1. **Answer: 15**

   Solution: Let d be the length of one lap in miles. Then he needs to complete the four laps in $\frac{4d}{10} = \frac{2d}{5}$ hours. He has already spent $\frac{3d}{9} = \frac{d}{3}$ hours on the first three laps, so he has $\frac{2d}{5} - \frac{d}{3} = \frac{d}{15}$ hours left. Therefore, he must maintain a speed of $\boxed{15}$ mph on the final lap.

2. **Answer:** $\mathbf{\frac{10}{11}}$

   Solution: After removing x from 10, and then increasing that amount by 10%, we must end up with at least the amount we started with, 10 pounds. That is, the maximum value of x must satisfy $\frac{11}{10}(10-x) = 10$. Solving for x, we get that $x = \boxed{\frac{10}{11}}$.

3. **Answer: 8960**

   Solution: All of Karl's favorite quadratics take the form $(x-r)(x-17)$, where $0 \leq r \leq 34$. The sum of the coefficients of any polynomial can be determined by evaluating the polynomial at $x = 1$. This gives $16r - 16$. $\sum_{r=0}^{34}(16r-16) = 16 \cdot \frac{34 \cdot 35}{2} - 16 \cdot 35 = \boxed{8960}$.

4. **Answer:** $\mathbf{\frac{68}{3}}$

   Solution: Substituting $x = 2$, we get that $f(2) + 2f(6) = 4$. Substituting $x = 6$, we get that $f(6) + 2f(2) = 36$. Solving for $f(2)$ and $f(6)$ gives us that $f(6) = -\frac{28}{3}$ and $f(2) = \boxed{\frac{68}{3}}$.

5. **Answer: 168**

   Solution: We have that b is a valid number if and only if $(x^2+2x+3)-(bx-17) = x^2+(2-b)x+20$ has exactly one real root. This means that $2 - b = \pm 2\sqrt{20}$, so $b = 2 \pm 2\sqrt{20}$. $b_1^2 + b_2^2$ is therefore $2(2^2) + 2(2\sqrt{20})^2 = 8 + 160 = \boxed{168}$.

6. **Answer:** $\mathbf{\frac{1+\sqrt{13}}{2}}$

   Solution: Note that $x^4-x^3-5x^2+2x+6 = (x^4-5x^2+6)-x(x^2-2) = (x^2-2)(x^2-3)-x(x^2-2) = (x^2-2)(x^2-x-3)$. The two largest candidate roots are therefore $\sqrt{2}$ and $\frac{1+\sqrt{13}}{2}$. Note that $\sqrt{13} > 3$, so $\frac{1+\sqrt{13}}{2} > 2 > \sqrt{2}$, so therefore the largest root is $\boxed{\frac{1+\sqrt{13}}{2}}$.

7. **Answer:** $\mathbf{\frac{546}{5}}$

   Solution: Observe that $f(a) = \sqrt[3]{20x+a}$ is an increasing function in a, so the only way that $f(f(a)) = a$ can be true is if $f(a) = a$. Solving $\sqrt[3]{20x+13} = 13$, we obtain $x = \boxed{\frac{546}{5}}$.

8. **Answer: −11/3**

   Solution: Let $f(x) = 4x^2 + 15x + 17$, $g(x) = x^2 + 4x + 12$, and $h(x) = x^2 + x + 1$. Then, the given equation becomes

$$\begin{aligned}\frac{f(x)}{g(x)} &= \frac{f(x)+h(x)}{g(x)+h(x)}\\ \implies & f(x)g(x)+f(x)h(x) = f(x)g(x)+g(x)h(x)\\ \implies & f(x)h(x) = g(x)h(x).\end{aligned}$$

Since $h(x) > 0$ for all real x, we may divide through by $h(x)$ to get

$$\begin{aligned}f(x) &= g(x)\\ &\implies 4x^2+15x+17 = x^2+4x+12\\ &\implies 3x^2+11x+5=0.\end{aligned}$$

The discriminant of this quadratic is

$$11^2 - 4\cdot 3\cdot 5 = 61 > 0,$$

so it has two real roots. By Vieta's, the sum of these roots is $\boxed{-11/3}$.

9. **Answer: 30**

Solution: Putting everything over a common denominator, we can rewrite the expression as

$$\frac{a^4(b-c)-b^4(a-c)+c^4(a-b)}{(a-b)(a-c)(b-c)} = \frac{a^4b-ab^4-a^4c+ac^4+b^4c-bc^4}{(a-b)(a-c)(b-c)}.$$

Notice that if $a=b$, the numerator becomes $a^5-a^5-a^4c+ac^4+a^4c-ac^4=0$; similarly if $a=c$ or $b=c$. This means that the numerator is in fact divisible by $(a-b)(a-c)(b-c)$. Factoring, we find that the above expression is equal to

$$\frac{(a-b)(b-c)(a-c)(a^2+b^2+c^2+ab+bc+ac)}{(a-b)(b-c)(a-c)} = a^2+b^2+c^2+ab+bc+ac$$

as long as the original expression was well-defined. But we have

$$a^2+b^2+c^2+ab+bc+ac = \frac{1}{2}\left((a+b)^2+(b+c)^2+(c+a)^2\right)$$

and plugging in the given values of a,b,c gives

$$\frac{1}{2}\left((2\sqrt{7})^2+(2\sqrt{3})^2+(2\sqrt{5})^2\right) = 2(7+3+5) = \boxed{30}.$$

10. **Answer: 6, $\frac{-1\pm\sqrt{13}}{2}$**

Solution: First of all, if $z=1$, then the expression is simply equal to $\boxed{6}$. Otherwise, let $\omega = z+z^3+z^4+z^9+z^{10}+z^{12}$. We find that

$$\omega^2 = z^2+z^6+z^8+z^5+z^7+z^{11}+2(z^4+z^5+z^{10}+z^{11}+1+z^7+z^{12}+1+z^2+1+z+z^3+z^6+z^8+z^9).$$

Applying the identity $z+z^2+z^3+\cdots+z^{12}=-1$, we arrive at $\omega^2 = -1-\omega+2(3-1) = 3-\omega$, and the solutions to the quadratic are $\omega = \boxed{\dfrac{-1\pm\sqrt{13}}{2}}$.