1. Nick is a runner, and his goal is to complete four laps around a circuit at an average speed of 10 mph. If he completes the first three laps at a constant speed of only 9 mph, what speed does he need to maintain in miles per hour on the fourth lap to achieve his goal?

2. A tree has 10 pounds of apples at dawn. Every afternoon, a bird comes and eats x pounds of apples. Overnight, the amount of food on the tree increases by 10%. What is the maximum value of x such that the bird can sustain itself indefinitely on the tree without the tree running out of food?

3. Karl likes the number 17. His favorite polynomials are monic quadratics with integer coefficients such that 17 is a root of the quadratic and the roots differ by no more than 17. Compute the sum of the coefficients of all of Karl's favorite polynomials. (A monic quadratic is a quadratic polynomial whose x^2 term has a coefficient of 1.)

4. Given that $f(x) + 2f(8 - x) = x^2$ for all real x, compute $f(2)$.

5. For exactly two real values of b, b_1 and b_2, the line $y = bx - 17$ intersects the parabola $y = x^2 + 2x + 3$ at exactly one point. Compute $b_1^2 + b_2^2$.

6. Compute the largest root of $x^4 - x^3 - 5x^2 + 2x + 6$.

7. Find all real x that satisfy $\sqrt[3]{20x + \sqrt[3]{20x + 13}} = 13$.

8. Find the sum of all real x such that
$$\frac{4x^2 + 15x + 17}{x^2 + 4x + 12} = \frac{5x^2 + 16x + 18}{2x^2 + 5x + 13}.$$

9. Let $a = -\sqrt{3} + \sqrt{5} + \sqrt{7}$, $b = \sqrt{3} - \sqrt{5} + \sqrt{7}$, $c = \sqrt{3} + \sqrt{5} - \sqrt{7}$. Evaluate
$$\frac{a^4}{(a-b)(a-c)} + \frac{b^4}{(b-c)(b-a)} + \frac{c^4}{(c-a)(c-b)}.$$

10. Given a complex number z such that $z^{13} = 1$, find all possible values of $z+z^3+z^4+z^9+z^{10}+z^{12}$.