

1. Compute $\lim_{x \rightarrow 3} \frac{x^2 + 2x - 15}{x^2 - 4x + 3}$.
2. Compute all real values of b such that, for $f(x) = x^2 + bx - 17$, $f(4) = f'(4)$.
3. Suppose a and b are real numbers such that

$$\lim_{x \rightarrow 0} \frac{\sin^2 x}{e^{ax} - bx - 1} = \frac{1}{2}.$$

Determine all possible ordered pairs (a, b) .

4. Evaluate $\int_0^4 e^{\sqrt{x}} dx$.
5. Evaluate $\lim_{x \rightarrow 0} \frac{\sin^2(5x) \tan^3(4x)}{(\log(2x + 1))^5}$.
6. Compute $\sum_{k=0}^{\infty} \int_0^{\frac{\pi}{3}} \sin^{2k} x dx$.
7. The function $f(x)$ has the property that, for some real positive constant C , the expression

$$\frac{f^{(n)}(x)}{n + x + C}$$

is independent of n for all nonnegative integers n , provided that $n + x + C \neq 0$. Given that $f'(0) = 1$ and $\int_0^1 f(x) dx = C + (e - 2)$, determine the value of C .

Note: $f^{(n)}(x)$ is the n -th derivative of $f(x)$, and $f^{(0)}(x)$ is defined to be $f(x)$.

8. The function $f(x)$ is defined for all $x \geq 0$ and is always nonnegative. It has the additional property that if any line is drawn from the origin with any positive slope m , it intersects the graph $y = f(x)$ at precisely one point, which is $\frac{1}{\sqrt{m}}$ units from the origin. Suppose further that f has a unique maximum value at some real number $x = a$. Find $\int_0^a f(x) dx$.
9. Evaluate $\int_0^{\pi/2} \frac{dx}{(\sqrt{\sin x} + \sqrt{\cos x})^4}$.
10. Evaluate $\lim_{n \rightarrow \infty} \left[\left(\prod_{k=1}^n \frac{2k}{2k-1} \right) \int_{-1}^{\infty} \frac{(\cos x)^{2n}}{2^x} dx \right]$.