\boxed{\frac{3}{8}}$.

16. **Answer: 2880**

    Solution: Imagine that there are six slots that people can fit into. Alice and Bob go into one slot, Claire and Derek go into another slot, and each of the remaining four people get a slot. There are $6! = 720$ ways for the six slots to be assigned, and then there are 2 ways for Alice and Bob to stand, and there are also 2 ways for Claire and Derek to stand, thereby giving $720 \times 2^2 = \boxed{2880}$ ways for all of them to pose for the picture.

17. **Answer:** $\mathbf{\frac{25\pi}{2} - 25}$

    Solution:

    We use the fact that the hypotenuse of any right triangle that is inscribed in a circle is actually a diameter of the circle.

    The area of the circle is 25π. The hypotenuse creates two semicircles of area $\frac{25\pi}{2}$ each. The legs divide one of these semicircles into three regions, including a right triangle with area $\frac{(5\sqrt{2})^2}{2} = 25$. The other two regions sum to $\frac{25\pi}{2} - 25$. Since $25 > \frac{25\pi}{2} - 25$, the sum of the areas of the two smallest regions is $\boxed{\frac{25\pi}{2} - 25}$.

18. **Answer: 40**

    Solution: If we plant apple trees, the first apple tree requires 1 square meter to grow. The second one requires $3 = 2^2 - 1^2$ square meters to grow, the third one requires $5 = 3^2 - 2^2$, and the fourth one requires $7 = 4^2 - 3^2$. If we plant apricot trees, each tree requires 5 square meters to grow. If we plant plum trees, the first plum tree requires 1 square meter whereas each subsequent one will require at least $7 = 2^3 - 1^3$ square meters. Thus, to take up the least amount of space, we should plant 3 apple trees, 6 apricot trees, and 1 plum tree for a total of $\boxed{40}$ square meters.

19. **Answer:** $\mathbf{\sqrt{13}}$

    Solution: Note that $\frac{2 \cdot 3}{2} = 3$, so therefore the triangle is a right triangle with legs 2 and 3. As a result, the third side length is, by the Pythagorean Theorem, $\sqrt{2^2 + 3^2} = \boxed{\sqrt{13}}$.

20. **Answer:** $\mathbf{\frac{11}{5}}$

    Solution: We can split the target into five concentric circles with radii 1, 2, 3, 4, and 5; the five corresponding regions have areas of π, 3π, 5π, 7π, 9π and are worth 5, 4, 3, 2, 1 points respectively. Thus the answer is $\frac{5 \cdot \pi + 4 \cdot 3\pi + 3 \cdot 5\pi + 2 \cdot 7\pi + 1 \cdot 9\pi}{25\pi} = \boxed{\frac{11}{5}}$.

21. **Answer: 273**

    Solution: The integers which are valid have a 1-1 correspondence to days in the first 9 months – this is straightforward to see for all positive integers that do not have a 1 in the hundreds place and just requires careful inspection of the case where 1 is in the hundreds place. There are $365 - 31 - 30 - 31 = \boxed{273}$ such days.

22. **Answer: 55**

Solution: There is clearly 1 minimally competent subset of size 1, which is just $\{1\}$. For size 2, any minimally competent subset must contain 2 and then one of 3 through 10 (not 1, because then the minimally competent subset of size 1 would be a proper subset), so there are 8 possibilities. For size k in general, we can see that a minimally competent subset of size k must contain k and then $k-1$ numbers, each larger than k. Thus, a minimally competent subset can contain at most 5 numbers.

The answer is then $\sum_{k=1}^{5}\binom{10-k}{k-1}$, which can be computed directly as $\boxed{55}$. We can also argue by induction that the number is equal to F_{10} (the 10th Fibonacci number), which may be easier to compute.

23. **Answer: −353**

Solution: Note that $a^4+b^4 = (a^2+b^2)^2-2(ab)^2 = ((a+b)^2-2ab)^2-2(ab)^2 = (7^2-34)^2-2(17)^2 = 15^2-2(17)^2 = \boxed{-353}$.

24. **Answer: $\mathbf{2-\sqrt{3}}$**

Solution: Orient the triangle such that the right angle of the triangle is at the origin, and such that the two legs point in the directions of the positive x- and y-axes. Note that the incenter is at (r,r), where r is the inradius of the circle, but since the area is $A = rs$, we have that $r = \frac{A}{s} = \frac{\frac{\sqrt{3}}{2}}{\frac{3+\sqrt{3}}{2}} = \frac{\sqrt{3}}{3+\sqrt{3}}$. The circumcenter is at either $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ or $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$.

The square of the distance between the incenter and the circumcenter is therefore $\boxed{2-\sqrt{3}}$.

25. **Answer: 13**

Solution: First, note that we can fill the first 3 columns with any permutation of the numbers 1 through 9. Thus, there are 9! ways to do this. Next, we must consider how many ways there are to place numbers in the remaining three columns. This problem can be broken into two parts: splitting the numbers into each of 3 rows, and permuting the numbers in each row. For the first part, either the rows switch positions without their contents mixing (2 ways) or each new row has one number from one row and two numbers from another row. In this second option, there are $3^3 = 27$ ways to split each original row into a single and a pair, and 2 ways to arrange these singles and pairs. So we have a total of $2 + 27 \cdot 2 = 56$ ways. For the second part, we note that each set of 3 elements in a row can be permuted in 3! ways, giving a total of $56 \cdot (3!)^3$ ways to fill the last 3 columns, given a particular permutation for the first three columns. Thus, $N = 9! \cdot 56 \cdot 6^3$, and therefore N is divisible by 2^k for $k \le 7+3+3 = \boxed{13}$.